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# FINITE-ELEMENT NONLINEAR TRANSIENT RESPONSE COMPUTER PROGRAMS PLATE 1 AND CIVM-PLATE 1 FOR THE ANALYSIS OF PANELS SUBJECTED TO IMPULSE OR IMPACT LOADS

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16 Abstract


Described in this report are two computer programs for predicting the transient large-deflection elastic-viscoplastic responses of thin single-layer initially-flat unstiffened or integrally-stiffened Kirchhoff-Love ductile-metal panels. The PLATE 1 program pertains to structural responses produced by prescribed externally-applied transient loading or prescribed initial-velocity distributions. The CIVM-PLATE 1 program concerns structural responses produced by impact of an idealized nondeformable fragment. Finite elements are used to represent the structure in both programs. Strain-hardening and strain-rate effects of initially-isotropic material are taken into account.

An appropriate analysis involving impulse-momentum relations is used to predict the after-impact velocity components of the fragment and of the impact-affected region of the panel from each impact; the procedure is termed the collision-imparted velocity method (CIVM). Local fragment-panel impact may be treated as perfectly-elastic, perfectly-plastic, or intermediate between these two extremes. Also, the effects of friction between the fragment and the impacted panel are taken into accout. The imparted-velocity information is used in conjunction with a finite-element structural response code to predict the resulting transient structural response in small increments $\Delta t$ in time. Similarly, the equations of motion for the fragment are solved in small steps in time.

Illustrative examples for both PLATE 1 and CIVM-PLATE 1 are described, input data are shown, and example solution data are given to enable the user to check the adaptation of these computer programs to his particular computing facilities.


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SUMMARY

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SECTION 1

INTRODUCTION

The PLATE and CIVM-PLATE computer programs are the first of a proposed series of codes which are intended to be made available to the aircraft industry for possible use in analyzing three-dimensional (3-d) structural response problems such as initially-flat plates subjected to prescribed initial velocity distributions and/or external forcing functions, or subjected to impact by an idealized rigid fragment. In particular, the CIVM-PLATE code may be used for the analysis of containment/deflection structures intended to cope with engine rotor-burst fragments. This computer program may also be applicable to crashworthiness problems which are of interest to the automobile and nuclear power plant industries.

The computer programs, written in FORTRAN IV, permit one to analyze the large, three-dimensional, elastic-plastic responses of longeron-stiffened or unstiffened initially-flat single-layer thin plates. In actuality, elastic-viscoplastic behavior is treated in this report since strain-rate effects are taken into account; hereafter in this report, this behavior is termed simply elastic-plastic for convenience. The PLATE code is designed to predict the response of plates subjected to prescribed initial velocity distributions and/or prescribed externally-applied forces. The CIVM-PLATE code is designed to predict the response of plates subjected to impact by a single idealized rigid spherical fragment. The plate may be subjected to a variety of restraint conditions, and linear-elastic line restoring springs may be specified. The plate element used in these programs is restricted to a rectangular planform; hence, the geometrical shape of the plate is restricted to a planform which can be modeled by rectangular elements. The material behavior may be elastic strain-hardening, and/or strain-rate sensitive.

The assumed-displacement finite-element model is used to obtain the spatial properties of the plate, and the temporal solution is accomplished by using the Houbolt finite-difference operator. For predicting the transient responses of plates to rigid-fragment impact (CIVM-PLATE code), energy and momentum considerations are employed in an approximate analysis to predict the collision-induced velocities which are imparted to the fragment and to the affected

plate segments. The presence of fragment/plate surface friction is taken into account. The collision-imparted-velocity method (CIVM) was originally developed for use in the CIVM-JET series of codes [1, 2, 3, 4]* for the analysis of 2-d ring structures subjected to fragment impact; the CIVM approach has been extended to 3-d impact analyses for the CIVM-PLATE code. The pertinent analytical development and the solution methods upon which the PLATE and CIVM-PLATE codes** are based are presented in Appendices A and B, respectively.

Section 2 contains a general description of the organization and capabilities of the PLATE and CIVM-PLATE codes including (1) plate geometry, supports, elastic restraints, and material properties, (2) external loading options and solution procedure for the PLATE code, and (3) fragment/plate collision interaction analysis and solution procedure used for the CIVM-PLATE code. Section 3 contains a description of the main program and subprograms associated with the PLATE and CIVM-PLATE codes, and a description of the flow/usage of these subprograms. The input data, user-prepared subprograms, and output information for the PLATE and CIVM-PLATE codes are presented in Sections 4 and 5, respectively. A complete FORTRAN IV listing of the two codes is given in Section 6. Illustrative examples, including input data and the resulting solution data, using the PLATE and CIVM-PLATE codes are given in Sections 7 and 8, respectively. Finally, Appendices A and B summarize the equations on which these programs are based.

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*Numbers in square brackets [ ] denote references included in the reference list.

**Throughout this report, the terms PLATE and CIVM-PLATE are used for simplicity rather than their correct names PLATE 1 and CIVM-PLATE 1; these codes accommodate large deflections and elastic-inelastic material behavior but various implementation features restrict their applicabilities to small strain. Later, finite-strain versions called PLATE 2 and CIVM-PLATE 2 will be prepared and released.

## SECTION 2

## GENERAL DESCRIPTION OF THE PLATE AND CIVM-PLATE PROGRAMS

### 2.1 Plate Geometry, Supports, Elastic Restraints, and Material Properties

The present subsection is devoted to a general description of the capabilities of the PLATE and the CIVM-PLATE program including (1) geometry, (2) supports, (3) elastic restraints, and (4) material properties. It is important to note that the capabilities of the PLATE and CIVM-PLATE programs are identical with respect to these categories. Thus, the present discussion will be restricted to the PLATE code; any differences between the two codes will be noted. Discussions of those features of PLATE and CIVM-PLATE which differ (i.e., solution procedure and external loading) are given in Subsections 2.2 through 2.5.

The PLATE computer code treats single layer unstiffened or longeron-stiffened initially-flat plates of uniform thickness. The Kirchhoff hypothesis is employed; that is transverse shear deformation is neglected. Thus, the total thickness of the plate should be small compared with the spanwise dimensions of the plate. A global XYZ rectangular Cartesian coordinate system with associated unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ is employed. The initially-flat plate is assumed to lie in a plane which is parallel to the global XY plane. The unit outward-normal vector of the plate, $\hat{n}$, is defined by the right-hand rule as $\hat{n} = \hat{i} \times \hat{j}$; the unit-outward normal direction of the plate is thus in the direction of the positive z axis, and is automatically chosen once the directions of the unit vectors $\hat{i}$ and $\hat{j}$ (in the global X and Y directions, respectively) have been chosen. The choice of the positive direction for the unit vector $\hat{i}$ and $\hat{j}$ also serves to define the "inner" and "outer" surfaces of the plate; the positive outward-normal of the plate is directed toward the "outer" surface of the plate. For analyses using the CIVM-PLATE code, the positive outward normal direction must be chosen such that fragment impact can occur only on the "inner" surface of the plate (see Fig. 1).

In the spatial finite-element analysis, the plate is represented by an assemblage of discrete (or finite) elements compatibly joined at the nodal stations. For the present codes, the plate elements are restricted to rectangular planform and uniform thickness; the geometry and nomenclature of the present four-noded rectangular plate elements are shown in Fig. 2, where a local (element) xyz coordinate system has been adopted such that the sides of the element are parallel to the x or y axis, the origin of the local xyz system ($z=0$) is located at the geometric midsurface of the plate. The plate structure may contain void regions (cutouts), but in all cases (whether or not cutouts are present) the plate structure must be modeled by a series of rectangular elements. This restriction limits the types of plate structures which can be analyzed using the present programs; in subsequent versions of the present codes, this restriction could be relaxed by adopting a more general planform for the basic plate element.

The behavior of each finite element is characterized by a knowledge of the six generalized displacements: $u,v,w$, $\theta = \partial w/\partial x$, $\psi = \partial w/\partial y$, and $\chi = \partial^2 w/\partial x \partial y$ at each of its four (corner) nodal stations where u, v, and w are the translational displacements of the reference surface in the x, y, and z directions, respectively. The displacement behavior within each finite-element is represented by a bilinear interpolation in x and y for the inplane displacements u and v and a bicubic interpolation in x and y for the transverse displacement w, anchored to the generalized nodal displacements at each node (see Appendix A for further details). The geometry of each finite-element is completely defined by the side length dimensions and the thickness of the element, which is assumed to be constant.

The present analysis may be applied to integrally-stiffened flat plates. The stiffeners must be oriented in the global X and/or Y direction, are assumed to be of constant cross section over each element, and must begin and terminate only at an element boundary (i.e., each stiffener must completely span one or more rectangular plate element in the finite-element mesh). The geometry and nomenclature for a typical stiffened plate element are given in Fig. 3. In this illustration, the stiffener is located along the inner surface of the plate element and is oriented in the Y direction: however, note that stiffeners may also be located on the outer surface of the plate and may also be oriented in the X direction.

The formulation of the spatial properties for the X and/or Y direction stiffeners follows the formulation used in Ref. 5 in which each stiffener is treated as a discrete structural component, with behavior similar to that of a beam element. The following assumptions have been made concerning the geometry and deformation behavior of the stiffeners:

(1) In general, the stiffeners may be of material different from that of the plate to which it is attached; however, perfect bonding is assumed at the stiffener/plate interface. Thus, displacements are assumed to be continuous across the interfaces.

(2) The stiffeners are characterized as being thin; that is, a typical cross-sectional dimension of the stiffener is small compared with a typical spanwise dimension of the plate, and exhibit Bernoulli-Euler behavior.

(3) The contributions of twisting strain of the stiffener about its axial direction and bending strain of the stiffener in the plane parallel to the plate midsurface have been neglected. Only the normal strain of the stiffener in its axial direction is taken into account.

Within this framework and by considering the compatibility of displacements at the stiffener/plate interface, the displacement behavior of the stiffener can be related to the generalized degrees of freedom at the nodes of the plate element to which it is attached by employing the displacement interpolation assumption of the plate. The spatial properties of the stiffener are thus defined by a knowledge of the material properties and cross-sectional dimensions of the stiffener, and the relative location and orientation of the stiffener on the plate element. The reader is invited to consult Appendix A for further details on the formulation of stiffener spatial properties.

Two types of support conditions for the structure are included in the present programs: prescribed nodal displacement conditions and line elastic restraints. The following six prescribed displacement boundary condition options have been included (see Fig. 4a):

| Boundary Condition | Constrained Degrees of Freedom along side Y = constant | along side X = constant |
|---|---|---|
| Free | None | none |
| Symmetry | $v=\psi=\chi=0$ | $u=\theta=\chi=0$ |
| Ideally Clamped | $u=v=w=\theta=\psi=\chi=0$ | $u=v=w=\theta=\psi=\chi=0$ |
| Pinned-Fixed | $u=v=w=\theta=\chi=0$ | $u=v=w=\psi=\chi=0$ |
| Pinned, Free-Sliding z-direction | $u=v=\theta=\chi=0$ | $u=v=\psi=\chi=0$ |
| Pinned, Free-Sliding (inplane) normal | $u=w=\theta=\chi=0$ | $v=w=\psi=\chi=0$ |

Elastic restraints are provided in the form of linear elastic line translation and torsional restoring springs which can be specified along any element boundary or interelement boundary. Five spring constants (three translational and two rotational) may be specified for each element boundary on which line restoring springs are present (see Fig. 4b). A global effective stiffness matrix supplied by the linear-elastic line restoring springs is evaluated, and derived from the virtual-work statement.

For the present programs, the plate structure and the stiffeners can be of different elastic, or elastic, perfectly-plastic, or elastic-strain-hardening behavior. A maximum of five different material types may be specified by the user to define the behavior of the stiffeners if different from that of the plate. The strain-rate effects of each material can also be taken into account. In the present analysis, the strain-hardening material is accounted for by using the "mechanical sublayer model" [6,7];* see Figs. A.1a-A.1c. A useful feature of this model is its accounting for nonlinear strain hardening, anisotropic strain hardening, the formation of corners on the yield surface, and the Bauschinger effect -- all of these are features that ductile metals exhibit. No other material model appears to represent these features as efficiently and faithfully as does the mechanical-sublayer model. Strain-rate effects are included as described in Subsection A.5.

## 2.2 External Loading Options in the PLATE Program

The PLATE computer program is designed to predict the large, elastic plastic deformation responses of stiffened or unstiffened flat plates subjected

*Also, see Subsections 3.3.3 and 3.3.4 of Ref. 14, as well as Refs. 5 and 11.

to either a prescribed initial velocity distribution or a prescribed externally-applied, time-varying force distribution. In the present finite-element scheme, initial velocity distributions are input as a vector of nodal velocities, at time t=0, corresponding to the generalized nodal degrees of freedom in the finite-element mesh. Prescribed externally-applied force distributions are input as a vector of nodal forces corresponding to the generalized nodal degrees of freedom in the finite-element mesh. In general, the spatial distribution of these forces can vary with time so that this vector of nodal forces must be redefined at each step of the finite-difference temporal solution.

Much of the user interaction required when using the PLATE code is in the form of preparation of input data cards; wherever possible, internal logic has been incorporated into the PLATE code to minimize the amount of user-supplied data. However, for the present plate analysis, it is impractical and computationally inefficient to attempt to provide the user with an all-inclusive set of initial velocity distribution options and prescribed force distributions (in both space and time). As a result, two computer subroutines (INCOND and EXTL) have been isolated as user-interactive subroutines for the generation of the vector of initial nodal generalized velocities and the vector of prescribed externally-applied nodal forces at each time step. The user will be required to write (in FORTRAN IV) the internal logic for these subroutines corresponding to the particular problem being considered. Clearly, some basic knowledge of computer programming and the finite-element method is required to generate these computer subroutines. Guidelines for preparation of these subroutines, including simple example subroutines, are given in Subsection 4.6.

## 2.3 Solution Procedure for the PLATE Program

The spatial finite-element approach is utilized in conjunction with the Principle of Virtual Work and D'Alembert's Principle to obtain the equations of motion of the plate structure which is permitted to undergo large, elastic-plastic, transient deformation. In the interest of conciseness and convenience in this report, the user is invited to consult Ref. 1 and Appendix A for a detailed derivation and discussion of the equation of motion utilized in the PLATE code. For present purposes, it suffices to note that the governing

equations of motion for the complete assembled discretized structure correspond to the conventional formulation of Ref. 1 and are written in the following form:

$$[M]\ \{\ddot{q}^*\} + [K]\ \{q^*\} + [K_s]\ \{q^*\} = \{F\} + \{F^{NL}\} \tag{2.1}$$

where

| | |
|---|---|
| $\{q^*\}$ and $\{\ddot{q}^*\}$ | are the global generalized displacement vector and acceleration vector, respectively. |
| $[M]$ | is the mass matrix of the assembled structure. |
| $[K]$ | is the linear elastic stiffness matrix of the assembled structure. |
| $[K_s]$ | is the global effective stiffness matrix supplied by the linear-elastic line restoring springs. |
| $\{F\}$ | is the global generalized load vector corresponding to prescribed externally-applied forces. |
| $\{F^{NL}\}$ | represent equivalent "generalized loads" arising from both large deflections and plastic strains. |

The matrix equations of motion are solved by employing the Houbolt finite-difference operator. In the following, the general solution process is described briefly.

First, information is provided to define the geometry of the plate and the location and geometry of the stiffeners (if any). Material constants for the plate and stiffeners are defined, and information related to the structural discretization is specified. Also defined are the time-step size, $\Delta t$, the location and elastic constants of the line restoring springs, and the prescribed displacement boundary conditions. Next, the mass matrix and stiffness matrix for the entire structure are evaluated by assembling the element mass and stiffness matrices. It should be noted that the PLATE program provides the user with the option of selecting either a lumped (diagonal matrix) mass modeling or a consistent (fully populated matrix) mass modeling.

Gaussian quadrature is used to evaluate these element matrices; the quadrature+ rule chosen yields an exact integration of these element matrices. Following this step, the global effective stiffness matrix supplied by the linear-elastic line restoring springs is evaluated. Then the proper prescribed displacement conditions are imposed on these assembled mass and stiffness matrices.

Before beginning the timewise solution of the equations of motion, the generalized nodal velocity vector, $\{\dot{q}^*\}_o$, at time $t_o$ and generalized nodal force vector corresponding to prescribed externally-applied forces $\{F\}_1$, at time $t_1$, must be specified. From this information, the generalized nodal displacements and displacement increments are calculated for the first time increment $\Delta t$ (i.e., from time $t_o$ to time $t_1$) from Eq. 2.1 written in the finite-difference form corresponding to the Houbolt operator. From a knowledge of the displacements, displacement increments, and current states of stress and strain, one can determine the strain increments, stress increments, the stresses and/or the plastic strains and the plastic strain increments through the use of the pertinent elastic-plastic stress-strain relations including the plastic yield condition and flow rule. From this information, one can calculate the equivalent generalized load vector arising from large deflections and plastic strains. Next, the generalized nodal load vector $\{F\}_2$ corresponding to prescribed externally-applied forces at time $t_2$ is formed. Then, the proper recurrence equations, which is the finite-difference representation of the equations of motion, are solved to obtain the generalized displacements and displacement increments of the next time increment (i.e., at time $t_2 = t_1 + \Delta t$). The process then proceeds cyclically for as many time steps as desired. For present purposes, the above general description is considered to be adequate; one may consult Appendix A for a more detailed discussion of the solution and evaluation process.

In the PLATE code, the user is required to specify the time-step size, $\Delta t$, to be used in the temporal solution of the equations of motion of the structure. The proper choice of the time-step size, $\Delta t$, requires some knowledge of the characteristics of the Houbolt operator. For linear structural analysis, the Houbolt operator is unconditionally stable; that is, no numerical instabilities will occur regardless of the value of $\Delta t$. However, as the time

+Numerical integration (quadrature) is employed for these evaluations since it is much more efficient and less likely to produce errors than if an analyst were to carry out and program (lengthy) expressions obtained by analytical evaluations.

step size is increased, the solution will tend to diverge from the "exact" solution because of the presence of false damping and of frequency distortion contributed by the Houbolt operator; this is a problem of "convergence". Usually, it is recommended that successive runs of the same problem be made, each with a time-step size half that of the previous run until the computed results of two successive runs are sufficiently close to each other (i.e., to represent a converged solution). For nonlinear structure analysis, where the effects of nonlinearities are included as equivalent loads (and these equivalent loads are obtained by extrapolation --- (see Appendix A), the Houbolt operator becomes conditionally stable [8,9]; that is, numerical instability will occur if the time-step size, $\Delta t$, is too large. At present, no rigorous rules exist for determining the maximum allowable $\Delta t$ for nonlinear analysis using the Houbolt operator. Thus, some numerical experimentation may be necessary in order to determine an appropriate value for $\Delta t$.

To assist the user in the selection of a value for $\Delta t$, a "reference" time-step size is calculated and output by the PLATE computer program. This reference time-step size, $\Delta t_{max}^{CD}$, is equal to the maximum time-step size which could be used <u>if</u> the Central Difference timewise operator were employed, and is calculated as $\Delta t_{max}^{CD} = 0.8(2/\omega_{max})$. In this expression, $\omega_{max}$ is the largest natural frequency contained in the (linear) mathematical model of the structure, and $\Delta t \leq 2/\omega_{max}$ is the stability criterion (for the Central Difference operator) of a corresponding linear dynamic system; the factor 0.8 is introduced, as in Refs. 1 through 3, to account for large-deflection effects. In the PLATE program, the value of $\omega_{max}$ is obtained by an iteration process applied to[+]

$$\omega^2 \; [M] \, \{q^*\} = [K[ \, \{q^*\} \tag{2.2}$$

Having obtained the value of $\Delta t_{max}^{CD}$, the actual time-step size to be used in the PLATE program can be taken as some multiple of $\Delta t_{max}^{CD}$ (i.e., $\Delta t = f \, \Delta t_{max}^{CD}$). Again, no firm rules exist for the selection of the factor f, but results presented in Ref. 1 suggest that values of up to f = 6 will yield converged and stable solutions. These results are by no means conclusive, but do provide a reasonable guideline for the initial choice of $\Delta t$. To ensure

[+]The inverse power method is employed to find $\omega_{max}$.

convergence and/or to eliminate instabilities, subsequent computer runs can be made using a value of $\Delta t$ half that of the previous run.

## 2.4 Fragment/Plate Collision Interaction Analysis for the CIVM-PLATE Program

The CIVM-PLATE computer program is designed to predict the large-deflection, elastic-plastic, transient responses of initially-flat stiffened or unstiffened plates subjected to impact by a single fragment. For analyzing the collision-induced transient responses of containment and/or deflector plates and fragment motions, the fragment is idealized as a non-deformable fragment of spherical configuration (Fig. 5). The modeled-fragment radius, mass, mass moment of inertia, and velocity components are specified by the user to correspond with those of the actual fragment.

The process called the collision-imparted velocity method (CIVM), previously developed for the 2-d ring/fragment collision interaction analysis of Refs. 1 through 4, has been extended for the present 3-d collision-interaction analysis. In this procedure, energy and momentum considerations are employed to predict the collision-induced velocities which are imparted to the fragment and to the impact-affected zone of the plate. Also, the following simplifying assumptions are invoked:

(1) The collision process is instantaneous and involves only the fragment and the impact-affected zone of the plate. The impact-affected zone is defined as the fraction of the plate that responds to fragment impact instantaneously with momentum changes. The size of the impact affected zone is estimated from the speed of a longitudinal elastic wave in the material and the computational time increment size $\Delta t$, or from alternate means (see Appendix B).

(2) In an overall sense, the fragment is treated as being rigid, but at the "immediate contact region" between the fragment and the impacted plate the collision is regarded as acting in a perfectly-elastic ($e = 1$), perfectly-inelastic ($e = 0$), or

an intermediate fashion $(0<e<1)$, where e represents the coefficient of restitution.

(3) The colliding surfaces of both the fragment and the plate may be either perfectly smooth ($\mu = 0$) or may be "rough" ($\mu \neq 0$), where $\mu$ denotes the coefficient of sliding friction. Hence, respectively, force and/or momentum (or velocities) are transmitted only in the normal-to-surface direction or in both the normal and tangential directions.

(4) Although fragment/plate collision will, in general, occur during a time step, $\Delta t$ (rather than at the beginning or end of a time cycle), the impact-induced velocity changes of the fragment and the impact-affected zone of the plate are imposed at the beginning of the cycle in which the collision occurs. This assumption eliminates the need to extrapolate displacement and velocity data between time steps.

(5) For the purpose of calculating the impact-induced velocity changes of the fragment and the impact-affected zone of the plate, the contact forces are the only ones considered to act on the impact-affected zone of the plate and in an anti-parallel fashion on the fragment. Any internal forces which the plate segments adjacent to the impact-affected zone may exert on that zone as a result of this instantaneous collision are considered to be negligible because this impact duration is so short as to preclude their "effective development". However, these internal forces are included in the calculation of the post-impact displacement of the plate.

(6) To avoid unduly complicating the analysis and to maintain program efficiency, each rectangular plate element is divided into two triangular plate elements in the

derivation of the impact inspections and equations; the global coordinates of the three corners of the triangle define the plane in which collision may occur (see Fig. 6). However, for transient response predictions, the plate continues to be modeled as a series of rectangular elements, and general 3-d plate deformation behavior is assumed.

(7) For the analysis of stiffened plates, only the geometry of the plate elements is considered in the impact inspection procedure. If impact occurs at a location on the plate occupied by a stiffener, the impact is still considered to occur at the inner surface of the plate. However, stiffener mass contributions have been included in the calculation of the post-impact velocity changes of the fragment and impact-affected zone of the plate. Thus, the subsequent motion of the plate and fragment is affected by the presence of stiffeners.

(8) To minimize the special options in impact inspection, all impacts must occur at the inner surface of the plate.

An information flow schematic of the CIVM procedure is shown in Fig. 7. Briefly, the analysis procedure indicated in Fig. 7 consists of the following principal steps:

1. Motions and Positions of Bodies

The motions of the fragment and of the containment and/or deflector plate are predicted and the (tentative) region of space occupied by each body at the end of a given time cycle is determined.

2. Collision Inspection

Next, an inspection is performed to determine whether a collision has occurred during the small increment ($\Delta t$) in time from the last instant in time at which the body locations were known (beginning of the present time cycle) to the present instant in time at which the body-location data are sought. A collision is determined to have occurred if the (tentative) regions

of space occupied by the plate and the fragment overlap. If a collision has not occurred during this $\Delta t$, one follows the motion of each body for another $\Delta t$, etc. However, if a collision has occurred, one proceeds to carry out a collision-interaction calculation.

3. Collision-Interaction Calculation

In this calculation energy and momentum conservation relations are employed in an approximate analysis to compute the collision-induced changes in (a) the velocities $v_{fx}$, $v_{fy}$, $v_{fz}$ (translational) and $\omega_{fx}$, $\omega_{fy}$, $\omega_{fz}$ (rotational) of the fragment and (b) the nodal velocities of the plate impact-affected elements. These velocity changes are imposed at the beginning of the present time cycle in which impact was determined to have occurred. Steps 1 and 2 are then repeated for the present time cycle until no plate/fragment overlap is found at the end of the present time cycle; the resulting plate and fragment locations are then the corrected-for-impact locations.

One then returns to step 1, and the process is repeated for as many time increments as desired. The details of this analysis procedure as well as various considerations and simplifying assumptions employed are discussed further in Appendix B.

## 2.5 Solution Procedure for the CIVM-PLATE Program

The global solution procedure employed in the CIVM-PLATE computer program is essentially the same as that employed in the PLATE computer program except that initial velocity distributions and prescribed, externally-applied forces are not included in the CIVM-PLATE program; these loading options are "replaced" by the collision inspection-interaction analysis. The governing equation of motion for the complete assembled discretized plate structure is thus obtained by dropping the external nodal loading vector from Eq. 2.1 and is given by

$$[M]\ \{\ddot{q}^*\} + [K]\ \{q^*\} + [K_s]\ \{q^*\} = \{F^{NL}\} \tag{2.3}$$

where the definition of the terms in Eq. 2.3 are the same as those given in Subsection 2.3.

In the present procedure, a diagonal "lumped" mass matrix is employed;

the use of a consistent (fully populated) mass matrix is not permitted in the CIVM-PLATE program. Lumped mass modeling is chosen as a consequence of the present collision-interaction analysis in which the element (and structure) mass properties are assumed to be lumped at the nodal points. Thus, for consistency, the mass properties of the plate structure used in the global timewise solution procedure should also be nodal lumped masses; the use of a consistent mass modeling in the global timewise solution would result in an energy mismatch following each impact. In addition, the use of a lumped mass matrix results in a decrease of the highest natural frequency for the assembled structure, compared with the use of a consistent mass matrix, and thus permits one to use a larger time step, $\Delta t$, for the structural response calculations (see discussion in Subsection 2.3). This fact, coupled with reduced storage requirements and additional savings of computation time in each time step because of the simple form of the mass matrix, makes the use of a lumped matrix computationally efficient.

The equations of motion are solved through the use of the Houbolt finite-difference operator whereby one obtains a recurrence equation which provides a solution step-by-step in finite time increments. The discussion of the stability of the Houbolt operator given in Subsection 2.3 also applies to the present impact analysis. However, as will be discussed in Appendix B, this operator must be modified for time cycles in which plate/fragment collision occurs. Experience to date with the CIVM-PLATE program suggest that this modified operator is less stable than the conventional Houbolt operator, and thus smaller time steps, $\Delta t$, must be used. Because (diagonalized) lumped masses are used in all runs of the CIVM-PLATE program, the reference time step size, $\Delta t^{CD}_{max}$, discussed in Subsection 2.3 is calculated and output for each execution of the program. It is recommended that a value of $\Delta t = 2*\Delta t^{CD}_{max}$ be specified on the initial run of a containment/deflector analysis. A more detailed discussion of this modified operator is given in Appendix B.

The general solution process employed in the CIVM-PLATE program is essentially identical to that employed in the PLATE program in terms of input of geometric, mesh, and boundary condition information, and generation of the assembled-structure mass (lumped mass modeling only) and stiffness matrices.

Additional input is required for the CIVM-PLATE program to define the initial location and velocity components, geometry, and collision parameters of the attacking fragment. In the following, the general timewise solution process (which differs somewhat from that used in the PLATE program) is described briefly.

The plate structure is assumed to be at rest at time $t_o$, and the position and velocity of the attacking fragment are known at time $t_o$. From this information the generalized nodal and fragment displacements and displacement increments are computed for the first time increment, $\Delta t$. Then the fragment-plate collision inspection-and-correction procedure is carried out. If one, or more, fragment-plate collisions have occurred during this $\Delta t$, the coordinates which locate the position of the fragment and the impact-affected nodes of the plate are thereby corrected from their tentative uncorrected-for-impact locations. Next, the strain increment developed from $t_o$ to $t_1$ at every Gaussian station (or point) required over and depthwise through each finite element are calculated. From a knowledge of the previous states of stress and strain, and the strain increments, one can determine the stress increments, the stresses, and/or the plastic strains and plastic strain increments through the use of the pertinent elastic-plastic stress-strain relations including the plastic yield condition and flow rule.[+] Next, one can calculate the equivalent generalized load vectors arising from large deflections and plastic strains. Then, the proper recurrence equations, which is the finite-difference representation of the equations of motion, are solved to obtain the plate-nodal generalized displacements and displacement increments of the next time increment. The pertinent equations of motion for the fragment are also solved to obtain the displacements and displacement increments of the next time increment for the fragment. This process then proceeds cyclically for as many time steps as desired. It should be noted that the plate structure remains at rest until the first plate-fragment collision occurs.

For present purposes, the above general description is considered to be adequate; one may consult Appendix B for a more detailed discussion of the solution and evaluation process, including flow charts.

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[+]The mechanical-sublayer material model is particularly convenient and efficient for these evaluations; see Figs. A.1a through A.1c.

SECTION 3

DESCRIPTION OF PROGRAMS AND SUBPROGRAMS

## 3.1 Organization of the PLATE and CIVM-PLATE Programs

The PLATE and CIVM-PLATE computer program package consists of two dummy main programs (one each for PLATE and CIVM-PLATE) and 34 subprograms. Of these 34 subprograms, 5 are used exclusively in the PLATE program, 8 are used exclusively in the CIVM-PLATE program, and the remaining 21 are used by both the PLATE and CIVM-PLATE programs.

The dummy main program and subroutines used exclusively in the PLATE program are

| | | |
|---|---|---|
| CMASSP | INCOND | MAINP |
| EXTL | MAIN | PRINT |

The dummy main program and subroutines unique to the CIVM-PLATE program are

| | | |
|---|---|---|
| COLLSN | IMAINP | IMPSS |
| IFRAG | IMPACT | INTRAC |
| IMAIN | IMPDS | IPRINT |

The 21 subroutines used by both the PLATE and CIVM-PLATE programs are

| | | | | |
|---|---|---|---|---|
| ARRT | FACT | MESHPM | RCON | SPRING |
| ASSEMK | GAUSS | OMULT | RPLATE | STRESA |
| BCONMK | LMASSP | PRTOP | SFDM | STRESC |
| EAROW | MESHPA | PSTRN | SOLV | STRESP |
| | | | | TSTEP |

A description of these main and subprograms including the definitions of calling arguments for each subprogram is given in Subsection 3.2.

The PLATE program consists, therefore, of a dummy main program (MAIN) and 26 subroutines. The dummy main program has been included only to define array

dimensions which are problem dependent, and to input those parameters used in the dimensioning process. MAIN then calls subroutine MAINP which is the master subroutine which controls the flow of the transient nonlinear analysis. The flow of subroutine MAINP with particular emphasis on subroutine usage is shown in Fig. 8. The information provided in Fig. 8 together with the description of subroutines given in Subsection 3.2 and the theoretical developments given in Appendix A, is intended to provide the user with a better understanding of the operation and execution of the PLATE program.

The CIVM-PLATE program consists of a dummy main program (IMAIN) and 29 subroutines. The dummy main program has been provided only to define array dimensions which are problem dependent, and to input those parameters used in the dimensioning process. IMAIN then calls subroutine IMAINP which controls the flow of the impact-induced transient nonlinear analysis. The flow of subroutine IMAINP with particular emphasis on subroutine usage is shown in Fig. 9. The information provided in Fig. 9, together with the description of subroutines given in Subsection 3.2 and theoretical developments given in Appendix B, is intended to provide the user with a better understanding of the operation and execution of the CIVM-PLATE program.

## 3.2 Program and Subprogram Descriptions

In this subsection, each of the programs/subprograms used in the PLATE and CIVM-PLATE computer codes is described. The programs/subprograms are grouped (and presented alphabetically) according to those used only in the PLATE code, those used only in the CIVM-PLATE code, and those common to both codes. In each case, the subroutine names and argument list (as it appears in that subroutine) are given; each of the subroutine arguments is defined and a description of the overall operation of the subroutine is given.

No attempt is made to describe the programming logic or to describe each programming step of each subroutine. Information of this form could be generated by a user facility by a careful comparison of programming logic and associated theory presented in Appendices A and B, as well as various programming conventions discussed throughout this report. Such a detailed understanding of the PLATE and/or CIVM-PLATE codes would facilitate modifications and/or additions to the codes, but is not necessary for effective use of the codes.

### 3.2.1 Main Program and Subroutines Unique to the PLATE Program

In the following, the main program and subroutines used only in the PLATE program are discussed.

SUBROUTINE CMASSP(NTYPE, XL, YL, TH, XF, SLOC, CSA, SMOI, EM, GBI, DENS)

Purpose: This subroutine computes the consistent mass matrix for a rectangular plate element, x-direction stiffener element, or y-direction stiffener element. The formation corresponds to Eqs. A.28 and A.29 (for plate elements), Eq. A.91 for x-direction stiffners, and Eq. A.91b for y-direction stiffeners; see Figs. 10-14. Gaussian quadrature is used to compute the required integral.

Arguments:

| | | |
|---|---|---|
| NTYPE | : | Type of element<br>= 1 for plate element<br>= 2 for x-direction stiffener<br>= 3 for y-direction stiffener |
| XL | : | The x-direction length of the element |
| YL | : | The y-direction length of the element |
| TH | : | The plate element thickness |
| XF | : | The stiffener offset distance (see Fig. 12) |
| SLOC | : | The location of the stiffener on its corresponding element (see Fig. 13) |
| CSA | : | The stiffener cross-sectional area |
| SMOI | : | The mass moment of inertia of the stiffener |
| EM(24,24) | : | The mass matrix of the element |
| GBI(24,24) | : | The matrix $\bar{\underset{\sim}{G}}^{-1}$ given by Eq. A.19 |
| DENS | : | The initial mass per unit volume of the element |

SUBROUTINE EXTL(ELV,ITT,NP,NODE,XGI,YGI)

Purpose: This user-prepared subroutine is used to generate the nodal generalized forces corresponding to prescribed externally-applied forces at time $t_m$ (= $m\Delta t$) as described in Subsection 4.6.2 and corresponding to the development in Subsection A.1.3.4.

Arguments:

| | | |
|---|---|---|
| ELV(I) | : | The user-defined nodal force vector at time t = ITT*DELTAT (where DELTAT = $\Delta t$). ELV(I) is the generalized force corresponding to the Ith generalized degree of freedom. |
| ITT | : | The cycle number |
| NP(I,J) | : | The global node number associated with the Ith element node of element J. |
| NODE(K) | : | The assembly list associating element degrees of freedom with global degrees of freedom. If K is defined as K=24*(J-1) + I, then NODE(K) is the global degree of freedom number associated with the Ith degree of freedom of element J. |
| XGI(I) }<br>YGI(I) } | | The global X and Y coordinates, respectively, of the Ith global node at time zero. |

SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)

Purpose: This user-prepared subroutine is used to specify the initial (time zero) conditions on the plate nodal displacements and velocities $\underset{\sim}{q}^*_o$ and $\dot{\underset{\sim}{q}}^*_o$, respectively, required for the finite-difference solution of the governing equations of motion (see Subsection A.3). Instructions for preparation of this subroutine are given in Subsection 4.6.1.

Arguments:

| | | |
|---|---|---|
| DISP(I) | : | The displacement at time zero of the Ith generalized nodal degree of freedom |

VEL(I) : The velocity at time zero corresponding to the Ith generalized nodal degree of freedom

NP(I,J), NODE(K), XGI(I), YGI(I) : See SUBROUTINE EXTL

MAIN

Purpose: The dummy main program of the PLATE computer code. Problem dependent array dimensions are specified in this dummy main program and certain of those variables used in this dimensioning process are input. Guidelines for preparation of the array dimensions are given in Subsection 4.5

Arguments: None

SUBROUTINE MAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,ELV,VEL,ICOL,
INUM,KROW,NDEX,STOR,STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,EPSSI,EPSSO,EPEEI,
EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,
MATXS,TSGEE,LNYS,YSPROP,MATYS,LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,
MNYST,NVSA,NCON)

Purpose: The primary subroutine of the PLATE program. This subroutine controls the flow of program input, structural response prediction, and program output.

Arguments:

DELD(I), DIS(IP), DISP(I), DISM1(I), DISM2(I) : The displacement increment and displacement vectors $\Delta \underset{\sim}{q}^*_{m+1} \equiv (\underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_{m})$, $\underset{\sim}{q}^*_{m+1}$, $\underset{\sim}{q}^*_{m}$, $\underset{\sim}{q}^*_{m-1}$, $\underset{\sim}{q}^*_{m-2}$, respectively (e.g. Eq. A.110). The Ith term in each vector corresponds to the Ith structural degree of freedom.

| | |
|---|---|
| FLN(I)<br>FLVA(I)<br>FLVM(I)<br>FLVP(I) | Vectors used to accumulate effective nonlinear and external nodal forces required to form the right-hand side of Eq. A.114 (corresponding to Ith structural degree of freedom). |
| ELV(I) : | See Subroutine EXTL |
| VEL(I) : | The velocity corresponding to the Ith structural degree of freedom |
| ICOL(K)<br>INUM(K)<br>KROW(K)<br>NDEX(K) | Vectors containing pointers used to locate terms in the assembled stiffness matrix which is stored in compacted vector form (see Subsection 4.1.5) |
| STOR(K) : | Temporary vector in which the current external force vector is stored |
| STF(J) : | Assembled stiffness matrix stored in compacted vector form (see Subsection 4.1.5) |
| AMASS(J) : | The assembled (structure) mass matrix stored either in compacted vector form (if consistent mass option is used) or as a vector of the diagornal terms (if lumped mass is used). |
| NP(I,J)<br>NODE(K) | See Subroutine EXTL |
| TAUSS(I,J,K)<br>TAUEE(I,J,K)<br>TAUSE(I,J,K) | The stresses $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$, respectively at the Jth depthwise/sublayer integration station (J is a function of both location and sublayer) at the Kth spanwise Gaussian integration station of the Ith plate element at the current time. |

| | |
|---|---|
| EPSSI(K,I) <br> EPEEI(K,I) <br> EPSEI(K,I) | The strains $\tilde{\varepsilon}_{xx}$, $\tilde{\varepsilon}_{yy}$, and $\tilde{\varepsilon}_{xy}$, respectively, at the Kth Gaussian integration station (K=1,2, ...9) on the <u>inner</u> surface of the Ith plate element. |
| EPSSO(K,I) <br> EPEEO(K,I) <br> EPSEO(K,I) | As above, at the <u>outer</u> surface of the Ith plate element |
| NBC(J) <br> BC(J) | A list of the NB constrained degree-of-freedom numbers and corresponding constrained values (BC(J) = 0.0 set internally) |
| RFM(J,K) : | Matrix used in the calculation of the reaction forces at constrained nodes, and contains the rows of the assembled stiffness matrix (from the first nonzero term to the last nonzero term in that row) corresponding to each of the constrained degrees of freedom. |
| ILAST(J) : | For row NBC(J) of the assembled stiffness matrix, the column in which the last nonzero term is found. |
| UCF1(J) <br> UCF2(J) | Vectors containing the right-hand side of Eq. A.114 (prior to constraint) corresponding to constrained degree of freedom NBC(J) |
| XG(I) <br> YG(I) <br> ZG(I) | The <u>current</u> global (X,Y,Z) coordinates, respectively, of structural node I |
| XGI(I) <br> YGI(I) | See Subroutine EXTL |
| TAGSS(I,J,K): | The normal stress, $\sigma_{xx}$, at the Jth depthwise/sublayer integration station at the Kth spanwise Gaussian integra-station of the Ith individual X-direction stiffener. |

LNXS(I) : The plate element numbers to which the Ith individual X-direction stiffener is attached.

XSPROP(J,I) : Geometric and material properties of the Ith individual X-direction stiffener. See Cards 28-34 (Subsection 4.2) for definition of J = 1,2,3, ... 6.

MATSX(I) : The material type for the Ith individual X-direction stiffener.

TSGEE(I,J,K) : The normal stress, $\sigma_{yy}$, at the Jth depthwise/sublayer integration station at the Kth spanwise Gaussian integration station of the Ith individual Y-direction stiffener.

LNYS(I), YSPROP(J,I), MATYS(I) : Same as above (LNXS,XSPROP,MATXS), now for individual Y-direction stiffeners.

LNRS(I), ISRS(I) : The plate element number and element side number, respectively, to which the Ith linear line restoring spring is attached.

SC(K,I) : The given (K = 1,2, ... 5) elastic spring constant ($K_u, K_v, K_w, K_\theta, K_\psi$) corresponding to the Ith line restoring spring.

MAXEL, MNSL, NBE, MBWE, MNXST, MNYST : Problem size and dimensioning variables input by the user. See Card 1 (Subsection 4.2) for the definition of these parameters.

MNSLT4 : MNSL*4 (maximum number of depthwise/sublayer integration stations).

NVSA(I) : A list of those global node numbers at which nodal average strain output is given.

NCON(J,I) : The element numbers (J = 1,2,3,4 in general) which are common to structure node number NVSA(I).

SUBROUTINE PRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,XGI,YGI, NP,GBI,STDEP,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,NMSUB,SUBW,TAUSS,TAUEE, TAUSE,MAXEL,NZS,NXST,LNXS,MATXS,XSPROP,TAGSS,MNXST,NYST,LNYS,MATYS,YSPROP, TSGEE,MNYST,NELES,LNRS,ISRS,SC,EK,IMASS,AMASS,VEL,FLVM,KROW,NDEX,WFORCE, CINETI,NNSA,NVSA,NCON)

Purpose: This subroutine controls the calculation (as necessary) and output of all quantities (displacements, strains, energies, etc.) requested by the user at regular intervals during the timewise solution for the PLATE program.

Arguments: The arguments DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO, XGI,YGI,NP,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,TAUSS,TAUEE, TAUSE,MAXEL,LNXS,MATXS,XSPROP,TAGSS,MNXST,LNYS,MATYS,YSPROP, TSGEE,MNYST,LNRS,ISRS,SC,AMASS,VEL,FLVM,KROW,NDEX,NVSA, and NCON are defined in Subroutine MAINP.

GBI(24,24) : See Subroutine CMASSP.

STDEP(9,24) : The matrix used to calculate strains from displacements for an element. The rows correspond to the vectors $\underset{\sim}{D}_1^T, \underset{\sim}{D}_2^T \ldots \underset{\sim}{D}_9^T$ of Eqs. A.10 evaluated at a specified location within the element.

NMSUB(I) : The number of mechanical sublayers used to define the uniaxial stress-strain curve for material type I.

SUBW(I,K) : The sublayer weighting factors $C_k$ (Eq. A.120b) corresponding to material type I.

| | | |
|---|---|---|
| NZS | : | 4*MNSL where MNSL is the maximum number of mechanical sublayers used to specify a material behavior and 4 depthwise integration stations are used. |
| NXST | : | The actual number of individual X-direction stiffeners. |
| NYST | : | The actual number of individual Y-direction stiffeners. |
| NELES | : | The number of element sides on which linear restoring springs are attached. |
| EK(24,24) | : | The equivalent stiffness matrix for an element corresponding to line restoring springs. The matrix is used here to calculate the energy stored in restoring springs. |
| IMASS | : | Indicates the type of mass matrix employed<br>= 1 for lumped (diagonalized) mass matrix<br>= 2 for consistent mass matrix. |
| WFORCE | : | Current value of the work done by the externally-applied forces. |
| CINETI | : | The initial kinetic energy imparted to the structure by the user-specified initial velocity distribution. |
| NNSA | : | The number of nodes at which averaged nodal strains are to be output. |

### 3.2.2 Main Program and Subroutines Unique to the CIVM-PLATE Program

In the following, the main program and subroutines used only in the CIVM-PLATE program are discussed.

SUBROUTINE COLLSN(IOPT,XG,YG,ZG,PEN,RF,THALF,NNSR,XF,YF,ZF,XN,YN,ZN,T,NN1,NN2,NN3)

Purpose: This subroutine performs the detailed collision inspection over a triangular subregion of a plate element. If collision (plate fragment overlap) is detected, the penetration distance, location of maximum penetration distance on the element, and appropriate coordinate transformation matrix are calculated. (See Subsection B.2.1).

Arguments:

| | | |
|---|---|---|
| IOPT | : | A parameter for subroutine usage.<br>= 1 if the entire check process is to be followed.<br>= 2 if the subroutine is to be terminated after calculation of the coordinate transformation matrix for the triangular subregion. |
| XG(I)<br>YG(I)<br>ZG(I) | | Current global X,Y,Z coordinates of the Ith structural node. |
| PEN(I) | : | The penetration distance between the fragment and the Ith triangular subregion. |
| RF | : | The radius of the spherical fragment. |
| THALF | : | The plate half thickness. |
| NNSR | : | The current triangular subregion being inspected. |
| XF<br>YF<br>ZF | | The current global X,Y,Z coordinates of the center of the spherical fragment |
| XN(I)<br>YN(I)<br>ZN(I) | | The global location ($x_n, y_n, z_n$ -- see Eq. B.12) of the point of maximum penetration for the Ith triangular subregion. |
| T(3,3) | : | The coordinate transformation matrix used to transform from global to local (impact) coordinates. See Eqs. B.7 and B.8. |
| NN1<br>NN2<br>NN3 | | The global node numbers of nodes 1, 2, and 3, respectively of the current triangular inspection region. |

SUBROUTINE IFRAG (PMASS,AMASS)

Purpose: In this subroutine the initial fragment position and velocity, fragment geometric properties, contact surface properties and any symmetry options are input and printed for verification. The fragment initial kinetic energy is calculated and the mass matrix used in impact-interaction calculations is formed.

Arguments:

PMASS(I) : The lumped (diagonal terms only) mass matrix corresponding only to translational degrees of freedom, used in impact-interaction calculations.

AMASS(I) : The lumped mass matrix for the complete plate structure.

IMAIN

Purpose: The dummy main program of the CIVM-PLATE computer code. Problem-dependent array dimensions are specified, and certain of those variables used in this dimensioning process are input. Guidelines for preparation of the array dimensions are given in Subsection 5.3.

Arguments: None.

SUBROUTINE IMAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,VEL,ICOL,INUM,KROW,NDEX,STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS,LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON,PMASS,VN,VNB,SI,NEFF,ALPHA,NIAN)

Purpose: The controlling main program for the CIVM-PLATE code. The flow of input, timewise calculation, and output is governed by this primary subroutine.

Arguments:

PMASS(I) : See Subroutine IFRAG

VN(J,I) : The translational velocity components in the global X,Y,Z coordinates (J = 1,2,3) for the Ith impact-affected node at the current time instant.

VNB(J,I) : The translational velocity components in the local $(\bar{x},\bar{y},N)$ impact coordinate system (J = 1,2,3) for the Ith impact-affected node at the current time instant.

SI(I) : For the current impact interaction calculation, the distance $|s_1|$ (see Eq. B.16) from the point of impact to the Ith impact-affected node.

NEFF(I) : For the current impact-interaction calculation the global node number of the Ith impact-affected node.

ALPHA(I) : The proportionality factor, $\alpha_1$ (see Eqs. B.19), for impulse distribution associated with the Ith impact-affected node for the current impact-interaction calculation.

NIAN : A user estimate of the maximum number of nodes which will be affected by any impact.

The remaining arguments are as defined for Subroutine MAINP given in Subsection 3.2.1.

SUBROUTINE IMPACT(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,NEFF,ALPHA, IMPCT1,NIAN)

Purpose: Controlling subroutine for impact inspection and interaction calculations for general (no symmetry) impact cases). For each step time, this subroutine controls inspection for maximum plate-fragment overlap. If impact has occurred, impact-affected nodes are identified and their pre-impact velocities are updated to post-impact velocities (similarly for the fragment) via the impact-interaction calculations.

Arguments:

$\left.\begin{array}{l}\text{XG(I)}\\ \text{YG(I)}\\ \text{ZG(I)}\end{array}\right\}$ See Subroutine COLLSN

DELD(I) : The current vector $\Delta \underset{\sim}{q}^*_{m+1} = \underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_m$ of nodal displacement increments.

VEL(I) : The velocity corresponding to the Ith degree of freedom at the beginning of the current time step (from $t_m$ to $t_{m+1}$).

IMPCT1 : Index indicating if impact has been detected during the current execution of Subroutine IMPACT.
= 0 if no impact has been detected.
= 1 if impact has been detected.

The remaining variables are as defined in Subroutine IMAINP.

SUBROUTINE IMPDS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,NEFF,ALPHA, IMPCT1,NIAN)

Purpose: Controlling subroutine for impact inspection and interaction calculations for double-symmetry impact cases. The underlying flow and purpose are the same as Subroutine IMPACT.

Arguments: All arguments are the same as in Subroutine IMPACT.

SUBROUTINE IMPSS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,NEFF,ALPHA, IMPCT1,NIAN)

Purpose: Controlling subroutine for impact inspection and interaction calculations for single-symmetry impact cases.

Arguments: See Subroutine IMPACT.

SUBROUTINE INTRACT(S1,S2,A0,B1,B3,COEFR,FRNC,PTILDA)

Purpose: The impulse coordinates $\tilde{p}_{\bar{x}}, \tilde{p}_{\bar{y}}, \tilde{p}_N$ at the instant of termination of the collision are calculated in this subroutine. The relations used to calculate these values correspond to Cases I through III (Eqs. B.25 through B.28) of Subsection B.2.2.

Arguments:

| | | |
|---|---|---|
| S1, S2, A0, B1, B3 | | Interaction constants, $S_1, S_2, A_0, B_1$, and $B_3$ defined by Eqs. B.22. |
| COEFR, FRNC | | The coefficient of restitution, e, and coefficient of friction, $\mu$, respectively, between the plate and rigid spherical fragment surfaces. |
| PTILDA(I) | : | The impulse coordinates $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, and $\tilde{p}_N$ (I = 1,2,3, respectively) at the instant of termination of the collision. |

SUBROUTINE IPRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEL,EPEEO,EPSEI,EPSEO,XGI,YGI,NP,GBI,STDEP,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,NMSUB,SUBW,TAUSS,TAUEE,TAUSE,MAXEL,NZS,NXST,LNXS,MATXS,XSPROP,TAGSS,MNXST,NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,EK,AMASS,VEL,NNSA,NVSA,NCON)

Purpose: This subroutine controls the calculation and output of all quantities (displacement, strain, energies, etc.) as requested by the user at regular intervals during the timewise solution for the CIVM-PLATE program.

Arguments: All arguments are as defined in Subroutine PRINT in Subsection 3.2.1.

### 3.2.3 Subroutines Used by both the PLATE and the CIVM-PLATE Program

In the following, the subroutines used by both the PLATE and the CIVM-PLATE program are discussed.

SUBROUTINE ARRT(STF,AMASS,F,ICOL,INUM,KROW,NDEX,NODE,NDPE)

Purpose: In this subroutine, the mesh nodal connectivity data in NODE(I) is used to establish the pointer arrays necessary to locate terms in the assembled stiffness matrix (and mass matrix if consistent mass is used) stored in compacted vector form. The number of words of storage required for the assembled stiffness matrix is calculated, output, and compared with the user estimate; if the user estimate is too small, a message is output and the run terminated. The assembled stiffness and mass (only for lumped mass) matrices and force vector are zeroed.

Arguments:

F(I) : A force-type vector of length NDT, where NDT is the total number of degrees of freedom for the current plate structure being analyzed.

NDPE : The number of degrees of freedom per element (NDPE - 24 for the present plate elements).

All remaining arguments are as defined in Subroutine MAINP of Subsection 3.2.1.

SUBROUTINE ASSEMK(EK,STF,INUM,NDPE,NODE,N)

Purpose: Perform assembly operation of the element stiffness matrix into the assembled stiffness matrix.

Arguments:

EK(24,24) : The stiffness matrix for the current element.

STF(I), INUM(I), NODE(I) : See Subroutine MAINP of Subsection 3.2.1.

NDPE : See Subroutine ARRT.

N : The current element number being assembled.

SUBROUTINE BCONMK(STF,BC,ICOL,INUM,NBC)

Purpose: Imposes the boundary condition constraints (zero displacements) on the assembled matrix $[\frac{2}{(\Delta t)^2}\underset{\sim}{M} + \underset{\sim}{K}]$ given in Eq. A.114, by zeroing the row and column (not including the diagonal terms) corresponding to each constrained degree of freedom.

Arguments: See Subroutine MAINP of Subsection 3.2.1.

SUBROUTINE EAROW(STF,NBC,ICOL,INUM,RFM,ILAST,NBE,MBWE)

Purpose: This subroutine extracts the rows of the assembled matrix $[\frac{2}{(\Delta t)^2}\underset{\sim}{M} + \underset{\sim}{K}]$ (from the first nonzero term to the last nonzero term of that row) corresponding to each constrained degree of freedom (prior to constraining) and stores these values in the matrix RFM(I,J). The user estimate of maximum bandwidth at a constrained degree of freedom is checked; if the user estimate is too small, a message is output and the run is terminated.

Arguments: See Subroutine MAINP of Subsection 3.2.1.

SUBROUTINE FACT(STIFM,NCOL,KROW,NDEX,IDET,NTAPE6,IC)

Purpose: Performs triple-factorization on the assembled matrix $[\frac{2}{(\Delta t)^2}\underset{\sim}{M} + \underset{\sim}{K}]$ in Eq. A.114, putting it in the form $\underset{\sim}{L}^T\underset{\sim}{D}\underset{\sim}{L}$ where $\underset{\sim}{L}$ is a lower triangular matrix and $\underset{\sim}{D}$ is a diagonal matrix. The original matrix is destroyed in this operation. Details of the triple-factoring operation may be found in numerous texts on numerical methods and finite-element methods.

Arguments:

| | |
|---|---|
| STIFM(I)<br>NCOL(I)<br>KROW(I)<br>NDEX(I)<br>IC(I) | These arrays correspond to arrays STF(I),ICOL(I), KROW(I),NDEX(I), and INUM(I), respectively, as defined in Subroutine MAINP of Subsection 3.2.1. |

IDET : A parameter set equal to the number of negative diagonals in the assembled matrix being factored. If the matrix is not positive definite, IDET = -1 is set

NTAPE6 : The device number used for printed output. NTAPE6=6 is used in the PLATE and CIVM-PLATE programs.

SUBROUTINE GAUSS(N,GS,GW)

Purpose: This subroutine defines the location of the Gaussian integration stations and the corresponding weighting factors in one dimensional space. The number of Gaussian stations used may be from 1 to 6.

Arguments:

N : The number of Gaussian stations to be used.

GS(I) : The location of the N Gaussian stations relative to a normalized coordinate $-1 \leq s \leq 1$.

GW(I) : The corresponding Gaussian weights at the N Gaussian stations.

SUBROUTINE LMASSP(NTYPE,XL,YL,TH,SL,CSA,SMOI,EML,DENS)

Purpose: This subroutine forms the lumped (diagonal) mass matrix for a plate or stiffener element.

Arguments:

NTYPE : Element type

= 1 for a plate element (unstiffened)

= 2 for an x-direction stiffener element

= 3 for a y-direction stiffener element

XL, YL, TH : The x-length, y-length, and thickness, respectively, of the plate element (if NTYPE=1), or of the plate element to which the stiffener is attached (if NTYPE=2 or 3).

SL : If NTYPE=2 or 3, the normalized $\xi$ location ($0 \leq \xi \leq 1$) of an x-direction stiffener on the plate element (NTYPE=2), or the normalized $\eta$ location ($0 \leq \eta \leq 1$) of a y-direction stiffener on the plate element (NTYPE=3).

CSA, SMOI : The stiffener cross-sectional area and mass moment of inertia, respectively (required only if NTYPE=2 or 3).

EML(24) : The lumped mass matrix for the element.

DENS : The original mass per unit volume for this element ($lb\text{-}sec^2/in^4$).

SUBROUTINE MESHPA(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,YGI,XSPROP,YSPROP, NXST,NYST,IMESH,MATXS,MATYS)

Purpose: This subroutine controls input, formation, and output of mesh, stiffener, and boundary condition information when the auto-mesh generation option is chosen. A detailed discussion of this subroutine is given in Subsection 4.1.2.

Arguments:

IMESH : See Card 2 of Subsection 4.2.

NXST, NYST : See Subroutine PRINT of Subsection 3.2.1.

All remaining arguments are as defined for Subroutine MAINP of Subsection 3.2.1.

SUBROUTINE MESHPM(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,YGI,XSPROP,YSPROP, NXST,NYST,IMESH,MATXS,MATYS)

Purpose: Mesh generation subroutine (similar to MESHPA) when the manual mesh generation option is chosen. A detailed discussion of this subroutine is given in Subsection 4.1.3.

Arguments: Same as Subroutine MESHPA.

SUBROUTINE OMULT(SQVCT,RWVCT,ACC,NCOL,KROW,NDEX)

Purpose: This subroutine performs the matrix multiplication (ACC)=(SQVCT)*(RWVCT).

Arguments:

| | | |
|---|---|---|
| SQVCT(I) | : | A square (nxn) symmetric vector stored in lower triangle compacted vector form, in a fashion consistent with the pointers established in Subroutine ARR. |
| RWVCT(I) | : | A vector of length n of known quantities. |
| ACC(I) | : | The vector product (ACC)=(SQVCT)*(RWVCT) of length n. |
| NCOL(I) | : | Pointer array corresponding to ICOL(I) in Subroutine MAINP. |
| KROW(I)<br>NDEX(I) | | See Subroutine MAINP in Subsection 3.2.1. |

SUBROUTINE PRTOP(NNSA,NVSA,NCON,NP)

Purpose: In this subroutine, the print options to be executed at regular user-specified time intervals are established. Relevant data corresponding to various of these options is also defined. Those options available and the associated input are given on Cards 51 through 62 in Subsection 4.2.

Arguments:

| | | |
|---|---|---|
| NNSA | : | See Card 61 in Subsection 4.2. |
| NVSA(I)<br>NCON(I,J)<br>NP(I,J) | | See Subroutine MAINP in Subsection 3.2.1. |

SUBROUTINE PSTRN(G11,G22,G12,S,D)

Purpose: The maximum principal strain and associated direction (in degrees, with respect to the X axis) are calculated. Only principal <u>tensile</u> strains are considered.

Arguments:

| | | |
|---|---|---|
| G11<br>G22<br>G12 | | The strain components $\tilde{\varepsilon}_{xx}, \tilde{\varepsilon}_{yy}$, and $\tilde{\varepsilon}_{xy}$, respectively, at a point on the plate structure. |
| S | : | The maximum principal tensile strain. |
| D | : | The direction (degrees) of the maximum principal tensile strain (with respect to the X axis). |

SUBROUTINE RCON(F,NBC)

Purpose: Subroutine which applies boundary constraints to the force vector. Terms in the force vector corresponding to constrained degrees of freedom are zeroed (it is assumed that all prescribed displacements are zero).

Arguments:

| | | |
|---|---|---|
| F(I) | : | Force vector. |
| NBC(I) | : | List of those degrees of freedom which are constrained. |

SUBROUTINE RPLATE(NTYPE,ANU,XL,YL,TH,SL,CSA,SLAM,SMOI,KT,GBI,YMOD)

Purpose: The stiffness matrix for an unstiffened rectangular plate element (NTYPE=1), or an x-direction stiffener element NTYPE=2), or a y-direction stiffener element (NTYPE=3) are formed in this subroutine. The pertinent theoretical basis for the plate element is found in Subsection A.1.3.3. The theoretical basis for stiffener elements is found in Subsection A.2.3.

Arguments:

| | | |
|---|---|---|
| NTYPE | : | See Subroutine LMASSP. |
| ANU | : | Poisson's ratio, $\nu$, for the element material type. |
| XL<br>YL<br>TH<br>SL<br>CSA | | See Subroutine LMASSP. |

SLAM : The distance (in the z direction) from the centerline of the stiffener to the midsurface of the plate (for NTYPE=2 or 3).

SMOI : See Subroutine LMASSP.

KT(24,24) : The calculated element stiffness matrix.

GBI(24,24) : The matrix $\bar{\underset{\sim}{G}}^{-1}$ relating generalized displacement parameters $\underset{\sim}{\alpha}$ to normalized nodal degrees of freedom $\bar{\underset{\sim}{q}}$; see Eq. A.19.

YMOD : Young's modulus for the element material type.

SUBROUTINE SFDM(SL,EL,XL,YL,GBI,STD)

Purpose: This subroutine generates the row matrices $\underset{\sim}{D}_1^T$, $\underset{\sim}{D}_2^T$, ... $\underset{\sim}{D}_9^T$ of Eqs. A.10 used to evaluate strain components (or increments in strain) at a specified location within an element in terms of the element nodal generalized displacements.

Arguments:

SL, EL : The normalized location $\xi = \frac{x}{a}$, $\eta = \frac{y}{b}$, respectively, at which $\underset{\sim}{D}_1^T$, $\underset{\sim}{D}_2^T$, ... $\underset{\sim}{D}_9^T$ are to be evaluated.

XL, YL : The dimensions a and b of the element in the x and y directions, respectively.

GBI(24,24) : See Subroutine RPLATE.

STD(9,24) : The strain in terms of displacement matrix. The rows correspond to $\underset{\sim}{D}_1^T$, $\underset{\sim}{D}_2^T$, ... $\underset{\sim}{D}_9^T$ of Eqs. A.10 and the column correspond to $q_1$, $q_2$, $q_3$, ... $q_{24}$ (the element generalized degrees of freedom).

SUBROUTINE SOLV(STIFM,G,SOL,NCOL,KROW,NDEX)

Purpose: This subroutine performs the forward and backward substitution operations required to complete the solution of a system of linear simultaneous equations by the triple-factoring method. This subroutine must be used in conjunction with Subroutine FACT.

Arguments:

| | | |
|---|---|---|
| STIFM(I) | : | The factored coefficient matrix obtained from Subroutine FACT. |
| G(I) | : | The right-hand side vector (known) of the system of equations. |
| SOL(I) | : | The solution vector of the system of equations. |
| NCOL(I), KROW(I), NDEX(I) | | See Subroutine FACT. |

SUBROUTINE SPRING(EK,SS,ISIDE,XL,YL)

Purpose: This subroutine calculates the effective element stiffness matrix corresponding to line translational and torsional linear restoring springs applied along the side of a rectangular plate element. The theoretical basis for these calculations is given in Subsection A.1.3.5.

Arguments:

| | | |
|---|---|---|
| EK(24,24) | : | The effective element stiffness matrix corresponding to line restoring springs applied to an element. |
| SS(5) | : | The five elastic spring constants ($K_u$,$k_v$,$k_w$,$k_\theta$,$k_\psi$) of the line restoring springs. |
| ISIDE | : | The element side number (1,2,3, or 4) to which the line restoring springs are attached. |
| XL, YL | | The x and y dimensions, respectively, of the plate element to which the line springs are attached. |

SUBROUTINE STRESA(DISP,DELD,FLVA,ITT,NODE,MAXEL,MAX2,TSGEE,PLASTW,XGI,YGI,
NP,STDEA,GBI,NMSUB,DSR,PSR,SUBW,SNOM,NXST,XSPROP,LNXS,MATXS,NZS)

Purpose: This subroutine calculates the effective nodal force vector for an x-direction stiffener element corresponding to nonlinear geometric and material effects. The expression for this vector is given by Eq. A.97.

Arguments:

| | | |
|---|---|---|
| DISP(I) | : | The displacement vector $\underset{\sim}{q}^*_{m+1}$ at the current time step |
| DELD(I) | : | The vector of displacement increments $\Delta\underset{\sim}{q}^*_{m+1} = \underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_{m}$ |
| FLVA(I) | : | The vector of equivalent nodal forces for the structure corresponding to geometric and material nonlinear effects. |
| ITT | : | The current cycle number. |
| NODE(I)<br>MAXEL | | See Subroutine MAINP in Subsection 3.2.1. |
| MAX2 | : | Corresponds to MNXST on Card 1 of Subsection 4.2. |
| TSGEE(I,J,K) | : | Matrix to store current stress values at Gaussian integration (sublayer stations, corresponding to matrix TAGSS(I,J,K) of Subroutine MAINP in Subsection 3.2.1. |
| PLASTW | : | A scalar corresponding to plastic work contributions. |
| XGI(I)<br>YGI(I)<br>NP(I,J) | | See Subroutine MAINP in Subsection 3.2.1. |
| STDEA(9,24) | : | Strains in terms of displacement matrix corresponding to STD(9,24) of Subroutine SFDM. |
| GBI(24,24) | : | See Subroutine RPLATE. |
| NMSUB(I) | : | See Subroutine PRINT in Subsection 3.2.1. |
| DSR(I)<br>PSR(I) | | The strain-rate parameters, D and p, respectively, (see Eq. A.122) of the Ith material type. |
| SUBW(I,K) | : | See Subroutine PRINT in Subsection 3.2.1. |
| SNOM(I,K) | : | The static yield stress $\sigma_{o_k}$ for the kth mechanical sublayer of the Ith material type. |
| NXST | : | See Subroutine PRINT of Subsection 3.2.1. |

XSPROP(J,I)
LNXS(I)
MATXS(I) } See Subroutine MAINP in Subsection 3.2.1.

NZS : See Subroutine PRINT in Subsection 3.2.1.

SUBROUTINE STRESC(DISP,DELD,FLVA,ITT,NODE,MAXEL,MAX2,TSGEE,PLASTW,XGI,YGI,NP,STDEC,GBI,NMSUB,DSR,PSR,SUBW,SNOM,NYST,YSPROP,LNYS,MATYS,NZS)

Purpose: Calculates the effective nodal force vector for a y-direction stiffener element corresponding to nonlinear geometric and material effects (Eq. A.98).

Arguments: Many of the arguments are identical to those of Subroutine STRESA. Only those which are different are defined here.

MAX2 : Corresponds to MNYST on Card 1 of Subsection 4.2.

TSGEE(I,J,K) : See Subroutine MAINP in Subsection 3.2.1.

STDEC(9,24) : Strains in terms of displacement matrix corresponding to STD(9,24) of Subroutine SFDM.

NYST : See Subroutine PRINT in Subsection 3.2.1.

YSPROP(J,I)
LNYS(I)
MATYX(I) } See Subroutine MAINP in Subsection 3.2.1.

SUBROUTINE STRESP(DISP,DELD,FLVA,ITT,NODE,MAXEL,TAUSS,TAUEE,TAUSE,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,PLASTW,XGI,YGI,NP,STDEP,GBI,NMSUB,DSR,PSR,SUBW,SNOM,NZS)

Purpose: Calculates the effective nodal force vector for a plate element corresponding to geometric and material nonlinear effects (Eq. A.35). The theoretical basis for these calculations is given in Subsection A.1.3.2.

Arguments:

| | | |
|---|---|---|
| DISP(I)<br>DELD(I)<br>FLVA(I)<br>ITT | | See Subroutine STRESA |
| NODE(I)<br>MAXEL<br>TAUSS(I,J,K)<br>TAUEE(I,J,K)<br>TAUSE(I,J,K)<br>EPSSI(K,I)<br>EPSSO(K,I)<br>EPEEI(K,I)<br>EPEEO(K,I)<br>EPSEI(K,I)<br>EPSEO(K,I) | | See Subroutine MAINP in Subsection 3.2.1. |
| PLASTW | : | See Subroutine STRESA. |
| XGI(I)<br>YGI(I)<br>NP(I,J) | | See Subroutine MAINP in Subsection 3.2.1. |
| STDEP(9,24) | : | Strains in terms of displacement matrix corresponding to STD(9,24) of Subroutine SFDM. |
| GBI(24,24) | : | See Subroutine RPLATE. |
| NMSUB(I) | : | See Subroutine PRINT in Subsection 3.2.1. |
| DSR(I)<br>PSR(I) | | See Subroutine STRESA |
| SUBW(I,K) | : | See Subroutine PRINT in Subsection 3.2.1. |
| SNOM(I,K) | : | See Subroutine STRESA. |
| NZS: | : | See Subroutine PRINT in Subsection 3.2.1. |

SUBROUTINE TSTEP(STF,AMASS,NBC,BC,ICOL,INUM,KROW,NDEX,TRIAL,SOLN)

Purpose: Subroutine used to calculate a reference time-step bound for the Houbolt operator. The maximum natural frequency $\omega_{max}$ of the system is calculated from $\omega^2[M]\{q^*\} = [K]\{q^*\}$ using an iterative technique.+ The central-difference time-step bound $\Delta t_{max}^{CD} = 2/\omega_{max}$ corresponding to linear systems is calculated and output. To account for nonlinear effects, a reference time-step bound of $0.8(\Delta t_{max}^{CD})$ is calculated and output. See Subsection 2.3 for a discussion of the use of these reference time-step bounds.

Arguments:

| | | |
|---|---|---|
| STF(J) | : | The assembled stiffness matrix stored in compacted vector form (see Subsection 4.1.5). |
| AMASS(I) | : | The assembled mass matrix for the structure, stored as a vector. |
| NBC(J)<br>BC(J)<br>ICOL(K)<br>INUM(K)<br>KROW(K)<br>NDEX(K) | | See Subroutine MAINP in Subsection 3.2.1. |
| TRIAL(I)<br>SOLN(I) | | Trial and updated eigenvector solutions used in the iteration scheme. |

+The inverse power method is used to calculate $\omega_{max}$.

## SECTION 4

## USE OF THE PLATE PROGRAM

### 4.1 Discussion of Conventions Adopted in the PLATE Program

The PLATE program is designed to predict the large-deflection, elastic-plastic, transient responses of stiffened or unstiffened flat plates subjected to prescribed initial velocity distributions and/or prescribed externally-applied time-varying load distributions. The user-interaction required to execute a run of the PLATE code falls into three categories:

(1) Preparation of input data cards
(2) Assigning of array dimensions in the dummy MAIN program
(3) Preparation of internal logic for subroutines INCOND and EXTL.

Instructions for preparation of input cards are given in Subsections 4.2 through 4.4, for array dimensioning in Subsection 4.5, and for preparation of subroutines INCOND and EXTL in Subsection 4.6.

The PLATE program provides the user with the option of generating structural discretization information by using an automated mesh generation procedure or a manual mesh generation procedure. If the automated mesh generation procedure is employed, substantially less input data are required, and bookkeeping tasks such as nodal and element numbering, generation of constrained degrees of freedom, etc. are performed internally according to well-defined conventions. The purpose of the present subsection is to define those problems for which automated mesh generation can be employed, and to discuss the numbering conventions used when automated mesh generation is employed. Also discussed are the schemes adopted for internal renumbering of stiffeners and storage of the assembled stiffness matrix.

#### 4.1.1 Use of Auto-Mesh Generation vs. Use of Manual Mesh Generation

An automated mesh generation procedure has been provided in the PLATE

program in an attempt to simplify the preparation of input data; this should, in turn, result in fewer input data card errors. However, the auto-mesh generation procedure can be used only when all of the following conditions are met:

(1) The planform of the plate structure must be rectangular (or square)

(2) The plate must have no void regions (or cutouts)

(3) Prescribed (zero) displacement boundary conditions must be imposed only along the outer boundaries of the rectangular plate structure

(4) Only one boundary condition option (i.e., free, or clamped, etc.) may be specified on each side of the rectangular plate structure, and the specified boundary condition must apply along the entire side of the rectangular plate structure

If one or more of these conditions is not met, then the manual mesh-generation procedure must be employed. In either case, it should be noted again that the basic plate element used in the PLATE program is of rectangular planform; when using either the automated or manual mesh generation procedure, the plate structure must be modeled by a series of compatibly-connected rectangular elements.

### 4.1.2 Conventions Adopted in the Auto-Mesh Generation Procedure

When the auto-mesh generation procedure is employed, the numbering of element nodes, global nodes, elements, and plate sides is performed internally according to well-defined conventions; these conventions are illustrated in Fig. 10. The rectangular plate lies in the global X-Y plane, with the X-axis to the right and the Y-axis up, so that the global Z-axis would be out of the paper. The four sides of the plate structure are numbered sequentially in a counterclockwise fashion beginning with the side of the plate having the minimum Y=constant value. Note also that the sides of the plate structure and the sides of all plate elements are parallel to the X or Y axis. Elements and nodes are numbered sequentially beginning at the lower, left-hand corner

of the plate; the first row of nodes (elements) is numbered left to right, followed by the second row of nodes (elements) numbered left to right, the third row of nodes (elements) numbered left to right, etc. until all rows of nodes (elements) have been processed. When this process is completed, the number assigned to the node at the upper right-hand corner of the plate structure will be equal to the total number of nodes in the finite-element mesh. Similarly, the number assigned to the element in the upper, right-hand corner of the plate structure will be equal to the total number of elements in the finite-element mesh.

Once the global node numbers have been assigned, the global degree-of-freedom numbers are assigned by taking $(u,v,w,\theta,\psi,\chi)$ at global node number 1, followed by $(u,v,w,\theta,\psi,\chi)$ at global node number 2, etc., until all global nodes have been processed. The total number of degrees of freedom in the finite-element mesh, NDT, is thus given by NDT=6*NNT, where NNT is the total number of nodes in the finite-element mesh. As an example of the correspondence between global degree-of-freedom number and displacement parameter, the global degree-of-freedom number (NDF) assigned to the w displacment parameter at global node number 9 would be NDF=(9-1)*6+3=51. In general, the global degree-of-freedom number assigned to displacement parameter J (J=1,2, 3, ... 6 corresponding to u, v, w, $\theta$, $\psi$, and $\chi$ at a node, respectively) at global node number I is given by NDF=(I-1)*6+J.

For purposes of assembling element matrices (stiffness, mass, loads, etc.) into the global (system) matrices, certain "nodal connectivity" information is required. In particular, the auto-mesh procedure determines for each element the global node number associated with each of the four element nodes. As shown in Fig. 10, the element nodes are numbered counterclockwise 1 through 4 beginning with the element node in the lower left-hand corner of the element. As an example, the nodal connectivity information for element 5 of Fig. 10 would be as follows:

| ELEMENT NODE NUMBER | GLOBAL NODE NUMBER |
|---|---|
| 1 | 6 |
| 2 | 7 |
| 3 | 11 |
| 4 | 10 |

This nodal connectivity information is internally generated in the auto-mesh procedure. However, as will be discussed shortly, when the manual mesh generation procedure is employed, the user is required to input this nodal connectivity information.

A thorough understanding of the above discussed numbering conventions is essential for the accurate preparation of input data cards, dimensioning of arrays, and preparation of subroutines INCOND and EXTL. The numbers assigned to global nodes, elements, degrees of freedom, plate sides, etc. are never modified in the PLATE program. Thus, all input and output refers to this original numbering scheme.

Two types of meshes can be generated using the auto-mesh procedure: (1) meshes for which the side lengths of elements in the mesh differ (non-uniform mesh), and (2) meshes for which all elements in the mesh have the <u>same</u> side dimensions (uniform mesh). The significance of this "option" is in terms of the generation of the global (X,Y) coordinates of each node, and the location of the origin of the global X-Y axis system. If a <u>uniform</u> mesh is employed, the origin of the global X-Y axis system is <u>automatically</u> chosen to be at global node number 1 (i.e., global node number 1 has (X,Y) coordinates of (0,0)), and the coordinates of each node are automatically generated by knowing the number of element subdivisions and the plate dimensions along sides 1 and 4 of the plate. If a non-uniform mesh is employed, the origin of the global X-Y axis system is chosen by the user, and in order to generate the global X-Y coordinates of each node in the finite element mesh, the user is required to input the global X coordinate of each node along side 1 of the plate structure and the global Y coordinate of each node along side 4 of the plate structure.

For the auto-mesh procedure, prescribed displacement boundary conditions are generated by choosing a boundary condition option (see Subsections 2.1 and 4.2) for each side of the plate. A list of constrained degrees of freedom is then internally generated by processing each of the plate sides in order and identifying those degrees of freedom at each node of side 1, <u>including</u> the end nodes of side 1, which are constrained as a result of the boundary condition option selected for side 1 of the plate. In general, this list of

constrained degrees of freedom will contain duplicates because the corner nodes of the plate are processed twice (e.g., if side 1 of the plate is ideally clamped and side 2 of the plate is pin-fixed, then the degree of freedom numbers corresponding to displacement parameters u, v, w, $\psi$, and $\chi$ at the node where sides 1 and 2 meet will appear twice in the list of constrained degrees of freedom). Thus, a sorting procedure is applied to this original list of constrained degrees of freedom, such that the final list of constrained degrees of freedom contains no duplicates and the constrained degree of freedom numbers appear in ascending order.

The previous discussion pertains to the auto-mesh generation procedure only; a similar discussion pertaining to the manual-mesh generation procedure is given in the next subsection. Also, no mention has been made of stiffeners in the present subsection; a discussion of conventions for stiffeners for both auto and manual mesh generation procedures is given in Subsection 4.1.4.

#### 4.1.3 Discussion of the Manual Mesh Generation Procedure

The manual mesh generation procedure has been included in the PLATE code to accommodate as general a plate structure as possible. The only restrictions are that (1) the plate be of uniform thickness, (2) the plate be modeled by compatibly-connected rectangular elements, (3) the plate be initially flat, and (4) the plate lie initially in a plane parallel to the global X-Y plane. The price to be paid for this somewhat more general analysis capability is the increased effort required to prepare the input data. In particular, the user must assign element and global node numbers, input the nodal connectivity information, input the global coordinates of each node in the mesh, and input each constrained degree of freedom. Each of these areas will now be discussed.

The user is required first to define the mesh pattern chosen for the plate structure under consideration. Then the elements are numbered (from 1 to NET, where NET is the total number of elements in the finite-element mesh) using any convenient scheme. Global node numbers (from 1 to NNT, where NNT is the total number of nodes in the finite-element mesh) are then assigned to each node on the plate structure. Note that in order to minimize

the computer core storage requirements for the assembled stiffness matrix and mass matrix (if consistent mass modeling is chosen), the global nodes should be numbered in such a way as to minimize the average semi-bandwidth of the assembled stiffness matrix (see Subsection 4.1.5 for a discussion of the storage of the assembled stiffness matrix). Having defined the element and global node numbers, the nodal connectivity information for each element in the finite-element mesh may be prepared; that is, the global node number associated with the Ith element node (I = 1,2,3,4) is defined. It is important to note that the element nodal numbering convention discussed in Subsection 4.1.2 and shown in Fig. 10 must be used when the manual mesh generation is employed. Global degree-of-freedom numbers are assigned internally as is done in the auto-mesh generation procedure; that is, by taking $(u,v,w,\theta,\psi,\chi)$ at global node 1, followed by $(u,v,w,\theta,\psi,\chi)$ at global node 2, etc. until all global nodes have been processed.

The user is also required to input the global X and Y coordinates of each global node in the finite-element mesh. The origin of the global XYZ coordinate system may be conveniently chosen by the user. The positive coordinate directions must be chosen so that (1) when viewing the finite-element-modeled plate from the top, the global X direction is to the right and the global Y direction is up and (2) all element sides are parallel to either the global X or Y axis.

Prescribed displacement constraint conditions on the plate structure are defined by specifying the global degree-of-freedom number associated with each generalized nodal displacement parameter which is to be constrained (i.e., zero displacement). The global degree-of-freedom numbering convention is related to the global node numbering scheme chosen by the user as previously discussed. Any degree of freedom at any node of the structure may be constrained. The list of constrained displacement parameters corresponding to various physical constraint conditions, presented in Subsection 2.1, may be of assistance to the user. The user is required to input the constrained degree of freedom numbers in ascending numerical order with no duplicates.

The preceding discussion is intended as a general overview of the

manual mesh generation procedure. Detailed information regarding the use of the manual mesh procedure is given in Subsection 4.3. Also, the topic of stiffeners has not been discussed in the present subsection; a unified discussion of stiffener conventions for both the auto-mesh and manual mesh generation procedures is presented in the next subsection.

#### 4.1.4 Numbering Convention for Stiffeners

The PLATE computer program accommodates the analysis of integrally-stiffened flat plates where the stiffeners are required to be in directions parallel to the global X or Y axis. These stiffeners are designated as being either "complete" or "additional" stiffeners. Both complete and additional stiffeners may be specified when using the auto-mesh generation procedure, whereas only additional stiffeners may be specified when using the manual mesh generation procedure.

In order for a stiffener to be designated a complete stiffener, it must (1) span the plate completely, (2) be of uniform cross section, and (3) be made of a single material. Illustrative examples of complete stiffeners are given in Fig. 11. The properties of a complete stiffener are characterized by a knowledge of the global location of the centerline of the stiffener, its cross-sectional dimensions, the distance from the midplane of the plate to the neutral axis of the stiffener (offset distance), and the material properties of the stiffener (see Figs. 3, 11, and 12). In order for a stiffener to be designated an additional stiffener it must (1) completely span one element in the finite-element mesh, (2) be of uniform cross section, and (3) be made of a single material. Input information required for additional stiffeners includes cross-sectional dimensions, offset distance, material type, element on which the stiffener is located, and the relative position of the stiffener on the element (see Fig. 13).

In the PLATE program, the numbering of X-direction stiffeners is performed independently of the Y-direction stiffeners; i.e., the X-direction stiffeners are numbered from 1 to the total number of X-direction stiffeners and the Y-direction stiffeners are numbered from 1 to the total number of Y-direction stiffeners. In subsequent discussions in this subsection

of stiffener numbering, conventions for only X-direction stiffeners will be considered; however, the user should note that the discussion applies equally to Y-direction stiffeners.

As discussed in Subsection 2.1, a stiffener is modeled as a beam-type "element" compatibly attached to a flat plate element along lines parallel to the global X or Y axes. The stiffness and mass properties (as well as equivalent loads in the timewise solution) of the stiffeners are obtained by evaluating the element stiffness and mass matrices for each individual stiffener spanning a single plate element. The element matrices are then assembled into the global stiffness and mass matrices. The positioning of the stiffener element matrices in the global matrices corresponds exactly to the positioning in the global matrices of the element stiffness and mass matrices of the plate element to which the stiffener is attached.

As a result of the above procedure, all stiffeners (complete and additional) are redefined as "individual" stiffeners and assigned individual stiffener numbers; this redefining and renumbering of stiffeners is performed <u>internally</u> in the PLATE program. The properties of an individual stiffener are identical to those for an additional stiffener (i.e., completely span one plate element, be of uniform cross section, and be made of a single material). Thus, a complete stiffener (spanning the plate structure) may be considered to be a series of individual stiffeners. The convention used for renumbering stiffeners as individual stiffeners will be discussed next, first for the auto-mesh procedure and then for the manual mesh generation procedure. It is essential that the user understand this renumbering procedure because subsequent input data, various array dimensions, and all program output refers to the individual stiffeners. Again, note that the following discussion refers only to X-direction stiffeners, but applies equally to Y-direction stiffeners.

When using the <u>auto-mesh</u> generation procedure, both complete and additional X-direction stiffeners may be specified. Complete stiffeners are implicitly user-numbered (from 1 to NAST, where NAST is the total number of complete X-direction stiffeners) by the order in which the input data for complete stiffeners are specified. Similarly, additional stiffeners are

specified. Similarly, additional stiffeners are implicitly user-numbered (from 1 to NAXS, where NAXS is the total number of additional X-direction stiffeners) by the order in which the input data for additional stiffeners are specified. The following notation will be useful in the discussion of the stiffener renumbering procedure:

CSn = nth complete X-direction stiffener
ASn = nth additional X-direction stiffener
ISn = nth individual X-direction stiffener
NEAD = total number of plate elements in X-direction

The numbering convention for individual stiffeners is illustrated in Fig. 14 and is defined as follows. First CS1 is subdivided into a series of NEAD individual stiffeners numbered left to right from 1 through NEAD. Next, CS2 is subdivided into a series of NEAD individual stiffeners numbered left to right from (NEAD+1) to 2*NEAD. This process is repeated until each of the NAST complete X-direction stiffeners has been subdivided into individual stiffeners. In general, jth subdivision (j=1,2,3 ... NEAD) of the kth complete X-direction stiffener (k=1,2,3, ... NAST) will be assigned the individual stiffener number ISn=(CSk-1)*NEAD+j. After the processing of all complete X-direction stiffeners, individual stiffener numbers from 1 through (NAST*NEAD) will have been assigned. Next, the additional X-direction stiffeners (recall that <u>additional</u> stiffeners have the same properties as individual stiffeners) are processed in the order in which input data for additional stiffeners have been specified. That is, the numbering of individual stiffeners continues beginning with AS1, then AS2, etc., until all NAXS additional X-direction stiffeners have been renumbered. In general, the kth additional X-direction stiffener is assigned the individual stiffener number ISn=(NAST*NEAD)+k. The numbering of individual X-direction stiffeners is completed once all additional X-direction stiffeners have been processed. The total number of individual X-direction stiffeners (NXST) is thus given by NXST=NAST*NEAD+NAXS.

The above discussion applies equally to Y-direction stiffeners with NEAD replaced by NECD (the total number of plate elements in the Y-direction), NAST replaced by NCST (total number of complete stiffeners in the Y-direction),

and NAXS replaced by NAYS (total number of additional stiffeners in the Y-direction). Also, CSn, ASn, and ISn are now assumed to correspond to Y-direction stiffeners. The renumbering of Y-direction stiffeners results in the assigning of individual Y-direction stiffener numbers from 1 through NYST, where NYST is the total number of individual Y-direction stiffeners and is given by NYST=NCST*NECD+NAYS.

When the manual mesh generation procedure is used, only additional X and/or Y-direction stiffeners may be specified (i.e., the option for complete stiffeners is not included in the manual mesh generation procedure). Also, it should be recalled that the definitions of additional and individual stiffeners are identical. As a result, there is no need for internal renumbering of stiffeners when the manual mesh generation procedure is employed. The numbering of individual X-direction stiffeners is identical to the user-numbering scheme for additional X-direction stiffeners implicitly chosen by the order in which the data for stiffeners are input; the same is true for individual Y-direction stiffeners. Thus, the user is directly providing input data for the individual X and/or Y-direction stiffeners when using the manual mesh generation procedure.

### 4.1.5 Storage of the Assembled Stiffness Matrix

A brief description of the manner in which the assembled stiffness matrix, $\underset{\sim}{K}$ is stored is presented in this subsection. An understanding of this storage technique is required in order to provide the correct dimensions for the assembled stiffness matrix and assembled mass matrix (if consistent mass modeling is chosen).

The assembled stiffness matrix is formed in such a manner that the ith row of this matrix corresponds to the ith global degree of freedom, and the jth column of this matrix corresponds to the jth global degree-of-freedom number. In all cases the assembled stiffness matrix is symmetric, so that only the lower triangular portion of $\underset{\sim}{K}$ need be stored. In general, this matrix is also banded and sparsely populated. A portion of a hypothetical assembled stiffness matrix is shown in Fig. 15a; the left-hand boundary of the shaded area represents the first nonzero entry in each row and the

right-hand boundary of the shaded area represents the last nonzero in each row. For present purposes the total bandwidth of the ith row in $\underset{\sim}{K}$ is defined as the total number of entries in the ith row between the first nonzero entry and the last nonzero entry (inclusive), and the semi-bandwidth of the ith row in $\underset{\sim}{K}$ is defined as the total number of entries in the ith row between the first nonzero entry and the diagonal term (inclusive). In the PLATE program, the assembled stiffness matrix is stored as a vector (one-dimensional array) by rows, in order, beginning in each row with the first nonzero entry and ending with the diagonal term(i.e., the semi-bandwidth entries of each row are stored, row by row, in a vector; the leading zero terms in each row are not stored). For example, the storage of the hypothetical $\underset{\sim}{K}$ matrix of Fig. 15a is shown in Fig. 15b.

In order to dimension the vector containing the assembled stiffness matrix properly, the user must obtain an estimate of, for example, the average semi-bandwidth. This requires some knowledge of the "assembly" process in the finite-element method (see, for example, Ref. 10). Alternatively, an abbreviated run of the PLATE program can be made to obtain the exact storage requirements for the assembled stiffness matrix; guidelines are provided in Subsection 4.8.

## 4.2 Input Information and Procedure for the PLATE 1 Code Using Auto-Mesh Generation

The information required to punch a set of data cards for a run of the PLATE 1 computer program is presented in a step-by-step manner in this section. The variables to be punched on the nth data card are shown, and to the right is the format to be used for that card; the definition of and some restrictions for each variable are given directly below. This is done for each card, in turn, until all are described. The notation "Card" implies that only one card is required to input the data; the notation "card(s)" implies that one or more cards may be required to input the data.

Card 1 FORMAT

MAXEL, MNSL, MNXST, MNYST, NBE, MBWE, MNC 8I10

where

| | |
|---|---|
| MAXEL | The total number of elements in the finite-element mesh. |
| MNSL | The maximum number of mechanical sublayers employed to model the material nonlinear behavior. Note that the maximum number of mechanical sublayers permitted per material type is 5. |
| MNXST | The maximum number of individual X-direction stiffeners employed. Note that the total number of individual X-direction stiffeners is defined to be the number of X-direction stiffeners spanning the plate times the number of elements in the X-direction, plus the number of additional X-direction stiffeners spanning individual elements. If no X-direction stiffeners are present, set MNXST=1. |
| MNYST | Same as MNXST, except referring to Y-direction stiffeners. |
| NBE | If reaction force output is requested (i.e., IOP5=1 on Card 51), set this variable equal to the estimated total number of constrained degrees of freedom after all duplicates have been eliminated. If no reaction force output is requested (i.e., IOP5=0 on Card 51), then set NBE=1. |
| MBWE | If IOP5=1 (on Card 51), set this variable equal to the estimated maximum <u>total bandwidth</u> of the assembled stiffness matrix at a constrained degree of freedom. For a particular row of the assembled stiffness matrix, this number is equal to the number of terms (including zeros) in that row starting with the first non-zero term and ending with the last non-zero term of that row (see Subsection 4.1.5). If IOP5=0, set MBWE=1. |
| MNC | The estimated total number of words of storage required for the assembled stiffness matrix |

stored as the lower triangle from the first nonzero term to and including the diagonal term of each row (see Subsection 4.1.5).

Card 2

IMESH, IMASS, IMCONT, IPUNCH　　　　2I4

where

IMESH　Indicates the type of mesh being employed

=0　For a structure whose planform is rectangular (with no voids) and for which a mesh of non-uniform elements is employed.

=1　For a structure whose planform is rectangular (with no voids) and for which a mesh of uniform (equal-size) elements is employed.

=2　For all plate structures having non-rectangular planform and/or containing voids.

If IMESH=0 or 1 is specified, the automatic mesh-generation subroutine is employed, and the following input cards are followed (Cards 3 through 43).

If IMESH=2, the manual mesh-generation subroutine is employed; Cards 3 through 43 are replaced by the input cards given in Subsection 4.3.

IMASS　Indicates the type of mass matrix to be employed

=1　For lumped (diagonal) mass matrix

=2　For consistent (fully populated) mass matrix.

IMCONT　Indicates whether present run is a continuation of a previous run.

=0　if present run is not a continuation run.

=1　if present run is a continuation run.

IPUNCH　Control parameter which indicates whether or not a deck of continuation cards is to be punched following the final cycle of the present run.

=0 if no continuation deck is desired.

=1 if continuation deck is desired.

Card 3

NEAD, NECD 2I4

where

NEAD The number of elements in the X-direction

NECD The number of elements in the Y-direction

Card 4a

EPAN, ANUP, DENSP, TH, XDIST 5D16.7

Card 4b

YDIST, ZPOS

where

EPAN SIG(1)/EPS(1) given on cards 46 and 47 for the plate material (psi).

ANUP Poisson's ratio of the plate material.

DENSP Mass per unit volume of the plate material (lb-sec$^2$/in$^4$)

TH Total thickness of the plate (in)

XDIST The dimension of the plate in the X-direction (in).

YDIST The dimension of the plate in the Y-direction (in).

ZPOS The Z-location of the plate midsurface (in)

If IMESH=1, skip Cards 5 and 6.

Card(s) 5

XG(I), I=1, (NEAD+1)[+] 5D16.7

where

XG(I) The global X coordinates of the <u>first</u> <u>row</u> of nodes in the X-direction, left to right (plate side 1).

Card(s) 6

YG(I), I=1,(NECD+1) 5D16.7

[+]The input notation XG(I), I=1, (NEAD+1) indicates that Card(s) 5 should contain the values of XG(1), XG(2), XG(3) ... XG(NEAD), XG(NEAD+1). This notation is employed throughout the description of input.

where

YG(I) The global Y coordinates of the first column of nodes in the Y-direction, bottom to top (plate side 4).

Card 7

NCSB(1), NCSB(2), NCSB(3), NCSB(4) 20I4

where

NCSB(I) The displacement boundary condition applied to the Ith side of the plate (sides are numbered counterclockwise beginning with the side parallel to the X axis having the lowest Y=constant value as discussed in Subsection 4.1.2).

=0 Free

=1 Symmetry

=2 Ideally clamped

=3 Pinned, fixed

=4 Pinned, free sliding Z-direction tangent to boundary

=5 Pinned, free sliding normal (inplane) to boundary

A list of the nodal degrees of freedom constrained as a result of each of the options is given in Subsection 2.1.

Card 8

NAST, NCST 20I4

where

NAST The total number of complete stiffeners spanning the plate in the X-direction (a maximum of 30 such stiffeners is permitted).

NCST The total number of complete stiffeners spanning the plate in the Y direction (a maximum of 30 such stiffeners is permitted).

If NAST=0 (no X-direction stiffeners spanning the plate), skip Cards 9 through 16. If NCST=0 (no Y-direction stiffeners spanning the plate), skip Cards 17 through 24.

Card(s) 9

APCST(I), I=1, NAST 5D16.7

where

APCST(I) The global Y location of the centerline of the Ith complete X-direction stiffener which spans the plate (subsequently referred to as the Ith complete stiffener). Note that if IMESH=1 (uniform mesh) the origin of the global axis system is automatically located at the first global node of the plate structure (Fig. 11).

Card(s) 10

RHC(I), I=1, NAST 5D16.7

where

RHC(I) The total thickness (in) of the Ith complete X-direction stiffener (Fig. 12).

Card(s) 11

RWC(I), I=1, NAST 5D16.7

where

RWC(I) The width (in) of the Ith complete X-direction stiffener (Fig. 12).

Card(s) 12

ECS(I), I=1, NAST 5D16.7

where

ECS(I) Young's modulus (psi) of the Ith complete X-direction stiffener.+

Card(s) 13

ANUCS(I), I=1, NAST 5D16.7

where

ANUCS(I) Poisson's ratio of the Ith complete X-direction stiffener

Card(s) 14

DENCS(I), I=1, NAST 5D16.7

+The user must for consistency and correctness provide a value for ECS(I) which equals SIG(1)/EPS(1) for the material of which the stiffener is composed.

where

DENCS(I) The mass per unit initial volume of the Ith complete X-direction stiffener ($lb\text{-}sec^2/in^4$).

Card(s) 15

XFCST(I), I=1, NAST 5D16.7

where

XFCST(I) The distance (in) <u>from</u> the centroid of the Ith stiffener <u>to</u> the plate midsurface. If the direction of the vector from the stiffener centroid to the plate midsurface is the same as the direction of the global Z axis, then the sign of this quantity is positive. If the direction of the vector is opposite, the sign of this quantity is negative (Fig. 12).

Card(s) 16

MATCS(I), I=1, NAST 20I4

where

MATCS(I) The material type for the Ith complete X-direction stiffener (an integer value from 1 through 5). Note that the user will specify up to 5 different material types (and their associated linear or nonlinear material behavior) on Cards 45 through 48; the value of MATCS(I) indicates which of these (user-numbered) material descriptions applies to the Ith complete X-direction stiffener.

Card(s) 17

APCST(I), I=1, NCST 5D16.7

Card(s) 18

RHC(I), I=1, NCST 5D16.7

Card(s) 19

RWC(I), I=1, NCST 5D16.7

Card(s) 20

ECS(I), I=1, NCST 5D16.7

Card(s) 21

ANUCS(I), I=1, NCST 5D16.7

Card(s) 22

DENCS(I), I=1, NCST 5D16.7

Card(s) 23

XFCST(I), I=1, NCST 5D16.7

Card(s) 24

MATCS(I), I=1, NCST 20I4

where

| | |
|---|---|
| APCST(I)<br>RHC(I)<br>RWC(I)<br>ECS(I)<br>ANUCS(I)<br>DENCS(I)<br>XFCST(I)<br>MATCS(I) | The geometric and material properties of the Ith complete Y-direction stiffener. The definitions of these quantities are those given in Cards 9 through 16, except that they now refer to complete Y-direction stiffeners. |

Card 25

NAXS, NAYS 20I4

where

NAXS The total number of additional (see Subsection 4.1.4) X-direction stiffeners to be located on individual user-specified elements. Note that each of these individual X-direction stiffeners is assumed to span the individual user-specified element on which it is located.

NAYS The total number of additional Y-direction stiffeners to be located on individual user-specified elements.

If both NAXS and NAYS are greater than zero, Cards 26 through 43 are required next. If NAXS=0, then Cards 26 through 34 are skipped. If NAYS=0, then Cards 35 through 43 are skipped.

Card(s) 26

LNXS(I), I=1, NAXS 20I4

where

LNXS(I) The element number on which the Ith additional X-direction stiffener is located.

Card(s) 27

MATXS(I), I=1, NAXS 20I4

where

MATXS(I) The material type for the Ith additional X-direction stiffener (see note for Card 16).

Card(s) 28

XSPROP(1,I), I=1, NAXS 5D16.7

Card(s) 29

XSPROP(2,I), I=1, NAXS 5D16.7

Card(s) 30

XSPROP(3,I), I=1, NAXS 5D16.7

Card(s) 31

XSPROP(4,I), I=1, NAXS 5D16.7

Card(s) 32

XSPROP(5,I), I=1, NAXS 5D16.7

Card(s) 33

XSPROP(6,I), I=1, NAXS 5D16.7

Card(s) 34

XSPROP(7,I), I=1, NAXS 5D16.7

where

XSPROP(J,I) The geometric and material properties for the Ith additional X-direction stiffener.

| | |
|---|---|
| J=1 | Thickness (in) |
| J=2 | Width (in) |
| J=3 | Young's modulus (psi)$^{+}$ |
| J=4 | Poisson's ratio |
| J=5 | Mass per unit initial volume (lb-sec$^2$/in$^4$) |
| J=6 | The normalized location, $\eta_s$, of the stiffener centerline on the user-specified element (a value from 0.0 to 1.0 inclusive). Note that for the present calculation a <u>local</u> x-y axis system is chosen whose origin is at <u>element</u> node number 1. The value of $\eta_s$ is calculated as the location $y_s$ of the stiffener centerline in the local x-y system divided by the dimension of the <u>element</u> in the Y-direction (Fig. 13) |
| J=7 | The distance (in) from the centroid of the Ith additional X-direction stiffener to the plate midsurface. See Card 15 for definition of sign convention. |

Card(s) 35

LNYS(I), I=1, NAYS 20I4

where

LNYS(I) The element number on which the Ith additional Y-direction stiffener is located

Card(s) 36

MATYS(I), I=1, NAYS 20I4

where

MATYS(I) The material type for the Ith additional Y-direction stiffener (see note for Card 16).

---

$^{+}$Footnote on page 59 applies.

Card(s) 37

YSPROP(1,I), I=1, NAYS 5D16.7

Card(s) 38

YSPROP(2,I), I=1, NAYS 5D16.7

Card(s) 39

YSPROP(3,I), I=1, NAYS 5D16.7

Card(s) 40

YSPROP(4,I), I=1, NAYS 5D16.7

Card(s) 41

YSPROP(5,I), I=1, NAYS 5D16.7

Card(s) 42

YSPROP(6,I), I=1, NAYS 5D16.7

Card(s) 43

YSPROP(7,I), I=1, NAYS 5D16.7

where

YSPROP(J,I) The geometric and material properties for the Ith additional Y-direction stiffener.

- J=1 Thickness (in)
- J=2 Width (in)
- J=3 Young's modulus (psi)+
- J=4 Poisson's ratio
- J=5 Mass density ($lb\text{-}sec^2/in^4$)
- J=6 Normalized location, $\xi_s$, of the stiffener centerline on the user-specified element. The value of $\xi_s$ is calculated as the location, $x_s$, of the stiffener centerline in the local x-y system divided by the dimension of the element in the X-direction.
- J=7 The distance (in) from the centroid of the stiffener to the plate midsurface. See Card 15 for definition of sign convention.

+Footnote on page 59 applies.

Card 44

NMAT 20I4

where

| | | |
|---|---|---|
| NMAT | The total number of material types to be utilized in the present run. A maximum of 5 different materials is allowed. | |

Card 45

NSUB 20I4

where

| | | |
|---|---|---|
| NSUB | The number of mechanical sublayers used to model the uniaxial stress-strain behavior of the first type of material. A maximum of 5 mechanical sublayers is allowed. | |

Card 46

SIG(J), J=1, NSUB 5D16.7

Card 47

EPS(J), J=1, NSUB 5D16.7

where

| | |
|---|---|
| SIG(J)<br>EPS(J) | The Jth coordinate pair of the stress, $\sigma$, and strain, $\varepsilon$, respectively, which is used to define the polygonal approximation of the uniaxial stress-strain diagram (see Appendix A) for the first material type. The units for SIG(J) are psi, and for EPS(J) are in/in. The number of values appearing on each of Cards 46 and 47 should be equal to the number of mechanical sublayers, NSUB, defined on Card 45. |

Card 48

DSR(1), PSR(1) 5D16.7

where

| | |
|---|---|
| DSR(1)<br>PSR(1) | The values of the constants D and p, respectively, used in the strain-rate sensitivity formula (see Appendix A) for the first material type. If no |

strain-rate sensitivity is required, set these parameters equal to zero.

Cards 45 through 48 are repeated for each of the NMAT (Card 44) material types to be defined (i.e., material type number 1 is defined by the first set of Cards 45 through 48, material type number 2 is defined by the second set of Cards 45 through 48, etc.). It is important to note that the plate material must always correspond to material type number 1; the material type of each of the complete and/or individual stiffeners is specified by the user (see Cards 16, 24, 27, and 31). It should also be noted that the value of MNSL specified on Card 1 is equal to the maximum value of NSUB used to define the various materials.

Card 49

ITIMEF, INCRT, IOUT 20I4

where

| | |
|---|---|
| ITIMEF | The time-step number (or cycle number) at which the present run will terminate. |
| INCRT | The number of cycles between regular printout (i.e., print every INCRT cycles). |
| IOUT | A print parameter which is numerically equal to the value of INCRT. |

Card 50

DELTAT, TIMEF 5D16.7

where

| | |
|---|---|
| DELTAT | The time-step size, $\Delta t$ (seconds), used in the timewise solution.[+] |
| TIMEF | The time (seconds) at which the present run will terminate. |

Note that a run of the PLATE computer code will terminate when either the cycle number exceeds the value of ITIMEF (Card 49) or the time exceeds the value of TIMEF (Card 50).

Card 51

IOP1, IOP2, IOP3, IOP4, IOP5, IOP6, IOP7, IOP8 20I4

[+] See Subsections 2.4 and 4.8.2 for guidance in selecting an appropriate $\Delta t$.

The value of these parameters governs the type of output to be given at every regular printout cycle (i.e., every INCRT cycles). A value of 0 indicates that the particular print option is to be skipped; a value of 1 indicates that a particular print option is to be exercised. The output associated with each of these print options is as follows:

| | |
|---|---|
| IOP1 | Displacements and current (global) X,Y coordinates for each node. |
| IOP2 | Strain components, principal (tensile) strain and direction at the upper and lower surfaces evaluated at the spanwise centroid of each element. |
| IOP3 | Strain components, principal (tensile) strain and direction at the nine Gaussian spanwise stations (evaluated at upper and lower surfaces at each spanwise location) for user-specified element numbers. |
| IOP4 | Strain components, principal (tensile) strain and direction, and elongations (in user-specified directions) at additional spanwise locations (evaluated at upper and lower surface at each spanwise location) specified by the user. |
| IOP5 | Reaction forces at all nodes where displacement constraint conditions have been imposed. |
| IOP6 | Normal strain at the upper and lower surfaces at the three Gaussian spanwise locations for user-specified X- and/or Y-direction stiffeners. |
| IOP7 | System energies (i.e., work input to structure, structure kinetic energy, structure elastic energy, structure plastic energy, and energy stored in elastic restoring springs). |
| IOP8 | Strain components, principal (tensile) strain and direction at the upper and lower plate surfaces at user-specified <u>nodes</u>. Note that these quantities are calculated by nodal averaging. |

Some or all of Cards 52 through 62 will be required if any of options IOP3, IOP4, IOP6, or IOP8 are to be exercised (i.e., if any of these parameters are input equal to 1). However, Cards 52 and 53 should be skipped if IOP3=0, Cards 54 through 57 should be skipped if IOP4=0, Cards 58 through 60 should be skipped if IOP6=0, and Cards 61 and 62 should be skipped if IOP8=0.

Card 52

NEGS 20I4

where

| | |
|---|---|
| NEGS | The total number of plate elements for which Gaussian station strain output is desired. The value of NEGS cannot exceed 200. |

Card(s) 53

LNGS(I), I=1, NEGS 20I4

where

| | |
|---|---|
| LNGS(I) | A vector containing the element numbers for which Gaussian station strain output is desired (NEGS element numbers must be specified). |

Card 54

NASP

where

| | |
|---|---|
| NASP | The total number of <u>additional</u> spanwise locations on the plate structure where strain printout is requested. A maximum of 50 additional spanwise strain points is permitted. |

Card 55

LNASP(I), SLASP(I), ELASP(I) I4,2D16.7

where

| | |
|---|---|
| LNASP(I) | The element number on which the Ith additional strain point is located. |
| SLASP(I) <br> ELASP(I) | The $\xi$ and $\eta$ coordinates, respectively, of the Ith additional strain point. The normalized coordinates are related to the local coordinates, x-y (whose origin is at node 1 of the element) by |

$$\xi = \frac{x}{a} \quad , \quad \eta = \frac{y}{b}$$

where a is the length of the element in the X direction

and b is the length of the element in the Y direction.

Card 55 is repeated for <u>each</u> additional strain point until all NASP additional strain points have been processed. Note that the numbering of the additional strain points follows the order in which their element and coordinate locations are input.

Card 56

NASPE 2014

where

NASPE The total number of additional strain points at which the directions for calculation of elongation are to be specified by the user (NASPE $\leq$ NASP). Note that elongations in the directions 0° and 90° (in the X and Y directions, respectively) are automatically calculated and output for each additional strain point. The purpose of Cards 56 and 57 is to allow the user to specify other directions for elongation calculation, if desired, at some or all of the additional strain points.

If NASPE=0, skip Card 57.

Card 57

NSP, DIR1, DIR2 I4,2D16.7

where

NSP The number of the additional strain point at which the directions for calculations of elongations are to be specified.

| | |
|---|---|
| DIR1, DIR2 | The two directions (degrees) in which elongations are to be calculated at the NSPth additional strain point. Angles are measured from the X axis to the direction desired, with counter-clockwise rotations being positive. |

Card 57 is repeated for each of the NASPE additional strain points at which the directions in which elongations are calculated are specified by the user.

Card 58

NXSS, NYSS 2014

where

| | |
|---|---|
| NXSS, NYSS | The total number of individual X-direction and Y-direction stiffeners, respectively, for which Gaussian station strain output is desired. (Note: the values of NXSS and/or NYSS cannot exceed 200.) |

Card(s) 59 (skip if NXSS=0)

INXSS(I), I=1, NXSS 2014

where

| | |
|---|---|
| INXSS(I) | The X-direction stiffener numbers for which strain output is desired. |

Card(s) 60 (skip if NYSS=0)

INYSS(I), I=1, NYSS 2014

where

| | |
|---|---|
| INYSS(I) | The Y-direction stiffener numbers for which strain output is desired. |
| Note: | The data input on Cards 58 through 60 must correspond to the stiffeners as renumbered internally (e.g., each stiffener defined as spanning the |

plate is now treated as a series of individual stiffeners spanning individual elements). The rules used for the internal renumbering of stiffeners are described in Subsection 4.1.4.

Card 61

NNSA 20I4

where

NNSA — The total number of nodes at which strain output is desired.

If the value of NNSA is set equal to the total number of nodes in the finite-element mesh, NNT, then Card 62 is skipped. If NNSA is less than NNT, then Card 62 must be input.

Card 62

NVSA(I), I=1, NNSA 20I4

where

NVSA(I) — The global node numbers at which strain output will be given.

Card 63

NELES 20I4

where

NELES — The total number of element boundaries on which linear-elastic line restoring springs are located.

If NELES=0, skip Cards 64 and 65.

Card 64

LNUM, ISIDE 20I4

where

LNUM, ISIDE — The global element number and local element side number (see Fig. 10) on which linear-elastic line restoring springs are located.

Card 65

SS(1), SS(2), SS(3), SS(4), SS(5) 5D16.7

where

SS(I) The five elastic spring constants ($k_u$, $k_v$, $k_w$, $k_\theta$, $k_\psi$) corresponding to the line restoring springs located on side ISIDE of element LNUM. The displacement direction and units for each of these constants is as follows:

I=1 Translation u in X direction (lb/in per in. of span)

I=2 Translation v in Y direction (lb/in per in. of span)

I=3 Translation w in Z direction (lb/in per in. of span)

I=4 Rotation $\partial w/\partial x$ (in-lb/rad per in. of span)

I=5 Rotation $\partial w/\partial y$ (in-lb/rad per in. of span)

Cards 64 and 65 are repeated for each of the NELES element boundaries on which line restoring springs are located. Note that the distribution of spring stiffnesses along an element boundary is assumed to be constant, and that line restoring springs which span entire sides of the plate, or large portions of a side (encompassing a number of consecutive element boundaries) must be treated as a series of restoring springs spanning individual elements.

The preceding input cards (1 through 65) apply to plate structures having rectangular planform and no voids, and to runs which are not continuations of previous runs. The input modifications required for plate structures having non-rectangular planform and/or voids is given in Subsection 4.3. The additional input required for continuation runs is discussed in Subsection 4.4.

## 4.3 Modification to Input when Using Manual Mesh Generation

The description of input given in Subsection 4.2 applies to stiffened or unstiffened flat plate structures with rectangular planform and with no void regions (e.g. cutouts). For such structures, the generation of mesh, geometry, and boundary constraint information has been automated (subroutine MESHPA) to minimize the amount of input information which must be prepared by the user. However, for flat plate structures having non-rectangular planform and/or containing void regions, an alternate scheme (subroutine MESHPM) has been employed in which the user is required to input directly nodal connectivity, nodal coordinate, and constrained degree of freedom information. Although input preparation is more cumbersome for such cases, this scheme does provide the user with the flexibility to analyze more general plate structures.

Before preparing the input data, the user is required to assign a global element number to each element in the finite-element mesh, and a global node number to each node in the finite-element mesh. Also, a global XYZ coordinate system should be conveniently located. The value of IMESH on Card 2 of Subsection 4.2 should be defined as IMESH=2. The following input cards (3' through 28') replace cards 3 through 43 of Subsection 4.2. For convenience, a prime is used for the card numbers presented in this subsection.

Card 3' FORMAT

NNT, NET 2014

where

NNT The total number of nodes in the finite-element mesh

NET The total number of elements in the finite element mesh.

Card 4'

EPAN, ANUP, DENSP, TH, ZPOS 5D16.7

where

EPAN SIG(1)/EPS(1) given on cards 46 and 47 for the plate material (psi).

ANUP Poisson's ratio of the plate material.

| | |
|---|---|
| DENSP | Mass per unit initial volume of the plate material (lb-sec$^2$/in$^4$). |
| TH | Total thickness of the plate (in). |
| ZPOS | The Z location of the midsurface of the plate (in). |

Card 5'

NP(I,1), NP(I,2), NP(I,3), NP(I,4) 20I4

where

NP(I,J) The global node number associated with the Jth element node (J=1,2,3,4) of the Ith element. The convention for element node numbering is discussed in Subsection 4.1.3.

Card 5' is repeated (I=1,2,3 ... NET) for each of the NET elements, in order, until all elements have been processed.

Card(s) 6'

XG(I), I=1, NNT 5D16.7

Card(s) 7'

YG(I), I=1, NNT 5D16.7

where

XG(I), YG(I) The global X and Y coordinates, respectively, of the Ith global node.

Card 8'

NB 20I4

where

NB The total number of constrained degrees of freedom (assuming no duplicates).

Card(s) 9'

NBC(I), I=1, NB 20I4

where

NBC(I) A vector containing the global degree of freedom numbers which are to be constrained (set equal to zero for all time steps). The degree of freedom numbers input on Card(s) 9' <u>must</u> appear in ascending order with no duplicates.

Card 10'

NXST, NYST 2014

where

NXST, NYST: The total number of individual X- and Y-direction stiffeners, respectively, located on individual user-specified elements.

If NXST=0, Cards 11' through 19' are skipped.

If NYST=0, Cards 20' through 28' are skipped.

Card(s) 11'

LNXS(I), I=1, NXST 20I4

Card(s) 12'

MATXS(I), I=1, NXST 20I4

Card(s) 13'

XSPROP(1,I), I=1, NXST 5D16.7

Card(s) 14'

XSPROP(2,I), I=1, NXST 5D16.7

Card(s) 15'

XSPROP(3,I), I=1, NXST 5D16.7

Card(s) 16'

XSPROP(4,I), I=1, NXST 5D16.7

Card(s) 17'

XSPROP(5,I), I=1, NXST    5D16.7

Card(s) 18'

XSPROP(6,I), I=1, NXST    5D16.7

Card(s) 19'

XSPROP(7,I), I=1, NXST    5D16.7

where

| | |
|---|---|
| LNXS(I)<br>MATXS(I)<br>XSPROP(J,I) | The geometric and material properties of the Ith individual X-direction stiffener; the definition of these quantities is identical to that given in Subsection 4.2, Cards 26 through 34. |

Card(s) 20'

LNYS(I), I=1, NYST    20I4

Card(s) 21'

MATYS(I), I=1, NYST    20I4

Card(s) 22'

YSPROP(1,I), I=1, NYST    5D16.7

Card(s) 23'

YSPROP(2,I), I=1, NYST    5D16.7

Card(s) 24'

YSPROP(3,I), I=1, NYST    5D16.7

Card(s) 25'

YSPROP(4,I), I=1, NYST    5D16.7

Card(s) 26'

YSPROP(5,I), I=1, NYST    5D16.7

Card(s) 27'

YSPROP(6,I), I=1, NYST 5D16.7

Card(s) 28'

YSPROP(7,I), I=1, NYST 5D16.7

where

| | |
|---|---|
| LNYS(I)<br>MATYS(I)<br>YSPROP(J,I) | The geometric and material properties of the Ith individual Y-direction stiffener; the definition of these quantities is identical to that given in Subsections 4.2, Cards 35 through 43. |

Note: When using the manual mesh generation option, all stiffeners are treated as individual stiffeners located on individual user-specified elements. As a result, no internal renumbering of stiffeners is done, and any future reference to stiffener numbers should correspond to the numbering scheme chosen by the user in preparation of data Cards 11' through 19' (for X-direction stiffeners) and Cards 20' through 28' (for Y-direction stiffeners).

This completes those input modifications required when using the manual mesh generation option (IMESH=2). The remaining cards follow exactly those given in Subsection 4.2 starting with Card 44.

## 4.4 Additional Input Required for Continuation Runs

An option has been provided in the PLATE code which allows the user to terminate a computer run at a specified time step, and continue the analysis in a subsequent computer run beginning with the time cycle at which the previous run was terminated. This continuation option allows the user to carry out long-time analyses in a series of continuation runs (without starting each run from time zero) with the opportunity to scan the data provided by each run before continuing the analysis, if desired.

In order to continue a previous run, a deck of continuation cards must be punched when the previous run terminates. This continuation deck

is obtained by setting IPUNCH=1 on input card number 1 (see Subsection 4.2), and is used as input for the continuation run (as described below). The parameter IMCONT is set to IMCONT=1 on input card number 1 to indicate that the present run is a continuation of a previous run.

If a continuation run is being made, the deck of continuation input cards should follow immediately after Card 65 of Subsection 4.2. It is important to note that input Cards 1 through 65 should be identical to those used in the previous run (which is being continued) with the following exceptions:

1. ITIMEF on Card 49 must be set equal to the final cycle number for the present continuation run. Note that the cycle number is <u>not</u> reset to zero for a continuation run, but, rather, is initialized to the value at the end of the previous run.
2. INCRT and IOUT on Card 49 may be changed to obtain printout at intervals different from the previous run.
3. TIMEF on Card 50 must be set equal to the final time for the present continuation run.
4. The parameters on Card 51 which govern the type of output to be given every regular printout cycle may be redefined (except for the option IOP5, which must have the same value as in the previous run). The information given in data cards 52 through 62 must also be altered to reflect any changes made on Card 51.

The continuation deck starts with Card 66 and contains the following information:

| Card 66 | Format |
|---|---|
| ITT, CINETI, WFORCE, POWMP1 | I6,3D15.7 |

where

ITT — The cycle number at which the previous run was terminated.

CINETI — The kinetic energy imparted to the plate by the

prescribed initial velocity distribution at time zero.

WFORCE The work input to the structure.

POWMP1 The power = $\underset{\sim}{F}_n \dot{\underset{\sim}{q}}_n$ at time $t_n$.

Card(s) 67

DIS(I), I=1, NDT 5D15.7

where

DIS(I) The total displacement (or rotation) of degree of freedom I at time cycle ITT.

Card(s) 68

DISP(I), I=1, NDT 5D15.7

where

DISP(I) The total displacement (or rotational) of degree of freedom I at time cycle (ITT-1).

Card(s) 69

VEL(I), I=1, NDT 5D15.7

where

VEL(I) The velocity of the Ith degree of freedom at time cycle ITT.

Card(s) 70

FLVA(I), I=1, NDT 5D15.7

where

FLVA(I) The equivalent load, corresponding to nonlinear effects, applied to degree of freedom I at time cycle ITT.

If IOP5=0, Card(s) 71 are skipped.

Card(s) 71

UCF2(I), I=1, NB 5D15.7

where

UCF2(I) The force applied to the Ith constrained degree of freedom at time cycle ITT.

Card(s) 72

STOR(I), I=1, NDT 5D15.7

where

STOR(I) The prescribed externally-applied force at degree of freedom I at time cycle ITT.

Card(s) 73

TAUSS(I,J,K), (I=1, NET), (J=1, MNSLT4), (K=1,9) 5D15.7

Card(s) 74

TAUEE(I,J,K) 5D15.7

Card(s) 75

TAUSE(I,J,K) 5D15.7

where

TAUSS(I,J,K), TAUEE(I,J,K), TAUSE(I,J,K) The stresses $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$, respectively at the Jth depthwise/sublayer integration station (J is a function of both location and sublayer) at the Kth spanwise Gaussian integration station of the Ith element at time cycle ITT.

If no X-direction stiffeners are present, Card(s) 76 will not be included.

Card(s) 76

TAGSS(I,J,K), (I=1, NXST), (J=1, MNSLT4), (K=1,3) 5D15.7

where

TAGSS(I,J,K) The normal stress, $\sigma_{xx}$, at the Jth depthwise/sublayer integration station at the Kth spanwise Gaussian integration station of the Ith individual X-direction stiffener at time cycle ITT.

If no Y-direction stiffeners are present, Card(s) 77 will not be included.

Card(s) 77

TSGEE(I,J,K), (I=1, NYST), (J=1, MNSLT4), (K=1,3) 5D15.7

where

TSGEE(I,J,K) The normal stress, $\sigma_{yy}$, at the Jth depthwise/sublayer integration station at Kth spanwise Gaussian integration station of the Ith individual Y-direction stiffener at time cycle ITT.

## 4.5 Guidelines for User-Prepared Array Dimensions

In addition to providing input data cards for the PLATE program, the user is required to provide appropriate dimensions in the dummy main program for a number of vectors and arrays used in the program. The sole purpose of the dummy main program is to provide dimensions for those arrays and vectors which are variably dimensioned in subroutines of the PLATE program. The dummy main program then calls subroutine MAINP which is the main controlling subroutine for PLATE program flow.

The justification for the variable dimensioning of arrays and the corresponding need to provide fixed dimensions in the present dummy main program is based on program efficiency. Computer core storage requirements contribute, along with computer execution time, to the total cost of a run. In general, those vectors and arrays which contribute most to the computer core storage requirements have dimensions which are based on certain problem parameters (e.g. number of elements, number of nodes, number of mechanical sublayers, etc.) and, thus, are problem dependent. For 3-d plate analyses, where the dimensions and thus storage requirements grow rapidly as the number of elements is increased, maximum program efficiency (in terms of computer core storage) is obtained by providing dimensions for these arrays (vectors) which correspond exactly to the pertinent parameters of the plate structure being analyzed.

A complete listing of the dummy main program for the PLATE program is given on the following pages. The user need be concerned only with that (first) portion of the dummy main program related to array (vector) dimensioning. Shown

are the names and associated dimensions (given as variables and constants where appropriate) of those arrays which must be user-dimensioned. The definition of each dimension variable is given directly below the full list of arrays (vectors) to be dimensioned.

The user is required to prepare a FORTRAN IV DIMENSION statement which contains <u>all</u> of the arrays and vectors shown with the appropriate numerical dimensions substituted in place of dimension variables. This DIMENSION statement should be inserted after the list of dimension variable definitions as shown in the program listing. The dimension variables NNT, NPT, MAXEL, MNSL, MNXST, MNYST, NRSS, and MNNSA may be determined exactly, whereas the dimension variables MNC, NB, NBE, and MBWE (which are more difficult to determine exactly) may be over-estimated on the first run of a given problem and then adjusted to the correct values in subsequent runs of the same problem.

In no case may a dimension variable be set equal to zero; the minimum value for a dimension variable (all dimension variables are integers) is 1. Thus, for example, if no X-direction stiffeners are present then the dimension variable MNXST (maximum number of individual X-direction stiffeners) is set equal to MNXST=1 (rather than zero). There is no restriction on the maximum value which can be assigned to a dimension variable.

Seven of the dimension variables, namely MAXEL, MNSL, MNXST, MNYST, NBE, MBWE, and MNC, are given as input to the PLATE program (see Card 1 of Subsection 4.2). The values specified for these variables on Card 1 of the input data cards must be the same as the values assigned to these variables for dimensioning purposes.

Finally, the user should be aware of the fact that improperly dimensioned arrays (vectors) can be a chief source of anomalous results for structural response. In particular, if the value of a dimension variable is smaller than the correct value, then data from the corresponding array or vector may be incorrectly accessed during program execution. In such cases, program execution often proceeds without apparent error, but response predictions will generally be incorrect. Thus, great care should be taken by the user in preparing the DIMENSION statement for the dummy main program. Examples of proper DIMENSION statements may be found in the sample problems of Section 7.

```
C   PLATE 1 COMPUTER CODE.
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS IS THE SMALL STRAIN VERSION.
C
C ILLUSTRATIVE EXAMPLE FOR PLATE PROGRAM - FULL MODEL.
C
C THIS DUMMY MAIN PROGRAM FOR THE PLATE PROGRAM REQUIRES SUBROUTINES:
C        ARRT,ASSEMK,BCONMK,CMASSP,EAROW,EXTL,FACT,GAUSS,
C        INCOND,LMASSP,MAINP,MESHPA,MESHPM,OMULT,PRINT,
C        PRTOP,PSTRN,RCON,RPLATE,SFDM,SOLV,SPRING,STRESA,
C        STRESC,STRESP,TSTEP.
C
C   DUMMY MAIN PROGRAM WHICH PROVIDES DIMENSIONS FOR THOSE ARRAYS
C   (VECTORS) WHOSE DIMENSIONS ARE PROBLEM DEPENDENT.
C   THE FOLLOWING ARRAYS MUST BE DIMENSIONED BY THE USER AS SHOWN:
C     DIMENSION DELD(NDT),DIS(NDT),DISP(NDT),DISM1(NDT),DISM2(NDT),
C     FLN(NDT),FLVA(NDT),FLVM(NDT),FLVP(NDT),ELV(NDT),VEL(NDT),ICOL(NDT)
C     ,INUM(NDT),KROW(NDT),NDEX(NDT),STOR(NDT),STF(MNC),AMASS(MDIM),
C     NP(4,MAXEL),NODE(24*MAXEL),TAUSS(MAXEL,4*MNSL,9),
C     TAUSE(MAXEL,4*MNSL,9),TAUEE(MAXEL,4*MNSL,9),EPSSI(9,MAXEL),
C     EPSSO(9,MAXEL),EPEEI(9,MAXEL),EPEEO(9,MAXEL),EPSEI(9,MAXEL),
C     EPSEO(9,MAXEL),NBC(NB),BC(NB),RFM(NBE,MBWE),ILAST(NBE),UCF1(NBE),
C     UCF2(NBE),XG(NNT),YG(NNT),ZG(NNT),XGI(NNT),YGI(NNT),
C     TAGSS(MNXST,4*MNSL,3),LNXS(MNXST),XSPROP(7,MNXST),MATXS(MNXST),
C     TSGEE(MNYST,4*MNSL,3),LNYS(MNYST),YSPROP(7,MNYST),MATYS(MNYST),
C     LNRS(NRSS),ISRS(NRSS),SC(5,NRSS),NVSA(MNNSA),NCON(4,MNNSA)
C
C   WHERE,
C   NNT     = TOTAL NUMBER OF NODES IN THE ASSEMBLED FINITE-ELEMENT
C             MODEL.
C   NDT     = TOTAL NUMBER OF DEGREES OF FREEDOM IN THE ASSEMBLED
C             FINITE-ELEMENT MODEL = 6*NNT.
C   MNC     = ESTIMATED NUMBER OF WORDS OF STORAGE FOR THE ASSEMBLED
C             STIFFNESS MATRIX (LOWER TRIANGLE STORED BY ROWS FROM FIRST
C             NONZERO TERM).
C   MDIM    = DIMENSION OF ASSEMBLED MASS MATRIX.
C           = NDT IF LUMPED MASS OPTION IS CHOSEN.
C           = MNC IF CONSISTENT MASS OPTION IS CHOSEN.
C   MAXEL   = NUMBER OF ELEMENTS IN FINITE-ELEMENT MODEL.
C   MNSL    = MAXIMUM NUMBER OF MECHANICAL SUBLAYERS EMPLOYED.
C             NOTE-- MNSL MUST BE LESS THAN OR EQUAL TO 5.
C   NB      = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM PRIOR
C             TO ELIMINATION OF DUPLICATES.
C   NBE     = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM AFTER
C             ALL DUPLICATES HAVE BEEN ELIMINATED (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
C   MBWE    = ESTIMATED MAXIMUM BANDWIDTH OF ASSEMBLED STIFFNESS MATRIX
C             AT A CONSTRAINED DEGREE OF FREEDOM (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
```

```
C   MNXST  = MAXIMUM NUMBER OF X-DIRECTION STIFFENERS.
C          = 1 IF NO X-DIRECTION STIFFENERS.
C   MNYST  = MAXIMUM NUMBER OF Y-DIRECTION STIFFENERS.
C          = 1 IF NO Y-DIRECTION STIFFENERS.
C   NRSS   = TOTAL NUMBER OF ELEMENT SIDES ON WHICH LINE RESTORING
C            SPRINGS ARE LOCATED.
C          = 1 IF NO RESTORING SPRINGS ARE PRESENT.
C   MNSLT4 = 4*MNSL
C   MNNSA  = MAXIMUM NUMBER OF NODES REQUESTED FOR STRAIN OUTPUT (IF
C          IOP8=1). = 1 IF IOP8 = 0
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DELD(390),DIS(390),DISP(390),DISM1(390),DISM2(390),
     2FLN(390),FLVA(390),FLVM(390),FLVP(390),ELV(390),VEL(390),ICOL(390)
     3,INUM(390),KROW(390),NDEX(390),STOR(390),STF(14037),AMASS(14037),
     4NP(4,48),NODE(1152),TAUSS(48,12,9),TAUSE(48,12,9),TAUEE(48,12,9),
     5EPSSI(9,48),EPSSO(9,48),EPEEI(9,48),EPEEO(9,48),EPSEI(9,48),
     6EPSEO(9,48),NBC(60),BC(60),RFM(60,60),ILAST(60),UCF1(60),UCF2(60),
     7XG(65),YG(65),ZG(65),XGI(65),YGI(65),TAGSS(1,12,3),LNXS(1),
     8XSPROP(7,1),MATXS(1),TSGEE(1,12,3),LNYS(1),YSPROP(7,1),MATYS(1),
     9LNRS(1),ISRS(1),SC(5,1),NVSA(6 ),NCON(4,6 )
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C DEFINE CARD READER, LINE PRINTER AND CARD PUNCH UNIT NUMBERS.
      MREAD = 5
      MWRITE = 6
      MPUNCH = 7
      READ(MREAD,500)MAXEL,MNSL,MNXST,MNYST,NBE,MBWE,MNC
 500  FORMAT(8I10)
      WRITE(MWRITE,600)
      WRITE(MWRITE,610) MAXEL
      WRITE(MWRITE,620) MNSL
      WRITE(MWRITE,630) MNXST
      WRITE(MWRITE,640) MNYST
      WRITE(MWRITE,650) NBE
      WRITE(MWRITE,660) MBWE
      WRITE(MWRITE,670) MNC
 600  FORMAT('1 PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) :  USER INPUT
     + FOR ARRAY DIMENSIONS')
 610  FORMAT(' ','MAXEL  =',I10)
 620  FORMAT(' ','MNSL   =',I10)
 630  FORMAT(' ','MNXST  =',I10)
 640  FORMAT(' ','MNYST  =',I10)
 650  FORMAT(' ','NBE    =',I10)
 660  FORMAT(' ','MBWE   =',I10)
 670  FORMAT(' ','MNC    =',I10)
      MNSLT4=MNSL*4
      CALL        MAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,ELV,
     2VEL,ICOL,INUM,KROW,NDEX,STOR,STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS,
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON)
      CALL EXIT
```

```
      END                                                      MAIN0100
```

NOTE: 1. The dimension 6400 for arrays STF and AMASS was obtained by the procedure described in items 1 through 4 on page 100. Also, see Figs. 16 and 17a.

2. For detailed instructions on establishing dimensions and input data, see the example problems discussed in Section 7.

## 4.6 Guidelines for User-Prepared Subprograms

The timewise solution of the governing equations of motion for the structure (Eq. 2.1) is accomplished by employing the Houbolt finite-difference operator. As shown in detail in Appendix A, this results in the following matrix recurrence equation

$$[\frac{2}{(\Delta t)^2} \underset{\sim}{M} + \underset{\sim}{K} + \underset{\sim}{K}_s] \underset{\sim}{q}^*_{m+1} = \underset{\sim}{F}_{m+1} + \underset{\sim}{F}^{NL}_{m+1} + \frac{2}{\Delta t} \underset{\sim}{M} (\dot{\underset{\sim}{q}}^*_m + \frac{1}{\Delta t} \underset{\sim}{q}^*_m) \tag{4.1}$$

where $\underset{\sim}{M}$, $\underset{\sim}{K}$, $\underset{\sim}{K}_s$, $\underset{\sim}{F}$, $\underset{\sim}{F}^{NL}$, and $\underset{\sim}{q}^*$ are defined in Subsection 2.3, and $\dot{\underset{\sim}{q}}^*$ is the vector of nodal genealized velocities. The subscripts m+1 and m refer to quantities evaluated at discrete time instants $t_{m+1} = (m+1)\ \Delta t$ and $t_m = m\Delta t$, where m is the time step number. The matrices $\underset{\sim}{M}$, $\underset{\sim}{K}$, $\underset{\sim}{K}_s$, and vector $\underset{\sim}{F}^{NL}$ are internally generated and are assumed to be known.+ However, the vector $\underset{\sim}{F}_{m+1}$, which is the vector of prescribed externally-applied nodal loads at time $t_{m+1}$ must be provided by the user to represent the spatial and temporal distribution of the external load being applied to the plate structure.

In general, the displacement vector $\underset{\sim}{q}^*_m$ is known from the solution of Eq. 4.1 at time $t_m$, and the velocity vector, $\dot{\underset{\sim}{q}}^*_m$ is calculated from the Houbolt finite-difference expression and is thus also known. However, values of the displacement and velocity vector at time zero, $\underset{\sim}{q}^*_0$ and $\dot{\underset{\sim}{q}}^*_0$, required for the solution of displacements $\underset{\sim}{q}^*_1$ at time $t_1$ are not known and must be provided by the user as initial conditions on the governing matrix equation of motion (which is a second-order differential equation). These initial (time zero) conditions on the displacement and velocity vectors must correspond to the spatial distribution of displacement and velocity for the problem under consideration.

Subroutines INCOND and EXTL have been included in the PLATE program to provide, respectively, the values of $\underset{\sim}{q}^*_0$ and $\dot{\underset{\sim}{q}}^*_0$ (initial conditions), and the external loading vector, $\underset{\sim}{F}_{m+1}$, at each time step. Because of the many feasible spatial distributions for $\underset{\sim}{q}^*_0$, $\dot{\underset{\sim}{q}}^*_0$, and $F_{m+1}$ and temporal distribution of $\underset{\sim}{F}$, the writing of self-contained, automated logic for subroutines INCOND and EXTL has been deemed inefficient and impractical. Instead, the user is required to

+The vector $\underset{\sim}{F}^{NL}_{m+1}$ is estimated by linear extrapolation from earlier known values ($\underset{\sim}{F}^{NL}_m$ and $\underset{\sim}{F}^{NL}_{m-1}$) by $\underset{\sim}{F}^{NL}_{m+1} \doteq 2\ \underset{\sim}{F}^{NL}_m - \underset{\sim}{F}^{NL}_{m-1}$.

write the internal logic for these two subroutines corresponding to the problem under consideration.

Guidelines for the preparation of subroutines INCOND and EXTL are given in Subsections 4.6.1 and 4.6.2, respectively. Both subroutines must be prepared for each execution of the PLATE program even if the values of $\underset{\sim}{q}_0^*$, $\dot{\underset{\sim}{q}}_0^*$ and/or $\underset{\sim}{F}$ are zero. For the sake of simplicity, no READ statements should appear in either subroutine; all data should be specified in the user-prepared internal logic. FORTRAN IV WRITE statements should, however, be included in these subroutines to verify input data. A simple example of internal logic is given in each of Subsections 4.6.1 and 4.6.2; additional examples may be found in the sample problem of Section 7.

It is assumed that the user has a working knowledge of the FORTRAN IV computer language and thus no specific discussion of the programming language has been included. Although maximum flexibility has been maintained by requiring the user to prepare these subroutines, automated versions of subroutines INCOND and EXTL could be prepared by the user (or programming personnel) to accommodate a variety of initial condition and external loading options often encountered in a given analysis or design group.

### 4.6.1 User-Prepared Subroutine INCOND (PLATE Program)

The user is required to write (in FORTRAN IV) a subroutine which generates the initial velocity for each degree of freedom, VEL(I), I=1, NDT, and the initial displacement, DISP(I), I=1, NDT, for each degree of freedom in the finite-element mesh. This subroutine is called prior to the first time cycle for each run of the PLATE code. The subroutine should be prepared as follows:

```
SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DISP(1),VEL(1), NP(4),NODE(1), XGI(1),YGI(1)
COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      Insert User-Code to generate
      VEL(I),I=1,NDT
      DISP(I),I=1,NDT
RETURN
END
```

The first four cards and final two cards of subroutine INCOND must be as shown.

The arrays NP, NODE, XGI, and YGI, and variables NDT, NET are available to the user and contain the following information:

| | |
|---|---|
| NP(I,J) | The global node number associated with the Ith element node of element J. |
| NODE(K) | The assembly list associating element degrees of freedom with global degrees of freedom. If K is defined as K = 24*(J-1)+I, then NODE (K) is the global degree of freedom number associated with the Ith element degree of freedom of element J. |
| XGI(I), YGI(I) | The global X and Y coordinates, respectively, of the Ith global node at time zero. |
| NDT | The total number of degrees of freedom in the finite-element mesh. |
| NET | The total number of elements in the finite-element mesh. |

The information in these arrays may be used as required but must not be altered by the user-supplied internal logic.

The user should note that the conventions adopted for global node numbering are preserved for the present subroutine. Also note that in most cases (such as impulsively-loaded plates) the vector DISP(I), I=1,NDT is set equal to zero (zero initial displacement) and certain of the degrees of freedom are given an initial velocity. For example, the following set of statements generates zero initial displacement for all degrees of freedom, and an initial velocity of 2794 in/sec for the transverse w degree of freedom for nodes 1 through 9:

```
      DO  10  I=1,NDT
      DISP(I)  = 0.0D0
10    VEL(I)   = 0.0D0
```

```
       DO     20  I=1,9
       II         = (I-1)*6+3
20     VEL(II)    = 0.2794D+04
```

### 4.6.2 User-Prepared External-Loading Subroutine EXTL (PLATE Program)

The user is required to write a computer subroutine which generates the vector of prescribed externally-applied forces (ELV(I), I=1,NDT) applied to each global degree of freedom at time $t_m$. It should be emphasized that this vector of externally-applied (time-dependent) nodal forces corresponds to the total load applied at that instant of time. This subroutine is called for each cycle starting with the first cycle in the timewise solution. Thus, the user is required to define the spatial distribution of the external forces at each time instant (i.e., at each time t = ITT*DELTAT, where ITT is the current cycle number and DELTAT is the time step size, $\Delta t$). The subroutine should be prepared as follows:

```
      SUBROUTINE EXTL (ELV,ITT,NP,NODE,XGI,YGI)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ELV(1), NP(4,1), NODE(1), XGI(1), YGI(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,ITIMEF,INCRT,IOUT,TIME,ITIME
           Insert user-code to generate the spatial distribution
           of the external forces, ELV(I), I=1, NDT, at time
           t=ITT*DELTAT. Note that if no external forces are
           present in the current run of the PLATE code, the
           vector ELV(I), I=1, NDT must be set equal to zero.
      RETURN
      END
```

The arrays (vectors) NP,NODE,XGI, and YGI, and variables NDT and NET retain the definitions given in Subsection 4.6.1 (it should be emphasized, however, that XGI(I) and YGI(I) are the global X and Y coordinates, respectively, of the Ith global node at time zero) and the parameters ITT and DELTAT have been defined in the preceding discussion. It should also be noted that the contents of the vector ELV(I), I=1, NDT are modified after the call to subroutine EXTL.

As an example, assume that a nodal force, F, of constant amplitude, but varying in time, of the form

$$F = 100 \sin(25t)$$

is applied to the transverse w degree of freedom at nodes 1 through 5. The following set of statements could be inserted in subroutine EXTL to generate the necessary external loading vector at each time step:[+]

```
      DO   10   I=1,NDT
10    ELV(I)      = 0.0D0
      DO   20   I=1,5
      II=(I-1)*6+3
20    ELV(II)=100.0D0*SIN(25.0D0*ITT*DELTAT)
```

## 4.7 Description of the Output

The printed output begins with a reiteration of the program input which defines the problem being solved. Although portions of the output associated with mesh generation are different for the auto-mesh versus manual mesh options, sufficient information is given so that the user can verify the accuracy of the input data provided. The initial output which is generated on each run of the PLATE program is summarized in the following categories:

### Dimension Parameter Data

The variables input on Card 1 are output to verify these dimension parameters.

### Finite Element Mesh Information (Auto-Mesh Generation)

Geometric and material data for the plate and general mesh subdivision information is first reiterated. Next, the nodal connectivity information (which is the set of global node numbers assigned to the 4 nodes of each element) is output. The user-selected boundary condition for each side of the plate is then printed, and the specific constrained degrees of freedom are output in ascending order with no duplicates. Finally, if X and/or Y direction stiffeners are present, data for each stiffener are printed. These data are given in terms of each individual (as renumbered -- see Subsection 4.1.4)

[+] This information could be supplied in any other allowable FORTRAN form.

stiffener and include the element number on which the stiffener is located, material type, thickness, width, Young's modulus, Poisson's ratio, material density, normalized location on the element, and the offset distance of the stiffener.

Finite-Element Mesh Information (Manual-Mesh Generation)

This output is similar in form to that obtained when the auto-mesh generation procedure is employed. In this case, however, most of the output is simply a reiteration of data input by the user. This output should be used to verify the accuracy of the user-prepared input data.

Mass Modeling Chosen

A message is printed indicating which mass model (lumped or consistent) has been chosen by the user.

Storage of Global Stiffness Matrix

The actual number of words of storage required for the assembled stiffness is output. If too little storage has been allowed by the user, the run is terminated and a message indicating this is printed.

Stress-Strain Data for Each Material Type

The total number of material types selected by the user is output and the following data are then printed for each material type: the coordinate pairs (stress, strain) which define the piecewise linear approximation to the actual stress-strain curve, the yield stress of each mechanical sublayer, and the strain rate parameters for that material type.

Time-Step Information

The time-step size, $\Delta t$, final time and final increment number, and the number of cycles between regular printouts for the present run are output.

Print Options

A list is printed of those quantities which will be output at each regular print cycle. This information corresponds to those print options exercised by the user as well as all pertinent data necessary for any given option. For example, if option IOP3 is exercised, a list is printed of

plate element numbers for which Gaussian station strain output is requested.

Line Restoring Spring Data

If line restoring springs are present, input data corresponding to each linear-elastic line restoring spring is reiterated. These data include the element number and element side number on which the line restoring spring is located, and the five spring constants input by the user.

Reference Time-Step Size

An eigen-analysis is performed and the following reference data are generated and output: the highest natural frequency, $\omega_{max}$, for the equivalent linear finite-element model, a reference time step size, $(\Delta t)_{Ref.} = 2/\omega_{max}$ (which is the stability criterion for the Central Difference operator) of a corresponding linear dynamic system, and finally a reference time-step size, $(\Delta t^{CD}_{max})_{Ref} = 0.8(\Delta t)_{Ref}$; the factor 0.8 is introduced to account for large-deflection effects when using the Central Difference operator [1-3]. The PLATE program, however, utilizes the Houbolt finite-difference operator, and this final reference time step size, $(\Delta t^{CD}_{max})_{Ref}$, may be used to choose the actual time-step size as discussed in Subsections 2.3 and 4.8.2.

Dimensions Related to Reaction Force Output (If Applicable)

If reaction force output has been selected (IOP5), an operation is performed which extracts and stores in array RFM the rows of assembled stiffness matrix corresponding to the constrained degrees of freedom. During this operation, the extracted row of the assembled stiffness matrix which has the largest bandwidth is identified. This row number (which must correspond to a constrained degree of freedom number) and the corresponding bandwidth are printed. If the calculated maximum bandwidth at a constrained degree of freedom (printed) is larger than the user estimates (MBWE on data input Card 1; also used to dimension array RFM) of this quantity, the run is terminated and a message to this effect is printed; the user should make appropriate changes to MBWE on Card 1 of input data and to the dimension of RFM.

This completes the initial output generated by the PLATE program. All other output is in the form of structural response data at user-specified intervals; these data are also output prior to the first time cycle to verify initial geometry and displacement conditions, and at the end of the first time cycle. The data output corresponds to those output options selected by the user, and only that output selected by the user will appear. However, for the sake of completeness, the output generated by each of the print options is shown. The following output is generated every INCRT time cycles:

```
*** INCR. NO. =          TIME =          SEC.

NODE    U    V    W    PSIX   PSIY    TWIST    X-POS.    Y-POS.    Z-POS.
 1
 2
 3
 .
 .
 .

REACTION FORCES AT CONSTRAINED NODES+

NODE   RX(LBS)   RY(LBS)   RZ(LBS)   MX(LBS-IN)   MY(LBS-IN)   MXY(IN-IN-LBS)
4
13
24
.
.
.
.
```

+Note that the "reaction force" MXY multiplied by its associated generalized displacement $\chi$ results in a quantity with units of energy:IN-LB.

```
STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

      EPS-X STRAIN   EPS-Y STRAIN   SHEAR STRAIN   PRINC.STRAIN(T)   DIRECTION(DEG)
ELEM.INNER OUTER     INNER OUTER    INNER OUTER    INNER OUTER       INNER OUTER

  1
  2
  3
  .
  .
  .
```

```
STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

       EPS-X STRAIN   EPS-Y STRAIN   SHEAR STRAIN   PRINC.STRAIN(T)   DIRECTION(DEG.)
NODE   INNER OUTER    INNER OUTER    INNER OUTER    INNER OUTER       INNER OUTER
  7
 10
 14
 18
  .
  .
  .
```

```
STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT GAUSSIAN STATIONS
    FOR SELECTED ELEMENTS
RESULTS FOR ELEMENT NUMBER [LNUM]
    EPS-X  STRAIN  EPS-Y STRAIN   SHEAR STRAIN   PRINC. STRAIN(T)   DIRECTION(DEG.)
STA.INNER OUTER    INNER OUTER    INNER OUTER    INNER OUTER        INNER OUTER

 1
 2
 3
 .
 .
 .
```

(This set of statements is repeated for each user-specified element.)

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG.(DIR.1) | ELONG.(DIR.2) | PRINC.STRN(T) | DIRECTION(DEG) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | | | | | | | |
| 1 | OUTER | | | | | | | |
| 2 | INNER | | | | | | | |
| 2 | OUTER | | | | | | | |
| 3 | INNER | | | | | | | |
| . | | | | | | | | |
| . | | | | | | | | |
| . | | | | | | | | |

STRAIN, EPS-XX, AT GAUSSIAN STATIONS ON SPECIFIED X-DIRECTION STIFFENERS

| STIFFENER | STATION 1 | | STATION 2 | | STATION 3 | |
|---|---|---|---|---|---|---|
| NUMBER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 16 | | | | | | |
| 18 | | | | | | |
| 21 | | | | | | |
| . | | | | | | |
| . | | | | | | |
| . | | | | | | |

STRAIN, EPS-YY, AT GAUSSIAN STATION ON SPECIFIED Y-DIRECTION STIFFENERS

| STIFFENER | STATION 1 | | STATON 2 | | STATION 3 | |
|---|---|---|---|---|---|---|
| NUMBER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 2 | | | | | | |
| 3 | | | | | | |
| 10 | | | | | | |
| 13 | | | | | | |
| . | | | | | | |
| . | | | | | | |
| . | | | | | | |

SYSTEM ENERGIES (IN-LB)

| | | |
|---|---|---|
| WORK INPUT TO STRUCTURE | = | [EWORK] |
| STRUCTURE KINETIC ENERGY | = | [CINET] |
| STRUCTURE ELASTIC ENERGY | = | [ELAST] |
| STRUCTURE PLASTIC ENERGY | = | [PLASTW] |
| ENERGY STORED IN ELASTIC RESTRAINTS | = | [SPREN] |

where

| | |
|---|---|
| INCR. NO. | = Current cycle number |
| TIME | = Current time in seconds |
| NODE | = Global node number at which the quantity is being evaluated. For displacement output, all nodes all processed; for reaction force output, those nodes which are constrained are processed; and for nodal-average strain output, only those nodes specified by the user are processed. |
| U<br>V<br>W<br>PSIX<br>PSIY<br>TWIST | = The current displacement u, v, w, $\theta = \partial w/\partial x$, $\psi = \partial w/\partial y$, and $\chi = \partial^2 w/\partial x \partial y$, respectively, at the node. |
| X-POS<br>Y-POS<br>Z-POS | = The current global X, Y, and Z coordinates of a node. |
| RX (LBS)<br>RY (LBS)<br>RZ (LBS)<br>MX (LBS-IN)<br>MY (LBS-IN)<br>MXY (LBS-IN-IN) | = The generalized reaction forces, $R_x$, $R_y$, and $R_z$, and moments, $M_x$, $M_y$, and $M_{xy}$ at a constrained node. |

| | |
|---|---|
| EPS-X, EPS-Y, SHEAR | = The strain components $\tilde{\varepsilon}_{xx}$, $\tilde{\varepsilon}_{yy}$, and $\tilde{\varepsilon}_{xy}$, respectively; the printed shear strain $\tilde{\varepsilon}_{xy}$ is the engineering value -- which is twice its tensorial counterpart. |
| PRINC.STRAIN(T) | = The (maximum) principal tensile strain component. If all principal strains are compressive, a value of 0.0 is printed. |
| DIRECTION(DEG.) | = The direction in degrees, measured from the positive global X direction, of the maximum principal tensile strain. |
| INNER | = Indicates strain component is evaluated at the inner surface of the plate (or stiffener). |
| OUTER | = Indicates strain component is evaluated at the outer surface of the plate (or stiffener). |
| LNUM | = Element number for which Gaussian station strain output is given. |
| STA. | = Gaussian station number at which strain data is output. A 3 by 3 spanwise Gaussian integration rule is used and the strains are printed at each of these nine Gaussian stations. The location of these stations on an element is given in terms of normalized coordinates $\xi$ and $\eta$ and the local (element) x, y coordinate locations (assuming element node number 1 has local coordinates (x,y) = (0,0) can be obtained from the formulas |

$$x = a(1+\xi)/2$$
$$y = b(1+\eta)/2$$

where a and b are the x and y dimensions of the element $\xi$ and $\eta$ take on the following values at each station:

| STATION NUMBER | $\xi$ VALUE | $\eta$ VALUE |
|---|---|---|
| 1 | -0.774597 | -0.774597 |
| 2 | 0.0 | -0.774597 |
| 3 | 0.774597 | -0.774597 |
| 4 | -0.774597 | 0.0 |
| 5 | 0.0 | 0.0 |
| 6 | 0.774597 | 0.0 |
| 7 | -0.774597 | 0.774597 |
| 8 | 0.0 | 0.774597 |
| 9 | 0.774597 | 0.774597 |

POINT NO. = The number of the additional strain point; this number is implicitly assigned by the user by the order in which additional-strain point information is input.

ELONG. (DIR. 1), ELONG. (DIR. 2) = The relative elongation in the two user-specified directions

STIFFENER NUMBER = The user-specified individual X or Y-direction stiffener number for which strain data are output.

STATION 1, STATION 2, STATION 3 = Spanwise Gaussian station on the X or Y-direction stiffener at which strain is calculated.

EWORK = Work input to the structure.

CINET = Current kinetic energy stored in the structure.

ELAST = Current elastic strain energy stored in the structure

PLAST = Total plastic work done on the structure (mechanical work dissipated during plastic flow).

SPREN = Total energy stored in linear elastic restoring springs.

If continuation cards have been requested, a message is printed at the conclusion of the run indicating that a continuation deck has been punched and also indicating the cycle number at which the continuation will begin.

## 4.8 Guides and Restrictions for Code Usage

The sequence of steps necessary to run the PLATE program (i.e., preparation of input data cards, dimensioning of arrays in the dummy main program, and writing of internal logic for subroutines INCOND and EXTL) have been discussed earlier in this section. In principle, this information is sufficient to execute the PLATE program. However, in practice, use of the PLATE program may be unnecessarily costly unless the options and features of the PLATE program are used efficiently.

The purpose of the present subsection is twofold: first, to discuss various techniques for efficient code usage in terms of computation time, computer core storage, and program cost, and second, to discuss the proper interpretation of program output. The present discussion is intended only to point out certain key areas for efficient code usage; additional areas will be identified by the user through experience.

### 4.8.1 Dimension of the Assembled Stiffness Matrix

The largest single block of storage required in the PLATE program is for the assembled stiffness matrix, $\underset{\sim}{K}$. If consistent mass modeling is employed, an additional block of storage equal to that required for $\underset{\sim}{K}$, is also required for the assembled mass matrix, $\underset{\sim}{M}$.

The manner in which the assembled stiffness matrix is stored has been discussed in Subsection 4.1.5. The total storage requirements for $\underset{\sim}{K}$ are directly proportional to the total number of nodes in the finite-element mesh and the average semi-bandwidth of the assembled stiffness matrix. The total number of nodes can be reduced by using fewer elements in the mesh; generally, the mesh size will be dictated by geometry and accuracy considerations. However, the user should be alerted to the fact that an excessively-refined mesh will lead to excessive computer core storage requirements and computation time (the amount of computation time per time cycle is directly related to the total number of elements).

For a fixed mesh arrangement, the total storage requirements for $\underset{\sim}{K}$ can be reduced by minimizing the average semi-bandwidth of $\underset{\sim}{K}$. For a rectangular

plate, this is accomplished by numbering the global nodes so that the difference between the lowest and highest global node number assigned to any given element is minimized. When the auto-mesh generation routine is used, global nodes are numbered sequentially left to right for the first row of nodes, then left to right for the second row of nodes, etc. Thus, the global nodes are numbered "fast" in the X-direction and "slow" in the Y-direction. The average semi-bandwidth will be minimized if the "fast" numbering direction has fewer nodes than the "slow" numbering direction. When using the auto-mesh generation routine, the side of the rectangular plate having the fewest nodes should always be aligned parallel to the global X-direction (the fast numbering direction).[+] A similar argument can be made when using the manual mesh generation routine; number global nodes sequentially choosing the direction with fewest nodes as the "fast" numbering direction.[+]

The dimension, MNC, of K (vector STF in the PLATE program) can be estimated if the user has a knowledge of the assembly process in the finite-element method. Alternately, the PLATE program can be used directly to obtain the exact value of MNC for each new analysis as follows:

1. Prepare all input data, internal logic for subroutines INCOND and EXTL, and dimensions in the dummy main program for the analysis being considered, except
2. Set the value of MNC on input Card 1 equal to 1 and dimension STF(1) (and AMASS(1) if consistent mass modeling is elected).
3. Next run the PLATE program. The exact value of MNC will be calculated and output, and the run will terminate since the correct value of MNC is larger than the specified value (of 1 as per step 2) of MNC.
4. Change the value of MNC on Card 1 and the dimension of STF (and AMASS if consistent mass modeling is elected) to the correct value as calculated on the abbreviated run. Then proceed with the desired analysis.

If the user provides an estimate of MNC which is larger than the correct value, the analysis will proceed and the run will not be terminated. However,

---

[+] These guidelines have been followed in the present examples; see, for example, Figs. 17 and 19.

the correct value of MNC will always be calculated and printed as output so that the value of MNC used on Card 1 and for the dimension of STF (and AMASS if IMASS = 2) can be adjusted to the correct value for subsequent runs. Use of the correct value for MNC is advised to minimize storage requirements.

### 4.8.2 Selection of Time-Step Size

The total computation time is directly related to the number of time cycles, which is in turn related to the time step size $\Delta t$; as $\Delta t$ is decreased, more time cycles will be required to obtain solutions up to a given final time. Alternatively, the value of $\Delta t$ cannot be set arbitrarily large because of solution convergence and stability considerations (see Subsection 2.3).

To aid the user in the selection of $\Delta t$, the PLATE program will calculate and output a reference value for $\Delta t$. For each new structural analysis or each new mesh arrangement of the same structure, it is recommended that an abbreviated run of the PLATE program be made to obtain the reference $\Delta t$ as follows:

1. Prepare all input data cards, internal logic for subroutines INCOND and EXTL, and dimensions in the dummy main program for the actual analysis, except
2. Set ITIMEF=1 on Card 49 so that the program will terminate after one cycle of execution. The value of $\Delta t$ (DELTAT on Card 50) can be set equal to any value.
3. When the PLATE program is run, the reference value of $\Delta t$ will be calculated and printed as output. In subsequent runs, this value of $\Delta t$ can be used as a reference; ITIMEF on Card 49 and DELTAT on Card 50 should be set equal to the desired values for the actual analysis.

The significance of the calculated reference $\Delta t$ and its relation to "allowable" values of $\Delta t$ has been discussed in Subsection 2.3. For initial runs of a given problem, it is recommended that the actual value of $\Delta t$ used be between 2 and 6 times the reference $\Delta t$. In most cases, some numerical

experimentation will be necessary to determine if the solution has converged. It is recommended that the larger factor be used on the initial run, followed by a run using half the previous value of $\Delta t$. This process is repeated until the solution has converged. This convergence check can be carried out over only a portion of the desired analysis time to save computing costs, and the full time analysis can be carried out once a suitable value of $\Delta t$ has been found.

Experience gained by the user with the PLATE program will help to minimize the amount of numerical experimentation necessary in later analyses. It is recommended that a record be kept of calculated vs. converged (allowable) $\Delta t$ values. For example, experience gained to date suggests that for impulsively-loaded plates, a value of $\Delta t$ four times the reference value will yield converged solutions.

#### 4.8.3 Use of Lumped Mass vs. Use of Consistent Mass

The PLATE program provides an option for using either lumped or consistent mass modeling. The choice of mass modeling is not arbitrary and the user should understand the relative advantages and disadvantages of each.

Consistent mass modeling is, as indicated by its name, the more consistent approach from a variational standpoint. The kinetic energy of the plate element is approximated by employing an interpolation polynomial for the nodal velocities which is identical to that used for the nodal displacements. As shown in Appendix A, this approach leads to an element mass matrix which is in general fully populated; i.e., the kinetic energy includes coupling effects between various nodal velocities. The important fact is that the kinetic energy is based on an interpolation of velocities which is consistent with the interpolation of displacements used to define the potential energy.

On the other hand, lumped mass modeling is based on average element velocities rather than on interpolated element velocities. The average element velocities are calculated from a weighted average of the nodal velocities and a lumped (diagonal) mass matrix result. In effect, the total mass and mass moment of inertia of the element are calculated and then lumped at each node, the proportions going to each node being determined generally by element

geometry. For the present uniform-thickness, rectangular plate elements, equal portions (one-quarter of the total) are given to each node (see Appendix A for a detailed derivation).

The more important considerations, however, are computational efficiency and accuracy. The computer core storage requirement for an assembled consistent mass matrix is identical to that of the assembled stiffness matrix (i.e., MNC words of storage) whereas the assembled lumped mass matrix requires only NDT words of storage to store the diagonal entries.

In terms of computation time, slightly more time is required to perform operations with the banded assembled consistent mass matrix than is required with the diagonal assembled lumped mass matrix. Also, the reference $\Delta t$ is inversely proportional to the maximum frequency in the assembled finite-element model. The maximum frequency when using consistent mass modeling is larger than that calculated when using the present lumped mass modeling.[+] This could, in effect, reduce the maximum allowable time-step size for convergence and/or stability and thereby increase computation costs.

Because of its relative efficiency, lumped mass modeling is often used and comparative studies (e.g., Refs. 1 and 19-23) suggest that little or no loss in solution accuracy will be observed. Generally, computer core storage limitations dictate that lumped mass modeling be used. Theoretical-experimental correlation studies carried out using the PLATE program suggest that lumped mass modeling is both accurate and adequate when using relatively refined meshes. However, if computer storage limitations permit, the user may wish to carry out a series of comparative studies to assess the relative merits of the two mass models.

#### 4.8.4 Selection of Output Options

Numerous useful options have been provided for the user which calculate and print displacement, strain, reaction force, and system energy information at user-specified intervals. The definition of these eight options (IOP1, IOP2, ... IOP8) is given on Card 51 of Subsection 4.2. Each option is useful, but not always necessary, and the user should be aware of the cost vs. value of each option before it is specified. The purpose of the present subsection is to discuss the implications of each option in terms of lines of output,

[+] See Subsection 2.5.3 of Ref. 11 and Refs. 19-23.

computation time, and core storage. The question of the relative value of the output options (in particular for strain output) will be taken up in the next subsection.

In some installations, a portion of the cost of computing is related to the number of lines (or pages) of printout generated. In the following table, the number of lines of output generated by each output option at each printout cycle is given (note that these figures do not include headings):

| OPTION | DESCRIPTION | LINES/QUANTITY |
|---|---|---|
| IOP1 | Nodal displacements and current location | 1/Node |
| IOP2 | Element centroidal strains | 1/Element |
| IOP3 | Gaussian station strains on user-specified elements | 9/Specified Element |
| IOP4 | Strains at user-specified locations | 2/Specified Location |
| IOP5 | Reaction forces at constrained nodes | 1/Constrained Node |
| IOP6 | Gaussian station strains on user-specified stiffeners | 3/Specified Stiffener |
| IOP7 | System energies | 5 lines |
| IOP8 | Strains at user-specified nodes | 1/Specified Node |

In terms of computer core storage requirements, only Options IOP5 and IOP8 require additional storage (see Subsection 4.5); Option IOP8 requires 5*MNNSA additional words of storage and Option IOP5 requires NBE*(MBWE+3) additional words of storage, where the variables MNNSA, NBE, and MBWE are defined in Subsection 4.5. The use of Option IOP5 will require significantly more additional core storage than the use of Option IOP8.

The information printed when Options IOP1, IOP2, and IOP3 are exercised is available at the time of output and no additional computations are required. However, the information required for Options IOP4 through IOP8 must be calculated at each printout cycle before it can be printed; thus, additional computation time will be required if any of these options is exercised.

Experience to date suggests that these options do not add a significant amount of computation time, particularly if the number of printout cycles is only a small percentage of the total number of cycles (e.g. 1 printout cycle every 5 time cycles).

Total lines of printout and total additional computation time will be dependent on both the options selected as well as the total number of printout cycles requested. Both will often be dictated by the type of response information being sought and the degree of definition required in the time response. In general, it is recommended that displacement (IOP1) and system energy (IOP7) output be obtained for each run of the PLATE program. In view of increased storage and computation time, reaction force output (IOP5) should be sought only when necessary. Some form of strain output (IOP2, IOP3, IOP4, IOP6, and IOP8) should be obtained for each run of the PLATE program. A discussion of strain calculation and interpretation is given in the next subsection; it is important that the user understand these areas so that efficient use may be made of the strain output options.

### 4.8.5 Comments on Strain Calculation

The PLATE program provides options which allow the user to obtain strain output at element centroids, element Gaussian stations, nodal points, stiffener Gaussian stations, as well as other user-specified locations. This flexibility can be of great value to the user but care should be taken in the selection and interpretation of these strain results.

The strain-displacement relation employed for the present rectangular plate elements is given in Appendix A. Some nonlinear terms have been included in the membrane strains, but only linear terms are included in the bending behavior. Thus, the membrane nonlinearities have been assumed to be more significant than bending nonlinearities. The calculated distribution of strain may be quite different from element to element and in general will be different than the "exact" distribution; in the present finite-element model the predicted strain distribution approximates the actual strain distribution in an average (integral) sense, and not in a pointwise sense. Although the calculated strain distribution <u>may</u> agree with the exact distribution at one or more points within

an element, the location of these "best" points for monitoring strains cannot be readily determined. Also, strains will not be continuous along interelement boundaries or at nodes where two or more elements are joined.

Because of these facts, certain precautions should be taken by the user when assessing strain distributions in space and/or time. The type and amount of strain output will depend on the problem being considered and the region of the structure under closest scrutiny. If general strain distribution informa-

tion is sought over a large portion of the structure. If detailed stress distribution information at various time instants is sought over a small region of the structure, then both nodal average strains and Gaussian station strains for elements in this small region could be employed. In either case, when these calculated values are plotted, the analyst can then make a reasonable "faired" estimate of the "proper" distribution. It should be noted that severe strain gradients within an element do not necessarily indicate poor behavior of the solution; however, it is in these regions where the analyst must excercise the greatest caution in making a reasonable faired estimate of the proper distribution.

If strain time-history information is required at various points on the structure, these points may be specified as additional strain points (if the point does not already fall at or near a nodal point or element centroid) and the time histories can be obtained directly. Two recommendations should, however, be followed whenever strain time-history data are sought. First, it is recommended that spatial distributions near these points of interest be obtained at several time instants to assess whether or not the strain at the point of interest is in reasonable agreement with the curve-fitted (or faired) distribution in that region of the structure. Second, if a point of interest falls near a node, nodal average strain information at that node should be preferred over the strain information obtained <u>near</u> the node (which is not averaged); in general, for elements (such as those in the PLATE program) exhibiting discontinuous strains at interelement boundaries (and therefore also at nodes), average strain values at nodes (or interelement boundaries) should

be preferred over the strain values evaluated near the node (or interelement boundary) using only the strain distribution from one element. If these steps are followed, a reasonable engineering assessment of strain information should be obtained.

The equations in Appendix A have been developed within the assumption of large deflections but small strains. Thus, reliable results may not be obtained in localized regions where large strains are predicted. The actual strain level at which the "small strain" assumption becomes invalid has been examined and discussed in Ref. 14; the reader is referred to Ref. 14 for further information and guidance.

## SECTION 5

## USE OF THE CIVM-PLATE PROGRAM

### 5.1 Introduction

The CIVM-PLATE computer program is designed to predict the large-deflection, elastic-plastic, transient responses of initially-flat stiffened or unstiffened plates subjected to impact by a single idealized rigid spherical fragment. Much of the programming logic used in the CIVM-PLATE computer program is identical to that used in the PLATE program, as evidenced by the large number of computer subroutines common to both programs (Section 3).

The two key differences between the PLATE code and the CIVM-PLATE code are: (1) the PLATE program is designed to predict transient responses of initially-flat plate structures subjected to prescribed initial velocity distributions and/or prescribed externally-applied forces, whereas the CIVM-PLATE program is designed to predict transient responses of initially-flat plates subjected only to single fragment impact, and (2) the PLATE program allows the user to choose either consistent or diagonalized lumped mass modelling, whereas the CIVM-PLATE program uses only a diagonalized lumped mass model.

User interaction with the CIVM-PLATE program is reduced (by comparison with that required for the PLATE program) to the following two tasks:

(1) Preparation of input data cards
(2) Dimensioning of arrays in the dummy main program.

These two tasks are essentially identical to the corresponding tasks required when using the PLATE program. In the PLATE program, the user is required to prepare internal logic for subroutines INCOND and EXTL to define the initial velocity distribution and externally-applied force distribution. These types of loading are not included in the CIVM-PLATE program and thus no preparation of internal logic is required to use the CIVM-PLATE program; instead, the user is required to prepare additional input data cards (as described in Subsection 5.2) to define the geometry, initial position, and initial velocity components of the rigid spherical fragment, and several collision parameters.

With respect to structural geometry, nomenclature, and numbering conventions, material properties, time-step information, and output options, the CIVM-PLATE program is identical to the PLATE program. The discussions in Subsection 4.1 on the use of auto-mesh generation versus the use of manual mesh generation, conventions adopted in the auto-mesh and manual-mesh generation subroutines, numbering convention for stiffeners, and storage of the assembled stiffness matrix apply equally to the CIVM-PLATE program; the user may wish to review this information before attempting to run the CIVM-PLATE code. Also, a general overview of the collision-interaction analysis and solution procedure is given in Subsections 2.4 and 2.5, and the equations on which the CIVM-PLATE program is based are developed and discussed in Appendix B.

## 5.2 Input Information and Procedure

The structural geometry, nomenclature, material properties, output options and time step input required for a run of the CIVM-PLATE code is identical to that required for a run of the PLATE code. For a run of the CIVM-PLATE code using the auto-mesh generation procedure, cards 1 through 65 of Subsection 4.2 are input exactly as given in Subsection 4.2 with the following modifications/additions:

(1) Cards 1 and 2 of Subsection 4.2 are modified as shown below.

(2) Cards 66 through 72 are added following card 65 of Subsection 4.2

If the manual mesh generation procedure is used, the input data presented in Subsection 4.2 are modified exactly as described in Subsection 4.3; the above modifications/additions required for the CIVM-PLATE program still apply. The variables to be punched on the nth data card are shown, and to the right is the format to be used for that card; the definition of and some restrictions for each variable are given below.

| Card 1 (Replaces Card 1 of Subsection 4.2) | Format |
|---|---|
| MAXEL, MNSL, MNXST, MNYST, NBE, MBWE, MNC, NIAN | 8I10 |

where

| | |
|---|---|
| MAXEL, MNSL, MNXST, MNYST, NBE, MBWE, MNC | Array dimension parameters as defined on Card 1 of Subsection 4.2 |
| NIAN | An array dimension parameter used in the dummy main program and defined as the estimated maximum number of nodes which will be affected by a single impact. |

Card 2 (Replaces Card 2 of Subsection 4.2) 20I4

IMESH, IMCONT, IPUNCH

where

| | |
|---|---|
| IMESH, IMCONT, IPUNCH | Mesh, continuation, and punched output options as defined on Card 2 of Subsection 4.2 |

Card 66 (Follows Card 65 of Subsection 4.2)

XF, YF, ZF 5D16.7

where

| | |
|---|---|
| XF, YF, ZF | The global X, Y, and Z coordinates respectively, of the centroid of the spherical fragment at time zero. These coordinates must be chosen so that no plate/fragment overlap occurs at time zero. |

Card 67

VF(1), VF(2), VF(3) 5D16.7

where

| | |
|---|---|
| VF(1), VF(2), VF(3) | The translational velocity components (in/sec) of the fragment in the global X, Y, and Z directions, respectively, at time zero. |

Card 68

OMEGF(1), OMEGF(2), OMEGF(3) 5D16.7

where

| | |
|---|---|
| OMEGF(1)<br>OMEGF(2)<br>OMEGF(3) | The rotational velocity components (rad/sec) of the fragment about the global X, Y, and Z directions, respectively, at time zero. |

Card 69

RF, FMASS, FMOI 5D16.7

where

| | |
|---|---|
| RF | The radius of the spherical fragment (in). |
| FMASS | The total mass of the spherical fragment ($lb\text{-}sec^2/in$). |
| FMOI | The mass moment of inertia of the fragment about its c.g. ($lb\text{-}sec^2\text{-}in$). |

Card 70

COEFR, FRNC, LEFF 5D16.7

where

| | |
|---|---|
| COEFR | The coefficient of restitution between the fragment and the impacted plate inner surface; $0 \leq COEFR \leq 1$, 1 for perfectly elastic, 0 for perfectly inelastic, $0<COEFR<1$ for intermediate. |
| FRNC | The coefficient of friction between the fragment and the plate inner surface. |
| LEFF | The radius of a circle from the point of impact on the plate by the fragment to the "boundary of the impact-affected region for a single impact". |

= 0 means that $L_{eff}$ is assumed to be $\left[\frac{E}{\rho_o(1-\nu^2)}\right]^{1/2}\Delta t$ and is calculated internally, where E is the elastic modulus, $\nu$ is the Poisson ratio, $\rho_o$ is the mass per unit initial volume of the plate material, and $\Delta t$ is the time-step size.

= a specified value that is used in place of and supersedes the above estimate for $L_{eff}$; as argued in Subsection 2.5.2 of Ref. 11, a physically

plausible value is $L_{eff}$ = nh independent of the time step size, where h is the plate thickness and n may be specified as some positive number such as 2, 2.5 ... or some other selected value (n=2 is often used). Any other prescribed value could be selected by the user.

Card 71

NSYM, IPSS, ICUT 2014

where

NSYM A parameter which indicates the type of symmetry option elected for the present impact analysis
= 0 if no symmetry option is employed.
= 1 if the single symmetry option is employed.
= 2 if the double symmetry option is employed.

IPSS A parameter used in conjunction with the single symmetry option (if NSYM=1) to denote the plate side number which is a line of symmetry.
= 0 if NSYM=0 or NSYM=2 is employed
= 1 if plate side number 1 (i.e. the global X-axis) is the line of single symmetry.
= 4 if plate side number 4 (i.e. the global Y-axis) is the line of single symmetry.

ICUT A parameter which indicates whether or not impact inspection and correction is to be followed for the entire response history.
= 0 if impact inspection and correction is to be followed at all time instants.
= 1 if impact inspection and correction is to be terminated once the fragment kinetic energy is less than a specified fraction of the fragment kinetic energy at time zero.

The single symmetry option (i.e. NSYM=1) can be exercised only when the following conditions are satisfied:

(1) The plate is rectangular (or square) with no cutouts, and half of the plate is modeled (i.e. the analysis is carried out for one half of the plate with one boundary of the half-plate model being a symmetry boundary).

(2) Initial and subsequent fragment/plate collisions occur along the line of symmetry (plate side 1 if IPSS=1, or plate side 4 if IPSS=4).

(3) The initial position and velocity components for the fragment must be specified in a fashion consistent with the assumption of motion which is symmetric with respect to the global X axis (if IPSS=1) or with respect to the global Y axis (if IPSS=4).

These conditions and other conventions adopted in the CIVM-PLATE code dictate some of the input which must be specified as follows (if NSYM=1 on Card 71):

(1) IMESH on Card 2 must be set equal to 0 or 1 so that the auto-mesh generation subroutine is exercised.

(2) If IMESH=0 is specified, then the global X and Y coordinates of node 1 of the structure must be specified to be 0.0 (locating the origin of the global X-Y system at node 1). This is accomplished by setting XG(1)=0.0 on Card 5 and YG(1)=0.0 on Card 6.

(3) The symmetry boundary condition must be chosen for the plate side which is the line of symmetry. This is accomplished by setting NCSB(1)=1 on Card 7 if IPSS=1 (global X axis is the line of symmetry) or by setting NCSB(4)=1 on Card 7 if IPSS=4.

(4) The centroid of the fragment must be located along the line of symmetry. Thus, YF=0.0 must be specified on Card 66 if IPSS=1 or XF=0.0 must be specified on Card 66 if IPSS=4.

(5) The translational velocity components of the fragment must be consistent with the single symmetry option. If IPSS=1 (global X axis is the line of symmetry) then the Y-component must be zero; this is accomplished by setting VF(2)=0.0 on Card 67. If IPSS=4

(global Y axis is the line of symmetry) then the X-component must be zero; this is accomplished by setting VF(1)=0.0 on Card 67.

(6) The rotational velocity components of the fragment must also be consistent with the single symmetry option. If IPSS=1 then the components of angular velocity about the X and Z axis must be zero -- set OMEGF(1)=0.0 and OMEGF(3)=0.0 on Card 68. If IPSS=4 then the components of angular velocity about the Y and Z axes must be zero -- set OMEGF(2)=0.0 and OMEGF(3)=0.0 on Card 68.

It should be noted that the fragment mass, FMASS, and fragment mass moment of inertia, FMOI, input on Card 69 still correspond to the whole fragment. If NSYM=1, FMASS and FMOI are internally reduced for the half-plate (single symmetry) impact analysis.

The double-symmetry option (i.e. NSYM=2) can be exercised only when the following conditions are satisfied:

(1) The plate is rectangular (or square) with no cutouts, and a quarter of the plate is modeled (i.e. a doubly-symmetric finite-element mesh is used to model the plate).

(2) Initial and subsequent fragment/plate collisions occur at the center of the plate.

(3) The only nonzero velocity component of the fragment is the translational velocity in the global Z direction.

These conditions and other conventions adopted in the CIVM-PLATE code dictate some of the input which must be specified (when NSYM=2) as follows:

(1) IMESH on Card 2 must be set equal to 0 or 1 so that the auto-mesh generation subroutine is exercised.

(2) If IMESH=0 is specified, then the global X and Y coordinates of node 1 of the structure must be specified to be 0.0 (the origin of the global X-Y coordinates must be located at node number 1, which is the center of the plate). This is accomplished by setting XG(1)=0.0 on Card 5 and YG(1)=0.0 on Card 6.

(3) The symmetry boundary condition option must be chosen for plate sides 1 and 4; this is consistent with the convention of choosing node number 1 as the center of the plate. This is accomplished by setting NCSB(1)=1 and NCSB(4)=1 on Card 7.

(4) The global X and Y coordinates of the fragment centroid at time zero (XF, YF on Card 66) must be set equal to zero (i.e. the fragment is located directly below structural node number 1).

(5) The translational velocity components of the fragment in the X and Y directions (i.e. VF(1) and VF(2) on Card 67) must be set equal to zero; the only nonzero component of velocity is VF(3) in the Z-direction.

(6) The rotational velocity components of the fragment, OMEGF(1), OMEGF(2), and OMEGF(3) on Card 68 must be set equal to zero.

It should be noted that the fragment mass, FMASS, input on Card 69 still corresponds to the whole fragment. If NSYM=2, FMASS is internally reduced for the quarter model (doubly symmetric) impact analysis.

Card 72 (skip if ICUT=0)

| | |
|---|---|
| CUTR | D16.7 |

where

CUTR — A factor which, when multiplied times the initial fragment kinetic energy, determines when the impact inspection and correction process is to be terminated. That is, when the fragment kinetic energy at a particular instant of time is less than CUTR times the initial fragment kinetic energy, then no further impact inspection/correction calculations will be made. Note, however, that transient structural response calculations may be continued beyond final impact inspection/correction (additional discussions of this feature is given in Subsection 5.5.2).

The above input is complete for runs of the CIVM-PLATE which are not a continuation of a previous run (i.e. for cases when IMCONT=0 is specified

on Card 2). For runs which are a continuation of a previous run (i.e. IMCONT=1 is specified on Card 2), the punched deck of continuation cards from the previous run (obtained by setting IPUNCH=1 on Card 2 for the previous run) follows immediately after Card 72 (or Card 71 if ICUT=0 is specified on Card 71).

Some additional comments related to continuation runs should be observed. The discussion (in Subsection 4.4) of input data which must (or may) be changed for continuation runs of the PLATE code should also be followed for continuation runs of the CIVM-PLATE code. In addition, the following rules should be observed when preparing input data Cards 66 through 72 for continuation runs of the CIVM-PLATE program:

(1) Input data on Cards 66 through 70 must be identical to that used on the initial run of the present analysis (in which the deck of continuation cards was punched). This is necessary so that the initial (time zero) kinetic energy of the fragment may be calculated correctly. Input data provided in the deck of continuation cards will then update the fragment position and velocity components to the correct values prior to the first solution time cycle of the continuation run.

(2) The values of NSYM and IPSS on Card 71 must not be altered for continuation runs.

(3) The value of ICUT on Card 71 and/or the value of CUTR on Card 72 may be changed for continuation runs. The implications of and possible reasons for such a change are discussed in Subsection 5.5.2.

## 5.3 Guidelines for User-Prepared Array Dimensions

The second (and final) task required to run the CIVM-PLATE program is the assigning of fixed dimensions to arrays in the dummy main program associated with the CIVM-PLATE code. This task is essentially identical to the dimensioning task associated with the PLATE code, and the user should consult Subsection 4.5 for a general discussion of guidelines to be followed in the preparation of array dimensions.

A listing of a dummy main program for the CIVM-PLATE program is given on the next page. The arrays to be dimensioned are shown and their dimensions are given in terms of a set of problem-dependent parameters. The definitions of these parameters are also shown in the listing. The user is required to prepare a FORTRAN IV DIMENSION statement including all arrays shown in the sample DIMENSION statement; problem dependent parameters are replaced by the fixed value of each parameter for the problem being considered.

The user will note that all parameters, except NIAN, have also been used in the dummy main program associated with the PLATE program (again as a result of the general similarity between the two codes). Some additional discussion of the parameter NIAN is perhaps necessary. As indicated in the listing, the parameter NIAN is defined as the estimated maximum number of nodes which will be affected by a single impact, and may be estimated as follows. When plate/fragment impact occurs, the coordinates of the point of impact on the plate are determined. The impact-affected region of the plate is then defined as that region falling within a circle of radius $L_{eff}$. All nodes which fall within this circle of radius $L_{eff}$ are considered to be impact-affected nodes (see Appendix B for additional details).

In general, the precise location of all impacts is not known a priori. Thus, the user should first select $L_{eff}$ (see Card 70) and then assume that an impact will occur in the region of the plate having the most dense population of nodes (i.e. that region having the most highly refined mesh). The number of nodes falling within the impact-affected region for this hypothetical (worst-case) plate/fragment impact may be used as a base value for NIAN. Those nodes falling near to the impact-affected region could also be included in the estimate of NIAN to provide a liberal estimate of this parameter; in all cases, the value of NIAN should be overestimated. During each cycle in which impact occurs, the value of NIAN is checked against the actual number of impact-affected nodes. If the value of NIAN is less than the actual number of nodes affected, a message is printed and the run is terminated; the actual number of impact-affected nodes (for the cycle in which the run is terminated) is included in the printed message.

```
C   CIVM-PLATE 1 COMPUTER CODE.
C
C THIS DUMMY MAIN PROGRAM FOR THE CIVM PLATE PROGRAM REQUIRES
C THESE SUBROUTINES:
C          ARRT,ASSEMK,BCONMK,COLLSN,EAROW,FACT,GAUSS,IFRAG,
C          IMAINP,IMPACT,IMPDS,IMPSS,INTRAC,IPRINI,LMASSP,
C          MESHPA,MESHPM,OMULT,PROTOP,PSTRN,RCON,RPLATE,SFDM,
C          SOLV,SPRING,STRLSA,STRESC,STRESP,TSTEP.
C
C ILLUSTRATIVE EXAMPLE FOR CIVM PLATE PROGRAM - FULL MODEL.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C                                                     .
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C   DUMMY MAIN PROGRAM WHICH PROVIDES DIMENSIONS FOR THOSE ARRAYS
C   (VECTORS) WHOSE DIMENSIONS ARE PROBLEM DEPENDENT.
C   THE FOLLOWING ARRAYS MUST BE DIMENSIONED BY THE USER AS SHOWN;
C     DIMENSION DELD(NDT),DIS(NDT),DISP(NDT),DISM1(NDT),DISM2(NDT),
C     FLN(NDT),FLVA(NDT),FLVM(NDT),FLVP(NDT),VEL(NDT),ICOL(NDT),
C     INUM(NDT),KROW(NDT),NDEX(NDT),STF(MNC),AMASS(NDT ),NP(4,MAXEL),
C     NODE(24*MAXEL),TAUSS(MAXEL,4*MNSL,9),TAUSE(MAXEL,4*MNSL,9),
C     TAUEE(MAXEL,4*MNSL,9),EPSSI(9,MAXEL),EPSSO(9,MAXEL),EPEEI(9,MAXEL)
C     ,EPEEO(9,MAXEL),EPSEI(9,MAXEL),EPSEO(9,MAXEL),NBC(NB),BC(NB),
C     RFM(NBE,MBWE),ILAST(NBE),UCF1(NBE),UCF2(NBE),XG(NNT),YG(NNT),
C     ZG(NNT),XGI(NNT),YGI(NNT),
C     TAGSS(MNXST,4*MNSL,3),LNXS(MNXST),XSPROP(7,MNXST),MATXS(MNXST),
C     TSGEE(MNYST,4*MNSL,3),LNYS(MNYST),YSPROP(7,MNYST),MATYS(MNYST),
C     LNRS(NRSS),ISRS(NRSS),SC(5,NRSS),NVSA(MNNSA),NCON(4,MNNSA),
C     PMASS(NNT),VN(3,NIAN),VNB(3,NIAN),SI(NIAN),NEFF(NIAN),ALPHA(NIAN)
C
C   WHERE,
C   NNT     = TOTAL NUMBER OF NODES IN THE ASSEMBLED FINITE-ELEMENT MODEL
C   NDT     = TOTAL NUMBER OF DEGREES OF FREEDOM IN THE ASSEMBLED FINITE-
C             ELEMENT MODEL = 6*NNT.
C   MNC     = ESTIMATED NUMBER OF WORDS OF STORAGE FOR THE ASSEMBLED
C             STIFFNESS MATRIX (LOWER TRIANGLE STORED BY ROWS FROM FIRST
C             NONZERO TERM).
C   MAXEL   = NUMBER OF ELEMENTS IN FINITE-ELEMENT MODEL.
C   MNSL    = MAXIMUM NUMBER OF MECHANICAL SUBLAYERS EMPLOYED.
C             NOTE-- MNSL MUST BE LESS THAN OR EQUAL TO 5.
C   NB      = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM PRIOR
C             TO ELIMINATION OF DUPLICATES.
C   NBE     = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM AFTER
C             ALL DUPLICATES HAVE BEEN ELIMINATED (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
C   MBWE    = ESTIMATED MAXIMUM BANDWIDTH OF ASSEMBLED STIFFNESS MATRIX
C             AT A CONSTRAINED DEGREE OF FREEDOM (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
C   MNXST   = MAXIMUM NUMBER OF X-DIRECTION STIFFENERS.
C           = 1 IF NO X-DIRECTION STIFFENERS.
C   MNYST   = MAXIMUM NUMBER OF Y-DIRECTION STIFFENERS.
```

```
C             = 1 IF NO Y-DIRECTION STIFFENERS.                         IMAN0053
C   NRSS      = TOTAL NUMBER OF ELEMENT SIDES ON WHICH LINE RESTORING   IMAN0054
C               SPRINGS ARE LOCATED.                                    IMAN0055
C             = 1 IF NO RESTORING SPRINGS ARE PRESENT.                  IMAN0056
C   MNSLT4    = 4*MNSL                                                  IMAN0057
C   MNNSA     = MAXIMUM NUMBER OF NODES REQUESTED FOR STRAIN OUTPUT     IMAN0058
C             = (IF IOP8=1)                                             IMAN0059
C   NIAN      = ESTIMATED MAXIMUM NUMBER OF NODES WHICH WILL BE AFFECTED IMAN0060
C               BY A SINGLE IMPACT.                                     IMAN0061
      IMPLICIT REAL*8(A-H,O-Z)                                          IMAN0062
      DIMENSION DELD(450),DIS(450),DISP(450),DISM1(450),DISM2(450),     IMAN0063
     2FLN(450),FLVA(450),FLVM(450),FLVP(450),VEL(450),ICOL(450),        IMAN0064
     3INUM(450),KROW(450),NDEX(450),STF(16335),AMASS(450),              IMAN0065
     4NP(4,56),NODE(1344),TAUSS(56,12,9),TAUSE(56,12,9),TAUEE(56,12,9), IMAN0066
     5EPSSI(9,56),EPSSO(9,56),EPEEI(9,56),EPEEO(9,56),EPSEI(9,56),      IMAN0067
     6EPSEO(9,56),NBC(60),BC(60),RFM(1,1),ILAST(1),UCF1(1),UCF2(1),     IMAN0068
     7XG(75),YG(75),ZG(75),XGI(75),YGI(75),TAGSS(1,12,3),LNXS(1),       IMAN0069
     8XSPROP(7,1),MATXS(1),TSGEE(28,12,3),LNYS(28),YSPROP(7,28),MATYS(28IMAN0070
     9),LNRS(28),ISRS(28),SC(5,28),NVSA(8),NCON(4,8),PMASS(75),VN(3,75),IMAN0071
     A VNB(3,75),SI(75),NEFF(75),ALPHA(75)                              IMAN0072
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                               IMAN0073
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                IMAN0074
C DEFINE CARD READER, LINE PRINTER AND CARD PUNCH UNIT NUMBERS.         IMAN0075
      MREAD = 5                                                         IMAN0076
      MWRITE = 6                                                        IMAN0077
      MPUNCH = 7                                                        IMAN0078
      READ(MREAD,500)MAXEL,MNSL,MNXST,MNYST,NBE,MBWE,MNC,NIAN           IMAN0079
  500 FORMAT(8I10)                                                      IMAN0080
      WRITE(MWRITE,600)                                                 IMAN0081
      WRITE(MWRITE,610) MAXEL                                           IMAN0082
      WRITE(MWRITE,620) MNSL                                            IMAN0083
      WRITE(MWRITE,630) MNXST                                           IMAN0084
      WRITE(MWRITE,640) MNYST                                           IMAN0085
      WRITE(MWRITE,650) NBE                                             IMAN0086
      WRITE(MWRITE,660) MBWE                                            IMAN0087
      WRITE(MWRITE,670) MNC                                             IMAN0088
      WRITE(MWRITE,680) NIAN                                            IMAN0089
  600 FORMAT('1 CIVM-PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) : USER IMAN0090
     +INPUT FOR ARRAY DIMENSIONS')                                      IMAN0091
  610 FORMAT(' ','MAXEL  =',I10)                                        IMAN0092
  620 FORMAT(' ','MNSL   =',I10)                                        IMAN0093
  630 FORMAT(' ','MNXST  =',I10)                                        IMAN0094
  640 FORMAT(' ','MNYST  =',I10)                                        IMAN0095
  650 FORMAT(' ','NBE    =',I10)                                        IMAN0096
  660 FORMAT(' ','MBWE   =',I10)                                        IMAN0097
  670 FORMAT(' ','MNC    =',I10)                                        IMAN0098
  680 FORMAT(' ','NIAN   =',I10)                                        IMAN0099
      MNSLT4=MNSL*4                                                     IMAN0100
      CALL      IMAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,    IMAN0101
     2VEL,ICOL,INUM,KROW,NDEX,     STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE, IMAN0102
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,   IMAN0103
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS, IMAN0104
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON,    IMAN0105
```

```
 6PMASS,VN,VNB,SI,NEFF,ALPHA,NIAN)                                      IMAN0106
  CALL EXIT                                                             IMAN0107
  END                                                                   IMAN0108
```

<u>NOTE</u>: 1. The dimension 3000 for array STF and the dimension 84 for AMASS were obtained by the procedure described in items 1 through 4 on page 100. Also, see Figs. 19a and 19b.

2. For detailed instructions on establishing dimensions and input data, see the example problems discussed in Section 8.

## 5.4 Description of the Output

The printed output generated by the CIVM-PLATE program (as with the PLATE program) falls into two general categories: (1) initial output which reiterates user-supplied input and defines additional internally-generated data regarding geometry, material properties, boundary conditions, output options, and timewise solution parameters, and (2) solution output generated at user-specified time intervals and corresponding to those output options exercised by the user. In both of these categories, the output generated is essentially identical to that generated by the PLATE program; only those differences between the output generated by the two codes will be discussed.

All of the initial output generated by the PLATE program is also automatically generated by the CIVM-PLATE program. The reader should refer to Subsection 4.7 for a complete description of this initial output. In addition, the initial output now reiterates the data supplied for the fragment geometry, properties, initial location, and initial velocity components, as well as the collision parameters specified by the user. Also, the initial kinetic energy of the fragment is calculated and output. The user should note that a reference time step size will always be calculated and output because the CIVM-PLATE code always uses a diagonalized lumped mass model.

Solution data output at user-specified time intervals and consisting of that data requested by the user through the appropriate selection of output options is also essentially identical to that generated by the PLATE program; the user should consult Subsection 4.7 for a description of this output. In addition to the user-selected output data, the following data are printed at each regularly-scheduled print cycle:

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| [XF] | [YF] | [ZF] | [VF(1)] | [VF(2)] | [VF(3)] | [OMEGF(1)] | [OMEGF(2)] | [OMEGF(3)] |

where

$\left.\begin{array}{l}\text{XF}\\ \text{YF}\\ \text{ZF}\end{array}\right\}$ The current global X, Y, and Z components, respectively, which locate the c.g. of the spherical fragment.

| | | |
|---|---|---|
| VF(1)<br>VF(2)<br>VF(3) | } | The current translational velocity components in the global X, Y, and Z direction, respectively, of the fragment. |
| OMEGF(1)<br>OMEGF(2)<br>OMEGF(3) | } | The current angular velocity components about the global X, Y, and Z axes, respectively, of the fragment. |

The system energy output generated by the CIVM-PLATE code is slightly different from that generated by the PLATE code and appears as follows:

```
SYSTEM ENERGIES (IN-LB)
     FRAG TRANSLATIONAL KINETIC ENERGY     =  [CINFTN]
     FRAG ROTATIONAL KINETIC ENERGY        =  [CINFRT]
     WORK INPUT TO STRUCTURE               =  [EWORK]
     STRUCTURE KINETIC ENERGY              =  [CINET]
     STRUCTURE ELASTIC ENERGY              =  [ELAST]
     STRUCTURE PLASTIC ENERGY              =  [PLASTW]
     ENERGY STORED IN ELASTIC RESTRAINTS   =  [SPREN]
```

The parameters EWORK, CINET, ELAST, PLASTW, and SPREN are defined in Subsection 4.7, and the parameters CINFTN and CINFRT are defined, respectively, as the translational and the rotational kinetic energy of the fragment at the current time. Note that for the present impact analysis the current total work input to the plate structure is the difference between the initial (time zero) kinetic energy of the fragment and the current kinetic energy of the fragment. Also the total plastic work done on the structure is calculated indirectly by PLASTW = EWORK - (CINET + ELAST + SPREN). Thus, if the coefficient of restitution, e, has been assigned a value in the range $0<e<1$ (corresponding to an inelastic collision), then the energy loss in each inelastic collision will be included in the total plastic work which is output.

In addition to the above information which is printed at each desired time cycle, whenever there is an impact the following information is printed out:

IMPACT IT = [ITIME]    ELEMENT = [LNMP]    SUBREGION = [NSR]

GLOBAL IMPACT LOCATION (XN,YN,ZN) = [XN]    [YN]    [ZN]

PENETRATION DISTANCE (IN.) = [PEN]

THIS IS IMPACT NUMBER [NIMPCT]

where

| | | |
|---|---|---|
| ITIME | = | Time cycle number during which this impact occurred. |
| LNMP | = | Element number in which impact occurred. |
| NSR | = | Triangular subregion of element LNMP in which impact occurred (equal to 1 or 2 --- see Fig. 6). |
| XN<br>YN<br>ZN | | The global X, Y, and Z coordinates, respectively, which locate the point of plate/fragment impact. |
| PEN | = | Penetration distance which is the amount of plate/fragment overlap prior to impact correction. |
| NIMPCT | = | The total number of plate/fragment impacts, including the present impact, which have occurred. |

If the full model version of the impact subroutine is being used and if more than one penetration has occurred, the penetration distance (line #3 above) will not be printed out because the scheme which averages the impact location does not average the penetration distance.

If one of the symmetry impact options has been elected, this printed output also will include a message indicating which of the symmetry options is in effect.

## 5.5 Guides and Restrictions for Code Usage

### 5.5.1 General Guidelines

As a result of the similarity between the CIVM-PLATE and PLATE codes, most of the discussion of guidelines for use of the PLATE code (Subsection 4.8) also apply to the CIVM-PLATE code. In particular, the discussions on selection of output options, strain calculation, and dimensioning of the assembled stiffness matrix apply directly to the CIVM-PLATE program and will be commented upon only briefly in this subsection. The selection of a time-step size, $\Delta t$, for the CIVM-PLATE program is somewhat more involved and will be discussed in Subsection 5.5.2.

The output options available in the CIVM-PLATE code are identical to those available in the PLATE program. The discussion of output options presented in Subsection 4.8.4 should be followed in the selection of output options for the CIVM-PLATE program. It is recommended that displacement, system energy, and some form of strain output be selected for each run.

Strain calculation in the CIVM-PLATE program is identical to that of the PLATE program and the discussion in Subsection 4.8.5 should be followed. The recommendations in Subsection 4.8.5 should be followed more carefully when strain data are sought near the region of plate/fragment impact. Severe strain gradients will generally be found in this region; strain distributions in this region at various time instants should be used to supplement strain time-history data sought at locations in this region.

The technique outlined in Subsection 4.8.1 for determining the dimension, MNC, of the assembled stiffness matrix, STF, should again be used with the CIVM-PLATE program. Note, however, that this dimension now applies only to the assembled stiffness matrix (array STF) and does not apply to the assembled mass matrix since only lumped mass modeling is used in the CIVM-PLATE program.

### 5.5.2 Selection of a Time-Step Size

The selection of a suitable time-step size, $\Delta t$, in the CIVM-PLATE program is somewhat more involved as a result of the process used to impose impact-induced velocity changes on the plate structure and on the fragment. Since it is the velocity changes imposed on the structure which affect the choice of $\Delta t$, attention will be restricted to the plate structure in the present discussion.

As shown in Appendix B, the CIVM-PLATE program, in effect, employs two different timewise operators. For time cycles in which no plate-fragment impact occurs, the Houbolt operator is employed. For time cycles in which plate-fragment impact does occur, the nodal velocities in the impact-affected region of the plate are redefined to be the post-impact velocities (calculated by using impulse-momentum -- see Appendix B) and a modified operator is used; this modified operator may be viewed as a version of that in the Generalized Acceleration Method. In both cases, the equivalent loads corresponding to

large deflection and plasticity effects are approximated by using a linear extrapolation of their values at the previous two time instants.

To assess the stability of this modified operator compared with that of the Houbolt operator, a simple one degree-of-freedom cubic-hardening spring-mass system subjected to a prescribed initial velocity was analyzed using each of these operators; linear extrapolation was used to estimate the nonlinear equivalent loads. For the particular problem being considered, the solution obtained by using the modified operator became unstable for a $\Delta t$ value approximately half that of the $\Delta t$ value at which the solution using the Houbolt operator became unstable.[+] Quantitatively, these results suggest that for the same plate structure, an analysis by the CIVM-PLATE program will require a smaller $\Delta t$ than an equivalent analysis by the PLATE program.

Problems of stability of the modified operator are most likely to occur if the modified operator is used every cycle for many cycles (i.e. if impact occurs on many successive cycles). Such instabilities have been observed during theoretical-experimental correlation studies using the CIVM-PLATE code. Spurious oscillations in the predicted strain time-histories at several locations along a narrow plate subjected to normal impact at its center were observed at a time just prior to fragment rebound. In this case, impacts had occurred during each of approximately 200 time cycles prior to instability. Once impacts stopped (following fragment rebound), the strain time-histories stabilized, probably as a result of the improved stability of the Houbolt operator.

Additional study of the probable sources of this instability is needed. Although not conclusive, experience to date suggests that a value of $\Delta t$ no larger than twice the reference time step size which is calculated and output will yield stable solutions (recall that lumped mass modeling is used in the CIVM-PLATE program, and thus a reference time step size is calculated and output on each run of the program). However, the user should monitor the solution output, particularly strain data, to determine if an instability has occurred. If so, a smaller value of $\Delta t$ should be used, until the instability is removed.

---

[+] Note, however, that no iterations are being carried out within a given time step $\Delta t$ to achieve convergence.

An alternate, but less appealing, approach for eliminating this instability has also been included in the CIVM-PLATE program. If desired, the parameters ICUT on Card 71 and CUTR on Card 72 (see Subsection 5.2) can be specified such that impact inspection and correction will be terminated (also terminating the use of the modified operator) once the fragment kinetic energy is less than CUTR times the initial fragment kinetic energy. This approach is based on experience to date in which instabilities are generally found to occur when only approximately 5% of the initial kinetic energy is left in the fragment (just prior to fragment rebound). In principle, it could be argued that one is neglecting only 5% (for example) of the energy input to the plate structure and that, in fact, a portion of this energy is returned to the fragment as the plate and fragment rebound and the plate imparts kinetic energy to the fragment. However, when a value of CUTR = 0.05 was used on the previously-mentioned impact analysis where instability was observed, it was found that the predicted permanent strain levels, although stable, were substantially less than the experimentally determined strain levels; when a stable $\Delta t$ value was used instead of the cut-off option, predicted strain levels were found to be in essential agreement with experimentally-determined strain levels.

As a general rule, instabilities should be eliminated by reducing the value of $\Delta t$. As a last resort, the cut-off option can be exercised to advance the solution beyond the point of instability; however, in this case the analyst should be alerted to the possible error in predicted transient strain levels (after the option is imposed in the timewise solution) and permanent strain levels.

SECTION 6

COMPLETE FORTRAN IV LISTING OF THE
PLATE AND CIVM-PLATE PROGRAMS

## 6.1 Subprograms Common to both Codes

The following 21 subprograms (or subroutines) are common to both the PLATE and the CIVM-PLATE program:

| | | | | |
|---|---|---|---|---|
| ARRT | FACT | MESHPM | RCON | SPRING |
| ASSEMK | GAUSS | OMULT | RPLATE | STRESA |
| BCONMK | LMASSP | PRTOP | SFDM | STRESC |
| EAROW | MESHPA | PSTRN | SOLV | STRESP |
| | | | | TSTEP |

A FORTRAN IV listing of each follows.

```
      SUBROUTINE ARRT(STF,AMASS,F,ICOL,INUM,KROW,NDEX,NODE,NDPE)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.... ..ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
      DIMENSION AMASS(1)
      DIMENSION STF(1),F(1),ICOL(1),INUM(1),KROW(1),NDEX(1),NODE(1)
      COMMON/BAS/NDT,NET,MN,NBD,NIRREG,MNC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C IN THIS SUBROUTINE, THE MESH NODAL CONNECTIVITY DATA IN NODE(I) IS
C USED TO ESTABLISH THE POINTER ARRAYS NECESSARY TO LOCATE TERMS IN
C THE ASSEMBLED STIFFNESS MATRIX (AND MASS MATRIX IF CONSISTENT MASS
C IS USED) STORED IN COMPACTED VECTOR FORM  THE NUMBER OF WORDS OF
C STORAGE REQUIRED FOR THE ASSEMBLRD STIFFNESS MATRIX IS CALCULATED,
C OUTPUT, AND COMPARED WITH THE USER ESTIMATE, IF THE USER ESTIMATE IS
C TOO SMALL, A MESSAGE IS OUTPUT AND THE RUN IS TERMINATED
C THE ASSEMBLED STIFFNESS AND MASS (ONLY FOR LUMPED MASS) MATRICES AND
C FORCE VECTOR ARE ZEROED.
C
      DO 598 I=1,NDT
  598 ICOL(I)=NDT
      DO 500 I=1,NET
      NA=NDT
      II=(I-1)*NDPE
      DO 5300 J=1,NDPE
      IIJ=II+J
      NZ=NODE(IIJ)
 5300 IF(NA .GT. NZ) NA=NZ
      DO 500 J=1,NDPE
      IIJ=II+J
      NIIJ=NODE(IIJ)
  500 IF(ICOL(NIIJ).GT.NA) ICOL(NIIJ)=NA
 5601 INUM(1)=1
      DO 599 I=2,NDT
  599 INUM(I)=I-ICOL(I-1)+INUM(I-1)
      DO 591 I=1,NDT
  591 INUM(I)=INUM(I)-ICOL(I)
      MN=INUM(NDT)+NDT
      WRITE(MWRITE,515) MN
  515 FORMAT(/2X,' MAX SIZE IS ',I6)
      IF(MN-MNC) 521,521,5211
 5211 WRITE(MWRITE,520) MN
  520 FORMAT(/2X,'MN=',I6,' THE DIM OF STIF IS TOO SMALL')
      GO TO 9999
  521 CONTINUE
      NIRREG=0
      INDEX=0
      ISET=1
      DO 116 I=1,NDT
      L=ICOL(I)
      IF(ICOL(I)-ISET) 117,116,119
  119 ISET=ICOL(I)
      GO TO 116
  117 NIRREG=NIRREG+1
      IF(NIRREG-NDT/2) 711,711,712
```

```
  711 KROW(NIRREG)=I                                                    ARRT0059
      NDEX(NIRREG)=INDEX                                                ARRT0060
  116 INDEX=INDEX+I-L                                                   ARRT0061
  712 CONTINUE                                                          ARRT0062
      DO 20 I=1,MN                                                      ARRT0063
 20   STF(I)=0.0                                                        ARRT0064
      DO 10 I=1,NDT                                                     ARRT0065
      AMASS(I)=0.0                                                      ARRT0066
 10   F(I)=0 0                                                          ARRT0067
 9999 RETURN                                                            ARRT0068
      END                                                               ARRT0069
      SUBROUTINE ASSEMK(EK,   STF,      INUM,NDPE,NODE,N)               ASMK0000
      IMPLICIT REAL*8(A-H,O-Z)                                          ASMK0001
C                                                                       ASMK0002
C     ASRL TR 154-14.... ..ORIGINAL REPORT VERSION OF PROGRAM           ASMK0003
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980           ASMK0004
C                                                                       ASMK0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.ASMK0006
C                                                                       ASMK0007
      DIMENSION STF(1),INUM(1),EK(NDPE,NDPE),NODE(1)                    ASMK0008
      COMMON/BAS/NDT,NET,MN,NBD,NIRREG,MNC                              ASMK0009
C                                                                       ASMK0010
C THIS SUBROUTINE ASSEMBLES THE ELEMENT STIFFNESS MATRIX INTO THE ASSEMBASMK0011
C STIFFNESS MATRIX                                                      ASMK0012
C                                                                       ASMK0013
      NN=(N-1)*NDPE                                                     ASMK0014
      DO 100 I=1,NDPE                                                   ASMK0015
      KMA=NODE(NN+I)                                                    ASMK0016
      DO 100 II=1,I                                                     ASMK0017
      KNA=NODE(NN+II)                                                   ASMK0018
      KA=INUM(KMA)+KNA                                                  ASMK0019
      IF(KNA.GT KMA)KA=INUM(KNA)+KMA                                    ASMK0020
      STF(KA)=STF(KA)+EK(I,II)                                          ASMK0021
 100  CONTINUE                                                          ASMK0022
      RETURN                                                            ASMK0023
      END                                                               ASMK0024
      SUBROUTINE BCONMK(STF,      BC,ICOL,INUM,NBC)                     BCMK0000
      IMPLICIT REAL*8(A-H,O-Z)                                          BCMK0001
C                                                                       BCMK0002
C     ASRL TR 154-14. .... ORIGINAL REPORT VERSION OF PROGRAM           BCMK0003
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980           BCMK0004
C                                                                       BCMK0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.BCMK0006
C                                                                       BCMK0007
      DIMENSION STF(1),        BC(1),ICOL(1),INUM(1),NBC(1)             BCMK0008
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                               BCMK0009
C                                                                       BCMK0010
C THIS SUBROUTINE IMPOSES THE BOUNDARY CONDITION CONSTRAINTS (ZERO      BCMK0011
C DISPLACEMENTS) ON THE ASSEMBLED MATRIX (2M/(DELTA T SQUARED)+K)       BCMK0012
C GIVEN IN EQUATION A 114, BY ZEROING THE ROW AND COLUMN (NOT           BCMK0013
C INCLUDING THE DIAGONAL TERM) CORRESPONDING TO EACH CONSTRAINED        BCMK0014
C DEGREE OF FREEDOM                                                     BCMK0015
C                                                                       BCMK0016
      IF(NB EQ 0)RETURN                                                 BCMK0017
      DO 100 I=1,NB                                                     BCMK0018
      K=NBC(I)                                                          BCMK0019
      KA=K+1                                                            BCMK0020
      IF(NBC(I).GE.NDT)GO TO 10                                         BCMK0021
      DO 200 J=KA,NDT                                                   BCMK0022
```

```
      IF(ICOL(J) GT K)GO TO 200                                         BCMK0023
      KK=NBC(I)+INUM(J)                                                 BCMK0024
      STF(KK)=0 0                                                       BCMK0025
 200  CONTINUE                                                          BCMK0026
 10   KA=K-1                                                            BCMK0027
      IF((NBC(I).LE.1) OR.(ICOL(K).GT.KA))GO TO 100                     BCMK0028
      MCO=ICOL(K)                                                       BCMK0029
      DO 500 J=MCO,KA                                                   BCMK0030
      KK=J+INUM(K)                                                      BCMK0031
      STF(KK)=0.0                                                       BCMK0032
 500  CONTINUE                                                          BCMK0033
 100  CONTINUE                                                          BCMK0034
      RETURN                                                            BCMK0035
      END                                                               BCMK0036
      SUBROUTINE EAROW(STF,NBC,ICOL,INUM,RFM,ILAST,NBE,MBWE)            EROW0000
      IMPLICIT REAL*8(A-H,O-Z)                                          EROW0001
C                                                                       EROW0002
C    ASRL TR 154-14.... ..ORIGINAL REPORT VERSION OF PROGRAM            EROW0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            EROW0004
C                                                                       EROW0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.EROW0006
C                                                                       EROW0007
      DIMENSION STF(1),NBC(1),ICOL(1),INUM(1),ILAST(1),RFM(NBE,MBWE)    EROW0008
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                               EROW0009
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                EROW0010
C                                                                       EROW0011
C   THIS SUBROUTINE EXTRACTS THE ROWS OF THE ASSEMBLED MATRIX, STF      EROW0012
C   (STORED BY ROWS  FROM THE FIRST NONZERO TERM TO THE DIAGONAL)       EROW0013
C   CORRESPONDING TO EACH CONSTRAINED D.O.F..  THE MATRIX RFM(K,J)      EROW0014
C   CONTAINS THE K-TH ROW OF STF, AND IS USED TO COMPUTE REACTION FORCESEROW0015
C   IT IS ASSUMED THAT THE CONSTRAINED D.O.F. LIST CONTAINS NO          EROW0016
C   DUPLICATES                                                          EROW0017
C                                                                       EROW0018
      IF(NB EQ.0)RETURN                                                 EROW0019
      MBW=0                                                             EROW0020
      LNM=0                                                             EROW0021
      DO 50 INB=1,NB                                                    EROW0022
      K=NBC(INB)                                                        EROW0023
      NN=0                                                              EROW0024
C   EXTRACT TERMS IN K-TH ROW FROM FIRST NONZERO TERM TO DIAGONAL TERM. EROW0025
      KA=ICOL(K)                                                        EROW0026
      DO 10 J=KA,K                                                      EROW0027
      NN=NN+1                                                           EROW0028
      KK-J+INUM(K)                                                      EROW0029
 10   RFM(INB,NN)=STF(KK)                                               EROW0030
C   FIND ROW WHICH HAS LAST NONZERO TERM IN COL. K.                     EROW0031
      IL=K                                                              EROW0032
      IF(K.EQ NDT)GO TO 30                                              EROW0033
      KA-K+1                                                            EROW0034
      DO 20 I=KA,NDT                                                    EROW0035
      IF(K GE.ICOL(I))IL=I                                              EROW0036
 20   CONTINUE                                                          EROW0037
 30   ILAST(INB)=IL                                                     EROW0038
      MM=ILAST(INB)-ICOL(K)+1                                           EROW0039
      IF(MM.LT MBW)GO TO 32                                             EROW0040
      MBW=MM                                                            EROW0041
      LNM=K                                                             EROW0042
 32   CONTINUE                                                          EROW0043
      IF(K.EQ.NDT)GO TO 50                                              EROW0044
```

```
C   EXTRACT TERMS IN K-TH COL. FROM (K+1)ST ROW TO LAST NONZERO TERM
C   IN K-TH COL..
      DO 40 I=KA,IL
      NN=NN+1
      IF(K GE ICOL(I))GO TO 35
      RFM(INB,NN)=0.0
      GO TO 40
 35   KK=INUM(I)+K
      RFM(INB,NN)=STF(KK)
 40   CONTINUE
 50   CONTINUE
      WRITE(MWRITE,600)MBW,LNM
      IF(MBW GT.MBWE)WRITE(MWRITE,610)
      IF(MBW GT MBWE)CALL EXIT
 600  FORMAT('0','MAXIMUM BANDWIDTH OF',I6,2X,'IS FOUND FOR (CONSTRAINED
    2) DEGREE-OF-FREEDOM NUMBER',I6)
 610  FORMAT('0','*** USER ESTIMATE OF MAX. BANDWIDTH AT A CONSTRAINED D
    2 O.F  IS TOO SMALL---RUN HAS BEEN TERMINATED')
      RETURN
      END
      SUBROUTINE FACT(STIFM,NCOL,KROW,NDEX,IDET,NTAPE6,IC)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14 ... . ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
      DIMENSION STIFM(1),NCOL(1),KROW(1),NDEX(1),IC(1)
      COMMON/BAS/NROWS,NET,MN,NBD,NIRREG,MNC
C
C THIS SUBROUTINE PERFORMS TRIPLE-FACTORIZATION ON THE ASSEMBLED
C MATRIX (2M/(DELTA T SQUARED)+K) IN EQUATION A.114, PUTTING IT IN THE
C FORM (L TRANSPOSE)*D*L WHERE L IS A LOWER TRIANGULAR MATRIX AND D IS
C A DIAGONAL MATRIX.THE ORIGINAL MATRIX IS DESTROYED IN THIS OPERATION.
C DETAILS OF THE TRIPLE-FACTORING OPERATION MAY BE FOUND IN NUMEROUS
C TEXTS ON NUMERICAL METHODS AND FINITE-ELEMENT METHODS (EQUATION 9).
C
C     LOWER TRIANGULAR FACTOR OF K MATRIX TO BE COMPUTED AND STORED IN
C     STIFM
C     PROCESS COLUMN 1
      I=1
      IDET=0
      IF(STIFM(1)) 152,122,101
  152 IDET=IDET+1
  101 INDEX=0
      IROW=1
      TEST=1 0
      KN=1
      DO 103 I=2,NROWS
      KN=KN+I-NCOL(I)
      IF (NCOL(I)-1) 103,102,103
  102 STIFM(KN)=STIFM(KN)/STIFM(1)
  103 CONTINUE
      DO 121 I=2,NROWS
      IP1=I+1
      IM1=I-1
      SUM=0.0
      NCK=0
```

```
      III=NCOL(I)
      INDEX=INDEX+I-III
      IF (IM1-III) 150,140,140
C     DIAGONAL TERMS
  140 DO 104 J=III,IM1
      IJ=INDEX+J
      ICJJ=IC(J)+J
  104 SUM=SUM+STIFM(IJ)+STIFM(IJ)*STIFM(ICJJ)
  150 II=INDEX+I
      SUM=STIFM(II)-SUM
      IF(SUM) 151,122,105
  151 IDET=IDET+1
  105 TES=DABS(SUM/STIFM(II))
      IF(TES-TEST) 106,107,107
  106 TEST=TES
      IROW=I
  107 STIFM(II)=SUM
C     OFF DIAGONAL TERMS
      IF (I-NROWS) 108,121,121
  108 KNDEX=INDEX
  109 DO 116 K=IP1,NROWS
      KK=NCOL(K)
      KNDEX=KNDEX+K-KK
      SUM=0.0
      IF (KK-III) 110,130,130
  110 KK=III
  130 IF (IM1-KK) 112,131,131
  131 DO 111 J=KK,IM1
      IJ=INDEX+J
      KJ=KNDEX+J
      ICJJ=IC(J)+J
 111  SUM=SUM+STIFM(IJ)+STIFM(KJ)*STIFM(ICJJ)
  112 IF (I-KK) 114,115,115
  114 IF(NIRREG .LE  0) GO TO 121
      IF(NIRREG .GT. NROWS/2) GO TO 116
      GO TO 190
  115 KI=KNDEX+I
      STIFM(KI)=(STIFM(KI)-SUM)/STIFM(II)
  116 CONTINUE
      GO TO 121
  190 NCK=NCK+1
      IF(NIRREG  LT. NCK) GO TO 121
      IP1=KROW(NCK)
      IF(I  LT  NCOL(IP1)) GO TO 190
      IF(IP1  LT  K) GO TO 190
      KNDEX=NDEX(NCK)
      GO TO 109
  121 CONTINUE
      WRITE(NTAPE6,1000) IROW,TEST,IDET
 1000 FORMAT (42H0ROUNDING ERROR PARAMETER IN FACTORING ROW,I4,2H =,F12.
     18,5X,'NO OF NEGATIVE DIAG=',I5)
      RETURN
  122 WRITE(NTAPE6,1001) I
      IDET=-1
 1001 FORMAT (37H1 MATRIX NOT POSITIVE DEFINITE IN ROW,I4)
      WRITE(NTAPE6,1002) SUM
 1002 FORMAT(2X,'SQR DIAG TERM=',D15.8,/2X,'PART FACT K',//)
      RETURN
      END
```

```
      SUBROUTINE GAUSS(N,GS,GW)
      IMPLICIT REAL*8(A-H,O-Z)
C
C
C    ASRL TR 154-14. ..   ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
C THIS SUBROUTINE DEFINES THE LOCATION OF THE GAUSSIAN INTEGRATION
C STATIONS AND THE CORRESPONDING WEIGHTING FACTORS IN ONE DIMENSIONAL
C SPACE. THE NUMBER OF GAUSSIAN STATIONS USED MAY BE FROM 1 TO 6.
C
      DIMENSION GS(6),GW(6)
      GO TO(10,20,30,40,50,60),N
 10   GS(1)=0 0D0
      GW(1)=2.0D0
      RETURN
 20   GS1=-0 577350269189626D+00
      GS(1)=GS1
      GS(2)=-  GS1
      CW(1)=1 0D0
      CW(2)=1 0D0
      RETURN
 30     GS1=- 774596669241483D+00
      GS(1)=GS1
      GS(2)=0.000000000000000D+00
      GS(3)=-  GS1
        GW1=.555555555555556D+00
      GW(1)=GW1
        GW(2)= 888888888888889D+00
      GW(3)=  CW1
      RETURN
 40     GS1=-.861136311594053D+00
        GS2=- 339981043584856D+00
       GS(1)=GS1
       GS(2)=GS2
      GS(3)=-  GS2
      GS(4)=-  GS1
        GW1= 347854845137454D+00
        GW2= 652145154862546D+00
      GW(1)=GW1
      CW(2)=GW2
      GW(3)=  GW2
      GW(4)=  GW1
      RETURN
 50   GS1=-0 906179845938664D+00
        GS2=-.538469310105683D+00
      GS(1)=GS1
      GS(2)=GS2
      GS(3)=0.0D0
      GS(4)=-  GS2
      GS(5)=-  GS1
        GW1=.236926885056189D+00
        GW2= 478628670499366D+00
      GW(1)=CW1
      GW(2)=GW2
      GW(3)=.568888888888889D+00
      GW(4)=  GW2
```

```
      GW(5)=  GW1                                                        GAUS0059
      RETURN                                                             GAUS0060
 60     GS1=-.932469514203152D+00                                        GAUS0061
        GS2=-.661209386466265D+00                                        GAUS0062
      GS(1)=GS1                                                          GAUS0063
      GS(2)=GS2                                                          GAUS0064
        GS3=-.238619186083197D+00                                        GAUS0065
      GS(3)=GS3                                                          GAUS0066
      GS(4)=-  GS3                                                       GAUS0067
      GS(5)=-  GS2                                                       GAUS0068
      GS(6)=-  GS1                                                       GAUS0069
        GW1=.171324492379170D+00                                         GAUS0070
        GW2=.360761573048139D+00                                         GAUS0071
        GW3= 467913934572691D+00                                         GAUS0072
      GW(1)=GW1                                                          GAUS0073
      GW(2)=GW2                                                          GAUS0074
      GW(3)=GW3                                                          GAUS0075
      GW(4)=  GW3                                                        GAUS0076
      GW(5)=  GW2                                                        GAUS0077
      GW(6)=  GW1                                                        GAUS0078
      RETURN                                                             GAUS0079
      END                                                                GAUS0080
      SUBROUTINE LMASSP(NTYPE,XL,YL,TH,SL,CSA,SMOI,EML,DENS)             LMSP0000
      IMPLICIT REAL*8(A-H,O-Z)                                           LMSP0001
C                                                                        LMSP0002
C     ASRL TR 154-14  .. ..ORIGINAL REPORT VERSION OF PROGRAM            LMSP0003
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            LMSP0004
C                                                                        LMSP0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS. LMSP0006
C                                                                        LMSP0007
      DIMENSION EML(1)                                                   LMSP0008
C                                                                        LMSP0009
C THIS SUBROUTINE FORMS THE LUMPED (DIAGONAL) MASS MATRIX FOR A PLATE    LMSP0010
C OR STIFFENER ELEMENT                                                   LMSP0011
      GO TO(10,50,70),NTYPE                                              LMSP0012
C    LUMPED MASS FOR PLATE                                               LMSP0013
 10   AM1=XL*YL*TH                                                       LMSP0014
      AM2=AM1*TH*TH/12.0                                                 LMSP0015
      DO 40 I=1,4                                                        LMSP0016
      INC=6*(I-1)                                                        LMSP0017
      DO 20 J=1,3                                                        LMSP0018
      JJ=INC+J                                                           LMSP0019
 20   EML(JJ)=AM1/4.0                                                    LMSP0020
      DO 30 J=4,5                                                        LMSP0021
      JJ=INC+J                                                           LMSP0022
 30   EML(JJ)=AM2/4.0                                                    LMSP0023
 40   EML(INC+6)=0 0                                                     LMSP0024
      GO TO 90                                                           LMSP0025
C    LUMPED MASS FOR X-DIRECTION STIFFENER                               LMSP0026
 50   AM1=CSA*XL                                                         LMSP0027
      AM2=SMOI*XL                                                        LMSP0028
      D1=1 0-SL                                                          LMSP0029
      D2=SL                                                              LMSP0030
      DO 60 I=1,4                                                        LMSP0031
      FCT=D2                                                             LMSP0032
      IF(I EQ 1 OR.I EQ 2)FCT=D1                                         LMSP0033
      INC=6*(I-1)                                                        LMSP0034
      EML(INC+1)=AM1*FCT/2.0                                             LMSP0035
      EML(INC+2)=EML(INC+1)                                              LMSP0036
```

```
      EML(INC+3)=EML(INC+1)
      EML(INC+4)=AM2*FCT/2.0
      EML(INC+5)=EML(INC+4)
   60 EML(INC+6)=0.0
      GO TO 90
C  LUMPED MASS FOR Y-DIRECTION STIFFENER
   70 AM1=CSA*YL
      AM2=SMOI*YL
      D1=1.0-SL
      D2=SL
      DO 80 I=1,4
      FCT=D2
      IF(I.EQ.1.OR.I.EQ.4)FCT=D1
      INC=6*(I-1)
      EML(INC+1)=AM1*FCT/2.0
      EML(INC+2)=EML(INC+1)
      EML(INC+3)=EML(INC+1)
      EML(INC+4)=AM2*FCT/2.0
      EML(INC+5)=EML(INC+4)
   80 EML(INC+6)=0.0
   90 CONTINUE
C  MULTIPLY BY DENSITY FOR THIS ELEMENT.
      DO 100 I=1,24
  100 EML(I)=EML(I)*DENS
      RETURN
      END
      SUBROUTINE MESHPA(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,YGI,
     2XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER TY,PE,TY1,PE1,TY2,PE2
C
C   ASRL TR 154-14 ..  . ORIGINAL REPORT VERSION OF PROGRAM
C  COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
      DIMENSION NP(4,1),NODE(1),XG(1),YG(1),ZG(1),NBC(1),BC(1),NCSB(4),
     2NC(6),XSPROP(7,1) ,YSPROP(7,1) ,NSC(6,5),NEC(6,5),NBCSC(5),
     3NBCEC(5),XGI(1),YGI(1),LNXS(1),LNYS(1),RHC(30),RWC(30),ECS(30),
     4ANUCS(30),DENCS(30),SLCS(30),XFCST(30),APCST(30),MATCS(30),
     5MATXS(1),MATYS(1)
      DATA NSC/1,4,6,3*0,1,2,3,4,5,6,1,2,3,5,6,0,1,2,5,6,2*0,2,3,5,6,
     @2*0/
      DATA NEC/2,5,6,3*0,1,2,3,4,5,6,1,2,3,4,6,0,1,2,4,6,2*0,1,3,4,6,
     @2*0/
      DATA NBCEC/3,6,5,4,4/
      DATA NBCSC/3,6,5,4,4/
      DATA TY1,TY2,PE1,PE2/' SIN',' DOU','GLE ','BLE '/
      COMMON/PLATE/XDIST,YDIST,TH,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,
     2THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C  THIS ROUTINE FORMS MESH, BOUNDARY CONDITION, AND STIFFENER
C  INFORMATION FOR FLAT PLATE PROBLEM.  THE STIFFENERS ARE
C  ASSUMED TO BE ORIENTED IN THE X AND Y DIRECTIONS, SPANNING THE
C  ENTIRE PLATE   AT PRESENT A MAXIMUM OF 30 STIFFENERS IN EACH
C  DIRECTION IS PERMITTED   THE MESH IS MADE UP OF RECTANGULAR
C  PLATE BENDING ELEMENTS, IN A UNIFORM OR NONUNIFORM PATTERN.
```

```
C   PLATE IS OF UNIFORM THICKNESS.
C
C   IF IMESH=0 , A NONUNIFORM MESH IS ASSUMED.
C   IF IMESH=1 , A UNIFORM MESH IS ASSUMED.
C   GENERAL MESH INFORMATION
      READ(MREAD,500)NEAD,NECD
      READ(MREAD,510)EPAN,ANUP,DENSP,TH,XDIST,YDIST,ZPOS
      THALF=TH/2.0D0
      NET=NEAD*NECD
      NDPE=24
      NDPN=6
      NXST=0
      NYST=0
      NEAD1=NEAD+1
      NECD1=NECD+1
      NDT=NEAD1*NECD1*NDPN
      NNT=NEAD1*NECD1
C   FORM NP(I,J), I=LOCAL NODE NUMBER, J=ELEMENT NO.
C   X-DIR  FIRST, Y-DIR. NEXT
      LNUM=0
      DO 10 NE=1,NECD
      INCE=(NE-1)*NEAD1
      DO 10 NS=1,NEAD
      LNUM=LNUM+1
      NP(1,LNUM)=INCE+NS
      NP(2,LNUM)=NP(1,LNUM)+1
      NP(3,LNUM)=NP(2,LNUM)+NEAD1
   10 NP(4,LNUM)=NP(3,LNUM)-1
C   FORM NODE VECTOR--- ASSEMBLY LIST
      DO 30 I=1,NET
      INCI=(I-1)*NDPE
      DO 30 J=1,4
      INCJ=(J-1)*NDPN
      DO 30 K=1,NDPN
      INCIJK=INCI+INCJ+K
   30 NODE(INCIJK)=NP(J,I)*NDPN-NDPN+K
C   SPACE COORDINATES OF NODES.
      IF(IMESH EQ.1)GO TO 32
C   FOR NONUNIFORM MESH, READ Y=0 NODAL COORDS. AND X=0 NODAL COORDS.
      READ(MREAD,510) (XG(I), I=1,NEAD1)
      I2=NECD*NEAD1+1
      READ(MREAD,510) (YG(I), I=1,I2,NEAD1)
      GO TO 38
C   FOR UNIFORM MESH GENERATE THIS INFO AUTOMATICALLY
C   ASSUMING FIRST NODE AT (0,0)
   32 DX=XDIST/NEAD
      DY=YDIST/NECD
      HXL=DX/2 0
      HYL=DY/2 0
      DO 34 I=1,NEAD1
   34 XG(I)=(I-1)*DX
      DO 36 I=1,NECD1
      I2=NEAD1*I-NEAD
   36 YG(I2)=(I-1)*DY
C   FILL OUT XG, YG, ZG
   38 DO 40 IY=1,NECD1
      INCY=(IY-1)*NEAD1
      DO 40 IX=1,NEAD1
      I2=INCY+IX
```

```
      XG(I2)=XG(IX)
      YG(I2)=YG(INCY+1)
   40 ZG(I2)=ZPOS
      DO 41 I=1,NNT
      XGI(I)=XG(I)
   41 YGI(I)=YG(I)
C   BOUNDARY CONDITIONS
C   CONVENTION FOR FLAT-PLATE BOUNDARY CONDITION  OPTIONS,  NCBS(SIDE)
C        0 - FREE
C        1 - SYMMETRY
C        2 - IDEALLY CLAMPED
C        3 - PINNED-FIXED
C        4 - PINNED-FREE SLIDING Z-DIRECTION
C        5 - PINNED-FREE SLIDING/NORMAL (INPLANE)
      READ(MREAD,500)(NCSB(I),I=1,4)
      NBT=0
C   SIDE 1---Y=0, VARY X
      IF(NCSB(1).EQ 0) GO TO 50
      MM=NCSB(1)
      NN=NBCEC(MM)
      DO 42 NS=1,NEAD1
      INS=(NS-1)*NDPN
      DO 42 I=1,NN
      NBT=NBT+1
   42 NBC(NBT)=INS+NEC(I,MM)
C   SIDE 2--- X=XDIST, VARY Y
   50 IF(NCSB(2) EQ.0)GO TO 60
      MM=NCSB(2)
      NN=NBCSC(MM)
      DO 52 NE=1,NECD1
      INE=NE*NEAD1+NDPN-NDPN
      DO 52 I=1,NN
      NBT=NBT+1
   52 NBC(NBT)=INE+NSC(I,MM)
C   SIDE 3--- Y=YDIST, VARY X
   60 IF(NCSB(3) EQ 0)GO TO 70
      MM=NCSB(3)
      NN=NBCEC(MM)
      DO 62 NS=1,NEAD1
      INS=NEAD1*NECD*NDPN+(NS-1)*NDPN
      DO 62 I=1,NN
      NBT=NBT+1
   62 NBC(NBT)=INS+NEC(I,MM)
C   SIDE 4--- X=0, VARY Y
   70 IF(NCSB(4).EQ.0)GO TO 80
      MM=NCSB(4)
      NN=NBCSC(MM)
      DO 72 NE=1,NECD1
      INE=(NE-1)*NEAD1*NDPN
      DO 72 I=1,NN
      NBT=NBT+1
   72 NBC(NBT)=INE+NSC(I,MM)
   80 CONTINUE
C   SORT CONSTRAINED D O.F., LOWEST TO HIGHEST, ELIMINATING DUPLICATES.
      IF(NBT EQ.0)NC=0
      IF(NBT.EQ.0)GO TO 850
C   REMOVE DUPLICATE CONSTRAINED D.O.F.
      NB=1
      DO 820 I=2,NBT
```

```
      NI=NBC(I)
      DO 810 J=1,NB
      IF(NBC(J).EQ.NI)GO TO 820
  810 CONTINUE
      NB=NB+1
      NBC(NB)=NI
  820 CONTINUE
C   REARRANGE, LOWEST TO HIGHEST.
      NBT=0
  830 NBT=NBT+1
      NMIN=NDT+1
      DO 840 II=NBT,NB
      IF(NBC(II).GT.NMIN)GO TO 840
      NMIN=NBC(II)
      NI=II
  840 CONTINUE
      NSTOR=NBC(NBT)
      NBC(NBT)=NBC(NI)
      NBC(NI)=NSTOR
      IF(NBT.LT.NB)GO TO 830
      DO 845 I=1,NB
  845 BC(I)=0.0D0
  850 CONTINUE
C   INFORMATION FOR STIFFENERS.  FOR PRESENT ANALYSIS, EACH AXIAL AND/
C   OR CIRC  STIFFENER SPANS ENTIRE DIMENSION OF STRUCTURE.
      READ(MREAD,500)NAST,NCST
C   INFORMATION FOR X-DIRECTION STIFFENERS SPANNING PLATE.
      IF(NAST.EQ.0)GO TO 100
      READ(MREAD,510)(APCST(I),I=1,NAST)
      READ(MREAD,510)(RHC(I),I=1,NAST)
      READ(MREAD,510)(RWC(I),I=1,NAST)
      READ(MREAD,510)(ECS(I),I=1,NAST)
      READ(MREAD,510)(ANUCS(I),I=1,NAST)
      READ(MREAD,510)(DENCS(I),I=1,NAST)
      READ(MREAD,510)(XFCST(I),I=1,NAST)
      READ(MREAD,500)(MATCS(I),I=1,NAST)
      NN=0
      DO 115 NES=1,NAST
      DO 105 I=1,NECD
      I2=NEAD1*I+1
      APOS=YG(I2)
      IF(APOS.GE.APCST(NES))NROW=I
      IF(APOS.GE.APCST(NES))GO TO 110
  105 CONTINUE
  110 I1=NEAD1*(NROW-1)+1
      I2=NEAD1*NROW+1
      SLCS(NES)=(2.D0*APCST(NES)-YG(I1)-YG(I2))/(YG(I2)-YG(I1))
      DO 115 NS=1,NEAD
      NN=NN+1
  115 LNXS(NN    )=(NROW-1)*NEAD+NS
      NN=0
      DO 116 I=1,NAST
      DO 116 J=1,NEAD
      NN=NN+1
      MATXS(NN)=MATCS(I)
      XSPROP(1,NN)=RHC(I)
      XSPROP(2,NN)=RWC(I)
      XSPROP(3,NN)=ECS(I)
      XSPROP(4,NN)=ANUCS(I)
```

```
      XSPROP(5,NN)=DENCS(I)                                             MSPA0210
      XSPROP(6,NN)=(SLCS(I)+1.0D0)/2.0D0                                MSPA0211
  116 XSPROP(7,NN)=XFCST(I)                                             MSPA0212
      NXST=NAST*NEAD                                                    MSPA0213
C    INFORMATION FOR Y-DIRECTION STIFFENERS SPANNING PLATE.             MSPA0214
  100 IF(NCST EQ 0)GO TO 120                                            MSPA0215
      READ(MREAD,510)(APCST(I),I=1,NCST)                                MSPA0216
      READ(MREAD,510)(RHC(I),I=1,NCST)                                  MSPA0217
      READ(MREAD,510)(RWC(I),I=1,NCST)                                  MSPA0218
      READ(MREAD,510)(ECS(I),I=1,NCST)                                  MSPA0219
      READ(MREAD,510)(ANUCS(I),I=1,NCST)                                MSPA0220
      READ(MREAD,510)(DENCS(I),I=1,NCST)                                MSPA0221
      READ(MREAD,510)(XFCST(I),I=1,NCST)                                MSPA0222
      READ(MREAD,500)(MATCS(I),I=1,NCST)                                MSPA0223
      NN=0                                                              MSPA0224
      DO 95 NES=1,NCST                                                  MSPA0225
      DO 85 I=1,NEAD                                                    MSPA0226
      POS=XG(I+1)                                                       MSPA0227
      IF(POS.GE.APCST(NES))NCOL=I                                       MSPA0228
      IF(POS GE.APCST(NES))GO TO 90                                     MSPA0229
   85 CONTINUE                                                          MSPA0230
   90 SLCS(NES)=(2.D0*APCST(NES)-XG(NCOL+1)-XG(NCOL))/(XG(NCOL+1)-      MSPA0231
     &XG(NCOL))                                                         MSPA0232
      DO 95 NE-1,NECD                                                   MSPA0233
      NN=NN+1                                                           MSPA0234
   95 LNYS(NN     )=NCOL+NEAD*(NE-1)                                    MSPA0235
      NN=0                                                              MSPA0236
      DO 96 I=1,NCST                                                    MSPA0237
      DO 96 J=1,NECD                                                    MSPA0238
      NN=NN+1                                                           MSPA0239
      MATYS(NN)=MATCS(I)                                                MSPA0240
      YSPROP(1,NN)=RHC(I)                                               MSPA0241
      YSPROP(2,NN)=RWC(I)                                               MSPA0242
      YSPROP(3,NN)=ECS(I)                                               MSPA0243
      YSPROP(4,NN)=ANUCS(I)                                             MSPA0244
      YSPROP(5,NN)=DENCS(I)                                             MSPA0245
      YSPROP(6,NN)=(SLCS(I)+1.0D0)/2.0D0                                MSPA0246
   96 YSPROP(7,NN)=XFCST(I)                                             MSPA0247
      NYST=NCST*NECD                                                    MSPA0248
  120 CONTINUE                                                          MSPA0249
      J1=NXST+1                                                         MSPA0250
      J2=NXST+NAXS                                                      MSPA0251
C    INFORMATION FOR ADDITIONAL STIFFENERS ON INDIVIDUAL ELEMENTS.      MSPA0252
      READ(MREAD,500)NAXS,NAYS                                          MSPA0253
      IF(NAXS.EQ 0)GO TO 125                                            MSPA0254
      READ(MREAD,500)(LNXS(J),J=J1,J2)                                  MSPA0255
      READ(MREAD,500)(MATXS(J),J=J1,J2)                                 MSPA0256
      DO 122 I=1,7                                                      MSPA0257
  122 READ(MREAD,510)(XSPROP(I,J),J=J1,J2)                              MSPA0258
  125 IF(NAYS EQ 0)GO TO 128                                            MSPA0259
      J1=NYST+1                                                         MSPA0260
      J2=NYST+NAYS                                                      MSPA0261
      READ(MREAD,500)(LNYS(J),J=J1,J2)                                  MSPA0262
      READ(MREAD,500)(MATYS(J),J=J1,J2)                                 MSPA0263
      DO 127 I=1,7                                                      MSPA0264
  127 READ(MREAD,510)(YSPROP(I,J),J=J1,J2)                              MSPA0265
  128 NXST=NXST+NAXS                                                    MSPA0266
      NYST=NYST+NAYS                                                    MSPA0267
C    PRINT INFORMATION                                                  MSPA0268
```

```
      WRITE(MWRITE,600)
      WRITE(MWRITE,605)XDIST,YDIST,ZPOS,TH,EPAN,ANUP,DENSP,NEAD,NECD,
     2NDPE,NET,NDT,NXST,NYST
      WRITE(MWRITE,625)
      WRITE(MWRITE,626)
      DO 130 I=1,NET
  130 WRITE(MWRITE,627)I,(NP(J,I),J=1,4)
      WRITE(MWRITE,700)
      WRITE(MWRITE,701)
      I=0
      DO 135 NSIDE=1,4
      NN=NCSB(NSIDE)
      IF(NN.EQ.0)WRITE(MWRITE,900)NSIDE
      IF(NN.EQ.1)WRITE(MWRITE,901)NSIDE
      IF(NN.EQ.2)WRITE(MWRITE,902)NSIDE
      IF(NN.EQ.3)WRITE(MWRITE,903)NSIDE
      IF(NN.EQ.4)WRITE(MWRITE,904)NSIDE
      IF(NN.EQ.5)WRITE(MWRITE,905)NSIDE
      IF(NN .EQ. 1) I=I+1
  135 CONTINUE
      IF(I .EQ. 0) GO TO 13503
      NO=2*I
      IF(I .EQ. 2) GO TO 13501
      TY=TY1
      PE=PE1
      GO TO 13502
13501 TY=TY2
      PE=PE2
13502 WRITE(MWRITE,634) NO,TY,PE
13503 CONTINUE
      IF(NB.EQ.0)GO TO 136
      WRITE(MWRITE,635)NB
      WRITE(MWRITE,636)(NBC(I),I=1,NB)
  136 CONTINUE
      IF(NXST.EQ.0)GO TO 150
      WRITE(MWRITE,650)
      WRITE(MWRITE,651)
      DO 140 I=1,NXST
  140 WRITE(MWRITE,655)I,LNXS(I),MATXS(I),(XSPROP(J,I),J=1,7)
  150 IF(NYST.EQ.0)GO TO 170
      WRITE(MWRITE,660)
      WRITE(MWRITE,661)
      DO 160 I=1,NYST
  160 WRITE(MWRITE,655)I,LNYS(I),MATYS(I),(YSPROP(J,I),J=1,7)
  170 CONTINUE
  500 FORMAT(20I4)
  510 FORMAT(5D16.7)
  600 FORMAT('0','AUTO-GENERATED FINITE-ELEMENT MESH INFORMATION FOR STI
     2FFENED OR UNSTIFFENED FLAT-PLATE PROBLEM')
  605 FORMAT(///
     25X,'X-LENGTH(IN)                 =',D15.7/
     35X,'Y-LENGTH(IN)                 =',D15.7/
     45X,'Z-POSITION(IN)               =',D15.7/
     55X,'THICKNESS(IN)                =',D15.7/
     65X,'YOUNGS MODULUS(PSI)          =',D15.7/
     75X,'POISSONS RATIO               =',D15.7/
     85X,'DENSITY(LB-SEC**2/IN**4)     =',D15.7/
     95X,'NO. OF ELEM. IN X-DIRECTION =',I15/
     A5X,'NO. OF ELEM. IN Y-DIRECTION =',I15/
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     B5X,'NO  OF D O F /ELEM          =',I15/                        MSPA0328
     C5X,'TOTAL NO. OF ELEMENTS       =',I15/                        MSPA0329
     D5X,'TOTAL NO  OF D O F          =',I15/                        MSPA0330
     E5X,'NO. OF X-DIR  STIFFENERS    =',I15/                        MSPA0331
     F5X,'NO. OF Y-DIR. STIFFENERS    =',I15)                        MSPA0332
  625 FORMAT(' ','THE GLOBAL NODE NUMBERS ASSOCIATED WITH EACH ELEMENT AMSPA0333
     2RE AS FOLLOWS  ')                                              MSPA0334
  626 FORMAT(' ',5X,'ELEMENT',5X,'GLOBAL NODE NUMBERS')              MSPA0335
  627 FORMAT(' ',5X,I6,5X,4I6)                                       MSPA0336
  700 FORMAT('0','BOUNDARY CONDITIONS (SEE WRITEUP FOR CONVENTION FOR SIMSPA0337
     2DE NUMBER AND BOUNDARY CONDITION)')                            MSPA0338
  701 FORMAT('0','SIDE NUMBER',5X,'BOUNDARY CONDITION')              MSPA0339
  900 FORMAT(' ',I6,10X,'FREE')                                      MSPA0340
  901 FORMAT(' ',I6,10X,'SYMMETRY')                                  MSPA0341
  902 FORMAT(' ',I6,10X,'IDEALLY CLAMPED')                           MSPA0342
  903 FORMAT(' ',I6,10X,'PINNED-FIXED')                              MSPA0343
  904 FORMAT(' ',I6,10X,'PINNED-FREE SLIDING Z-DIRECTION')           MSPA0344
  905 FORMAT(' ',I6,10X,'PINNED-FREE SLIDING/NORMAL (INPLANE)')      MSPA0345
  650 FORMAT('0','INFORMATION FOR X-DIRECTION STIFFENERS')           MSPA0346
  651 FORMAT('0','STIFF  NO.',2X,'ON ELEM ',1X,'MAT. NO.',3X,'THICKNESS'MSPA0347
     2,10X,'WIDTH',4X,'YOUNGS MOD.',2X,'POISSON RATIO',8X,'DENSITY',7X, MSPA0348
     3'ETA-LOC.',6X,'OFFSET,ZF')                                     MSPA0349
  655 FORMAT(' ',I7,5X,I5,I6,3X,7D15.5)                              MSPA0350
  660 FORMAT('0','INFORMATION FOR Y-DIRECTION STIFFENERS')           MSPA0351
  661 FORMAT('0','STIFF. NO ',2X,'ON ELEM.',1X,'MAT. NO.',3X,'THICKNESS'MSPA0352
     2,10X,'WIDTH',4X,'YOUNGS MOD.',2X,'POISSON RATIO',8X,'DENSITY',7X, MSPA0353
     3'PSI-LOC.',6X,'OFFSET,ZF')                                     MSPA0354
   634 FORMAT(3X,'(SYSTEM ENERGIES ARE 1/',I1,' THEIR ACTUAL VALUES BECAUMSPA0355
     +SE OF THE',2A4,'SYMMETRY CONDITION)')                          MSPA0356
  635 FORMAT('0','THE FOLLOWING',I6,2X,'DEGREES OF FREEDOM ARE CONSTRAINMSPA0357
     2ED')                                                           MSPA0358
  636 FORMAT(' ',20I5)                                               MSPA0359
      RETURN                                                         MSPA0360
      END                                                            MSPA0361
      SUBROUTINE MESHPM(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,YGI,   MSPM0000
     2XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)                     MSPM0001
      IMPLICIT REAL*8(A-H,O-Z)                                       MSPM0002
C                                                                    MSPM0003
C    ASRL TR 154-14. .  . ORIGINAL REPORT VERSION OF PROGRAM         MSPM0004
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980         MSPM0005
C                                                                    MSPM0006
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.MSPM0007
C                                                                    MSPM0008
      DIMENSION NP(4,1),NODE(1),XG(1),YG(1),ZG(1),NBC(1),BC(1),LNXS(1), MSPM0009
     2LNYS(1),XGI(1),YGI(1),XSPROP(7,1),YSPROP(7,1),MATXS(1),MATYS(1) MSPM0010
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                            MSPM0011
      COMMON/PLATE/XDIST,YDIST,TH,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP, MSPM0012
     2THALF                                                          MSPM0013
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                             MSPM0014
C   MANUAL MESH GENERATION FOR FLAT-PLATE STRUCTURES HAVING NON-RECTANG-MSPM0015
C   ULAR PLANFORM, VOIDS, OR REQUIRING SPECIAL CONSIDERATIONS NOT    MSPM0016
C   INCLUDED IN AUTO- GENERATION ROUTINE.                            MSPM0017
C                                                                    MSPM0018
      READ(MREAD,500)NNT,NET                                         MSPM0019
      READ(MREAD,510)EPAN,ANUP,DENSP,TH,ZPOS                         MSPM0020
      NDT=NNT*6                                                      MSPM0021
      NDPE=24                                                        MSPM0022
      NDPN=6                                                         MSPM0023
      THALF=TH/2.0                                                   MSPM0024
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C     READ IN NODAL CONNECTIVITY INFORMATION.
      DO 10 I=1,NET
 10   READ(MREAD,500)(NP(J,I),J=1,4)
      DO 20 I=1,NET
      INCI=(I-1)*NDPE
      DO 20 J=1,4
      INCJ=(J-1)*NDPN
      DO 20 K=1,NDPN
      INCIJK=INCI+INCJ+K
 20   NODE(INCIJK)=NP(J,I)*NDPN-NDPN+K
C     READ IN NODAL X AND Y COORDINATES.
      READ(MREAD,510)(XG(I),I=1,NNT)
      READ(MREAD,510)(YG(I),I=1,NNT)
      DO 30 I=1,NNT
      XGI(I)=XG(I)
      YGI(I)=YG(I)
 30   ZG(I)=ZPOS
C     READ IN CONSTRAINED DEGREES OF FREEDOM IN ASCENDING ORDER, NO
C     DUPLICATES
      READ(MREAD,500)NB
      IF(NB.EQ.0)GO TO 45
      READ(MREAD,500) (NBC(I),I=1,NB)
      DO 40 I=1,NB
 40   BC(I)=0.0
 45   CONTINUE
C     READ IN INFORMATION FOR STIFFENERS.
      READ(MREAD,500)NXST,NYST
C     PROPERTIES FOR X-DIRECTION STIFFENERS.
      IF(NXST.EQ.0)GO TO 60
      READ(MREAD,500)(LNXS(I),I=1,NXST)
      READ(MREAD,500)(MATXS(I),I=1,NXST)
      DO 50 I=1,7
 50   READ(MREAD,510)(XSPROP(I,J),J=1,NXST)
 60   CONTINUE
C     PROPERTIES FOR Y-DIRECTION STIFFENERS.
      IF(NYST.EQ.0)GO TO 80
      READ(MREAD,500)(LNYS(I),I=1,NYST)
      READ(MREAD,500)(MATYS(I),I=1,NYST)
      DO 70 I=1,7
 70   READ(MREAD,510)(YSPROP(I,J),J=1,NYST)
 80   CONTINUE
C     PRINT INFORMATION.
      WRITE(MWRITE,600)
      WRITE(MWRITE,605)EPAN,ANUP,DENSP,TH,ZPOS,NDPE,NET,NDT,NXST,NYST
      WRITE(MWRITE,625)
      WRITE(MWRITE,626)
      DO 130 I=1,NET
 130  WRITE(MWRITE,627)I,(NP(J,I),J=1,4)
      IF(NB.EQ.0)GO TO 136
      WRITE(MWRITE,635)NB
      WRITE(MWRITE,636)(NBC(I),I=1,NB)
 136  CONTINUE
      IF(NXST.EQ.0)GO TO 150
      WRITE(MWRITE,650)
      WRITE(MWRITE,651)
      DO 140 I=1,NXST
 140  WRITE(MWRITE,655)I,LNXS(I),MATXS(I),(XSPROP(J,I),J=1,7)
 150  IF(NYST.EQ.0)GO TO 170
      WRITE(MWRITE,660)
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      WRITE(MWRITE,661)                                                 MSPM0084
      DO 160 I=1,NYST                                                   MSPM0085
  160 WRITE(MWRITE,655)I,LNYS(I),MATYS(I),(YSPROP(J,I),J=1,7)           MSPM0086
  170 CONTINUE                                                          MSPM0087
  500 FORMAT(20I4)                                                      MSPM0088
  510 FORMAT(5D16.7)                                                    MSPM0089
  600 FORMAT('0','USER-GENERATED FINITE-ELEMENT MESH INFORMATION FOR STIMSPM0090
     2FFENED OR UNSTIFFENED FLAT-PLATE PROBLEM')                        MSPM0091
  605 FORMAT(///                                                        MSPM0092
     65X,'YOUNGS MODULUS(PSI)           =',D15.7/                       MSPM0093
     75X,'POISSONS RATIO                =',D15.7/                       MSPM0094
     85X,'DENSITY(LB-SEC**2/IN**4)      =',D15.7/                       MSPM0095
     55X,'THICKNESS(IN)                 =',D15.7/                       MSPM0096
     45X,'Z-POSITION(IN)                =',D15.7/                       MSPM0097
     B5X,'NO  OF D O F /ELEM.           =',I15/                         MSPM0098
     C5X,'TOTAL NO. OF ELEMENTS         =',I15/                         MSPM0099
     D5X,'TOTAL NO. OF D O F.           =',I15/                         MSPM0100
     E5X,'NO. OF X-DIR. STIFFENERS      =',I15/                         MSPM0101
     F5X,'NO. OF Y-DIR. STIFFENERS      =',I15)                         MSPM0102
  625 FORMAT(' ','THE GLOBAL NODE NUMBERS ASSOCIATED WITH EACH ELEMENT AMSPM0103
     2RE AS FOLLOWS .')                                                 MSPM0104
  626 FORMAT(' ',5X,'ELEMENT',5X,'GLOBAL NODE NUMBERS')                 MSPM0105
  627 FORMAT(' ',5X,I6,5X,4I6)                                          MSPM0106
  650 FORMAT('0','INFORMATION FOR X-DIRECTION STIFFENERS')              MSPM0107
  651 FORMAT('0','STIFF  NO ',2X,'ON ELEM.',1X,'MAT. NO.',3X,'THICKNESS'MSPM0108
     2,10X,'WIDTH',4X,'YOUNGS MOD.',2X,'POISSON RATIO',8X,'DENSITY',7X, MSPM0109
     3'ETA-LOC.',6X,'OFFSET,ZF')                                        MSPM0110
  655 FORMAT(' ',I7,5X,I6,3X,7D15.5)                                    MSPM0111
  660 FORMAT('0','INFORMATION FOR Y-DIRECTION STIFFENERS')              MSPM0112
  661 FORMAT('0','STIFF  NO.',2X,'ON ELEM.',1X,'MAT. NO.',3X,'THICKNESS'MSPM0113
     2,10X,'WIDTH',4X,'YOUNGS MOD.',2X,'POISSON RATIO',8X,'DENSITY',7X, MSPM0114
     3'PSI-LOC.',6X,'OFFSET,ZF')                                        MSPM0115
  635 FORMAT('0','THE FOLLOWING',I6,2X,'DEGREES OF FREEDOM ARE CONSTRAINMSPM0116
     2ED')                                                              MSPM0117
  636 FORMAT(' ',20I5)                                                  MSPM0118
      RETURN                                                            MSPM0119
      END                                                               MSPM0120
      SUBROUTINE OMULT(SQVCT,RWVCT,ACC,NCOL,KROW,NDEX)                  OMLT0000
      IMPLICIT REAL*8(A-H,O-Z)                                          OMLT0001
C                                                                       OMLT0002
C    ASRL TR 154-14 .. .. ORIGINAL REPORT VERSION OF PROGRAM            OMLT0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            OMLT0004
C                                                                       OMLT0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.OMLT0006
C                                                                       OMLT0007
       DIMENSION SQVCT(1),RWVCT(1),NCOL(1),ACC(1),KROW(1),NDEX(1)       OMLT0008
      COMMON/BAS/NROWS,NET,MN,NB,NIRREG,MNC                             OMLT0009
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                OMLT0010
C                                                                       OMLT0011
C      TO FIND ACC OF (SQVCT)*(RWVCT)=(ACC)                             OMLT0012
C                                                                       OMLT0013
       INDEX=0                                                          OMLT0014
       NROWM=NROWS-1                                                    OMLT0015
       IF (NIRREG .GT  0) GO TO 200                                     OMLT0016
C      HIGH SPEED PRODUCT FOR REGULAR MATRICES                          OMLT0017
       DO 100 NN=1,NROWM                                                OMLT0018
       SUM=0.0                                                          OMLT0019
       IP1=NN+1                                                         OMLT0020
       KST=NCOL(NN)                                                     OMLT0021
```

```
      INDEX=INDEX+NN-KST
      DO 101 KPL=KST,NN
      IJ=INDEX+KPL
  101 SUM=SUM+SQVCT(IJ)*RWVCT(KPL)
C     NOW FOR THE COLUMN ELEMENTS
      JNDEX=IJ
      DO 102 KPL=IP1,NROWS
      IF(NN LT.NCOL(KPL))GO TO 100
      JNDEX=JNDEX+KPL-NCOL(KPL)
  102 SUM=SUM+SQVCT(JNDEX)*RWVCT(KPL)
  100 ACC(NN)=ACC(NN)+SUM
C     NOW FOR THE LAST ROW
  104 KADD=NCOL(NROWS)
      SUM=0 0
      INDEX=INDEX+NROWS-KADD
      DO 103 KPL=KADD,NROWS
      IJ=INDEX+KPL
  103 SUM=SUM+SQVCT(IJ)*RWVCT(KPL)
      ACC(NROWS)=ACC(NROWS)+SUM
     RETURN
C     MEDIUM SPEED PRODUCT FOR NIRREG .LE. NROWS/2
  200 IF (NIRREG .GT. NROWS/2) GO TO 201
      DO 105 NN=1,NROWM
      IP1=NN+1
      KST=NCOL(NN)
      INDEX=INDEX+NN-KST
      SUM=0 0
      DO 106 KPL=KST,NN
      IJ=INDEX+KPL
  106 SUM=SUM+SQVCT(IJ)*RWVCT(KPL)
      NCK=0
      JNDEX=IJ
  107 DO 108 KPL=IP1,NROWS
      IF(NN .LT. NCOL(KPL)) GO TO 109
      JNDEX=JNDEX+KPL-NCOL(KPL)
  108 SUM=SUM+SQVCT(JNDEX)*RWVCT(KPL)
      GO TO 105
  109 NCK=NCK+1
      IF (NCK .GT.NIRREG) GO TO 105
      IF (KPL  GE  KROW(NCK)) GO TO 109
      IP1=KROW(NCK)
      JNDEX=NDEX(NCK)+NN
      GO TO 107
  105 ACC(NN)=ACC(NN)+SUM
      GO TO 104
  201 DO 503 NN=1,NROWM
      IP1=NN+1
      K=NCOL(NN)
      INDEX=INDEX+NN-K
      SUM=0 0
      DO 502 KRX=K,NN
      IJ=INDEX+KRX
  502 SUM=SUM+SQVCT(IJ)*RWVCT(KRX)
      JNDEX=IJ
      DO 504 KRX=IP1,NROWS
      K=NCOL(KRX)
      JNDEX=JNDEX+KRX-K
      IF (JN  LT. K) GO TO 504
      SUM=SUM+SQVCT(JNDEX)*RWVCT(KRX)
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 504   CONTINUE                                                         OMLT0081
 503   ACC(NN)=ACC(NN)+SUM                                              OMLT0082
       GO TO 104                                                        OMLT0083
      END                                                               OMLT0084
      SUBROUTINE PRTOP(NNSA,NVSA,NCON,NP)                               PRTO0000
      IMPLICIT REAL*8(A-H,O-Z)                                          PRTO0001
C                                                                       PRTO0002
C     ASRL TR 154-14.    . .ORIGINAL REPORT VERSION OF PROGRAM          PRTO0003
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980           PRTO0004
C                                                                       PRTO0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.PRTO0006
C                                                                       PRTO0007
      DIMENSION NVSA(1),NCON(4,1),NP(4,1)                               PRTO0008
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                               PRTO0009
      COMMON/ASP/SLASP(50),ELASP(50),EDIR(2,50),NASP,LNASP(50)          PRTO0010
      COMMON/PRNT/IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8               PRTO0011
      COMMON/GSOUT/NEGS,LNGS(200)                                       PRTO0012
      COMMON/SSOUT/NXSS,NYSS,INXSS(200),INYSS(200)                      PRTO0013
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                PRTO0014
C    PRINT CONTROL FOR PROGRAM  (0=NO, 1=YES)                           PRTO0015
C    IOP1  NODAL DISPLACEMENTS AND LOCATION                             PRTO0016
C    IOP2  ELEMENT CENTROID STRAINS, PRINCIPAL STRAIN AND DIRECTION.    PRTO0017
C    IOP3  STRAINS, PRINCIPAL STRAIN AND DIRECTION AT G.S. OF SPEC.     PRTO0018
C          ELEMENTS.                                                    PRTO0019
C    IOP4  STRAINS, PRINC  STRAIN AND DIR., AND ELONGATION IN SPECIFIED PRTO0020
C          DIRECTIONS AT ADDITIONAL LOCATIONS                           PRTO0021
C    IOP5  REACTION FORCES AT CONSTRAINED NODES.                        PRTO0022
C    IOP6  STRAIN AT GAUSSIAN STATIONS FOR SPECIFIED X- AND/OR Y-       PRTO0023
C          DIRECTION STIFFENERS.                                        PRTO0024
C    IOP7  SYSTEM ENERGIES.                                             PRTO0025
C    IOP8  STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION AT SPECIFIEDPRTO0026
C          NODES (USING NODAL AVERAGING).                               PRTO0027
C                                                                       PRTO0028
      READ(MREAD,500)IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8            PRTO0029
C    IF IOP3=1, READ IN ELEMENTS SPECIFIED FOR G.S. STRAIN OUTPUT.      PRTO0030
      IF(IOP3 EQ.0)GO TO 10                                             PRTO0031
      READ(MREAD,500)NEGS                                               PRTO0032
      READ(MREAD,500)(LNGS(I),I=1,NEGS)                                 PRTO0033
  10  CONTINUE                                                          PRTO0034
C    IF IOP4=1, READ IN INFO FOR ADDITIONAL POINTS.                     PRTO0035
      IF(IOP4 EQ 0)GO TO 50                                             PRTO0036
      READ(MREAD,500)NASP                                               PRTO0037
      DO 20  I=1,NASP                                                   PRTO0038
  20  READ(MREAD,520)LNASP(I),SLASP(I),ELASP(I)                         PRTO0039
      DO 30  I=1,NASP                                                   PRTO0040
      EDIR(1,I)=0 0D0                                                   PRTO0041
  30  EDIR(2,I)=90.0D0                                                  PRTO0042
      READ(MREAD,500)NASPE                                              PRTO0043
      IF(NASPE EQ.0)GO TO 50                                            PRTO0044
      DO 40  I=1,NASPE                                                  PRTO0045
      READ(MREAD,520)NSP,DIR1,DIR2                                      PRTO0046
       EDIR(1,NSP)=DIR1                                                 PRTO0047
  40   EDIR(2,NSP)=DIR2                                                 PRTO0048
  50  CONTINUE                                                          PRTO0049
C    IF IOP6=1, READ IN STIFFENER NUMBERS FOR STRAIN OUTPUT.            PRTO0050
      IF(IOP6 EQ 0)GO TO 51                                             PRTO0051
      READ(MREAD,500)NXSS,NYSS                                          PRTO0052
      IF(NXSS GT.0)READ(MREAD,500)(INXSS(J),J=1,NXSS)                   PRTO0053
      IF(NYSS GT.0)READ(MREAD,500)(INYSS(J),J=1,NYSS)                   PRTO0054
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```
 51   CONTINUE                                                          PRTO0055
C   IF IOP8=1, READ IN NODE NOS. FOR NODAL AVERAGE STRAIN CALCULATION.  PRTO0056
      IF(IOP8 EQ.0)GO TO 58                                             PRTO0057
      READ(MREAD,500)NNSA                                               PRTO0058
C   IF NNSA=NNT, THEN ALL NODES ARE INCLUDED AUTOMATICALLY              PRTO0059
      NNT=NDT/6                                                         PRTO0060
      IF(NNSA EQ.NNT)GO TO 53                                           PRTO0061
      READ(MREAD,500)(NVSA(I),I=1,NNSA)                                 PRTO0062
      GO TO 55                                                          PRTO0063
 53   CONTINUE                                                          PRTO0064
      DO 54 I=1,NNT                                                     PRTO0065
 54   NVSA(I)=I                                                         PRTO0066
 55   CONTINUE                                                          PRTO0067
C   DEFINE ELEMS. CONNECTED AT EACH REQUESTED NODE.                     PRTO0068
      DO 57 I=1,NNSA                                                    PRTO0069
      NN=NVSA(I)                                                        PRTO0070
      DO 57 J=1,4                                                       PRTO0071
      NCON(J,I)=0                                                       PRTO0072
      DO 56 K=1,NET                                                     PRTO0073
      IF(NP(J,K) NE.NN)GO TO 56                                         PRTO0074
      NCON(J,I)=K                                                       PRTO0075
      GO TO 57                                                          PRTO0076
 56   CONTINUE                                                          PRTO0077
 57   CONTINUE                                                          PRTO0078
 58   CONTINUE                                                          PRTO0079
      WRITE(MWRITE,600)                                                 PRTO0080
      WRITE(MWRITE,610)                                                 PRTO0081
      IF(IOP1 EQ 1)WRITE(MWRITE,620)                                    PRTO0082
      IF(IOP5 EQ 1)WRITE(MWRITE,625)                                    PRTO0083
      IF(IOP5 EQ.2)WRITE(MWRITE,630)                                    PRTO0084
      IF(IOP8 EQ.0)GO TO 59                                             PRTO0085
      WRITE(MWRITE,635)NNSA                                             PRTO0086
      WRITE(MWRITE,650)(NVSA(I),I=1,NNSA)                               PRTO0087
 59   CONTINUE                                                          PRTO0088
      IF(IOP3.EQ 0)GO TO 60                                             PRTO0089
      WRITE(MWRITE,640)NEGS                                             PRTO0090
      WRITE(MWRITE,650)(LNGS(I),I=1,NEGS)                               PRTO0091
 60   CONTINUE                                                          PRTO0092
      IF(IOP4 EQ 0)GO TO 80                                             PRTO0093
      WRITE(MWRITE,660)NASP                                             PRTO0094
      WRITE(MWRITE,670)                                                 PRTO0095
      DO 70 I=1,NASP                                                    PRTO0096
 70   WRITE(MWRITE,680)I,LNASP(I),SLASP(I),ELASP(I),EDIR(1,I),EDIR(2,I) PRTO0097
 80   CONTINUE                                                          PRTO0098
      IF(IOP6.EQ 0)GO TO 84                                             PRTO0099
      IF(NXSS EQ 0)GO TO 82                                             PRTO0100
      WRITE(MWRITE,700)NXSS                                             PRTO0101
      WRITE(MWRITE,650)(INXSS(J),J=1,NXSS)                              PRTO0102
 82   CONTINUE                                                          PRTO0103
      IF(NYSS.EQ 0)GO TO 84                                             PRTO0104
      WRITE(MWRITE ,710)NYSS                                            PRTO0105
      WRITE(MWRITE,650)(INYSS(J),J=1,NYSS)                              PRTO0106
 84   CONTINUE                                                          PRTO0107
      IF(IOP7.EQ 1)WRITE(MWRITE,690)                                    PRTO0108
 500  FORMAT(20I4)                                                      PRTO0109
 520  FORMAT(I4,2D16.7)                                                 PRTO0110
 600  FORMAT('0','***** OUTPUT CONTROL INFORMATION *****')              PRTO0111
 610  FORMAT('0','THE FOLLOWING RESULTS WILL BE GIVEN AT EVERY REGULAR PPRTO0112
     2RINTOUT CYCLE.')                                                  PRTO0113
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```
620  FORMAT(' ',5X,'NODAL DISPLACEMENTS AND LOCATION')                      PRTO0114
625  FORMAT(' ',5X,'REACTION FORCES AT CONSTRAINED NODES')                  PRTO0115
630  FORMAT(' ',5X,'STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION APRTO0116
    2T CENTROID OF EACH ELEMENT')                                           PRTO0117
635  FORMAT(' ',5X,'STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION APRTO0118
    2T THE FOLLOWING',I5,2X,'NODES (OBTAINED BY NODAL AVERAGING):')        PRTO0119
640  FORMAT(' ',5X,'STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION APRTO0120
    2T EACH GAUSSIAN STATION FOR EACH OF THE FOLLOWING',I5,2X,'ELEMENTSPRTO0121
    3 ')                                                                    PRTO0122
650  FORMAT(' ',10X,20I5)                                                   PRTO0123
660  FORMAT(' ',5X,'STRAIN COMPONENTS, ELONGATION IN SPECIFIED DIRECTIOPRTO0124
    2NS, PRINCIPAL STRAIN AND DIRECTION AT THE FOLLOWING',I5,2X,'ADDTL.PRTO0125
    3 POINTS ')                                                             PRTO0126
670  FORMAT(' ',5X,'ADDTL  POINT',2X,'ON ELEM.',4X,'PSI-LOCATION',4X,       PRTO0127
    2 ETA-LOCATION',2X,'ELONG. DIR.-1(DEG)',2X,'ELONG. DIR.-2(DEG)')        PRTO0128
680  FORMAT(' ',5X,I9,5X,I6,2X,2D16.7,4X,D16.7,4X,D16.7)                    PRTO0129
690  FORMAT(' ',5X,'SYSTEM ENERGIES')                                       PRTO0130
700  FORMAT(' ',5X,'STRAIN AT GAUSSIAN STATIONS FOR THE FOLLOWING',I5,      PRTO0131
    22X,'X-DIRECTION STIFFENERS.')                                          PRTO0132
710  FORMAT(' ',5X,'STRAIN AT GAUSSIAN STATIONS FOR THE FOLLOWING',I5,      PRTO0133
    22X,'Y-DIRECTION STIFFENERS.')                                          PRTO0134
     RETURN                                                                 PRTO0135
     END                                                                    PRTO0136
     SUBROUTINE PSTRN(G11,G22,G12,S,D)                                      PSTN0000
     IMPLICIT REAL*8(A-H,O-Z)                                               PSTN0001
C                                                                           PSTN0002
C    ASRL TR 154-14 ..   .ORIGINAL REPORT VERSION OF PROGRAM                PSTN0003
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980                  PSTN0004
C                                                                           PSTN0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.PSTN0006
C                                                                           PSTN0007
C   CALCULATE MAX  PRINCIPAL STRAIN (TENSILE ONLY) AND CORRESPONDING        PSTN0008
C   DIRECTION WITH RESPECT TO X-AXIS (IN DEGREES).                          PSTN0009
     C=DSQRT((G11-G22)**2+G12*G12)                                          PSTN0010
     A1=(G11+G22+C)*0.5                                                     PSTN0011
     A2=(G11+G22-C)*0.5                                                     PSTN0012
     S=0.0                                                                  PSTN0013
     IF(A1.GT.S)S=A1                                                        PSTN0014
     IF(A2.GT.S)S=A2                                                        PSTN0015
     IF(G11.NE.G22)GO TO 10                                                 PSTN0016
     D=45.0                                                                 PSTN0017
     RETURN                                                                 PSTN0018
10   CONTINUE                                                               PSTN0019
     X1=G11-G22                                                             PSTN0020
     D=DATAN2(G12,X1)                                                       PSTN0021
C   DIVISION OF D BY 2 AND CONVERSION TO DEGREES                            PSTN0022
     D=D*28.64789D+00                                                       PSTN0023
     IF(S.EQ.0.0)D=0.0                                                      PSTN0024
     RETURN                                                                 PSTN0025
     END                                                                    PSTN0026
     SUBROUTINE RCON(F,NBC)                                                 RCON0000
     IMPLICIT REAL*8(A-H,O-Z)                                               RCON0001
C                                                                           RCON0002
C    ASRL TR 154-14..    ORIGINAL REPORT VERSION OF PROGRAM                 RCON0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980                RCON0004
C                                                                           RCON0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.RCON0006
C                                                                           RCON0007
     DIMENSION F(1),NBC(1)                                                  RCON0008
```

```
      COMMON/BAS/NDT,NET,MN,NBD,NIRREG,MNC                              RCON0009
C                                                                       RCON0010
C THIS SUBROUTINE APPLIES BOUNDARY CONSTRAINTS TO THE FORCE VECTOR      RCON0011
C TERMS IN THE FORCE VECTOR CORRESPONDING TO CONSTRAINED DEGREES OF     RCON0012
C FREEDOM ARE ZEROED (IT IS ASSUMED THAT ALL PRESCRIBED DISPLACEMENTS   RCON0013
C ARE ZERO).                                                            RCON0014
C                                                                       RCON0015
C   ZEROING OF RHS ASSUMING FIXED CONDITIONS                            RCON0016
      IF(NBD EQ 0)RETURN                                                RCON0017
      DO 100 I=1,NBD                                                    RCON0018
      NN=NBC(I)                                                         RCON0019
 100  F(NN)=0 0                                                         RCON0020
      RETURN                                                            RCON0021
      END                                                               RCON0022
      SUBROUTINE RPLATE(NTYPE,ANU,XL,YL,TH,SL,CSA,SLAM,SMOI,KT,GBI,YMOD)RPLT0000
      IMPLICIT REAL*8(A-H,O-Z)                                          RPLT0001
      DOUBLE PRECISION KT                                               RPLT0002
C                                                                       RPLT0003
C    ASRL TR 154-14. .  ..ORIGINAL REPORT VERSION OF PROGRAM            RPLT0004
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            RPLT0005
C                                                                       RPLT0006
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.RPLT0007
C                                                                       RPLT0008
      DIMENSION GBI(24,24),KTLXS(10),KTLYS(10),CB1(4,4),D1(4,13),GS(6), RPLT0009
     2GW(6),STOR1(4,13),STOR2(24,24),KT(24,24),T(24)                    RPLT0010
C                                                                       RPLT0011
C THE STIFFNESS MATRIX FOR AN UNSTIFFENED RECTANGULAR PLATE ELEMENT     RPLT0012
C (NTYPE=1), OR AN X-DIRECTION STIFFENER ELEMENT (NTYPE=2), OR A        RPLT0013
C Y-DIRECTION STIFFENER ELEMENT (NTYPE=3) ARE FORMED IN THIS            RPLT0014
C SUBROUTINE  THE PERTINENT THEORETICAL BASIS FOR THE PLATE ELEMENT     RPLT0015
C IS FOUND IN SUBSECTION A 1 3 3. THE THEORETICAL BASIS FOR STIFFENER   RPLT0016
C ELEMENTS IS FOUND IN SUBSECTION A.2.3.                                RPLT0017
C                                                                       RPLT0018
      DATA KTLXS/2,4,13,15,17,18,20,22,23,24/                           RPLT0019
      DATA KTLYS/7,8,14.16,17,19,21,23,22,24/                           RPLT0020
      DO 5 I=1,24                                                       RPLT0021
      DO 5 J=1,24                                                       RPLT0022
 5    KT(I,J)=0.0                                                       RPLT0023
C   FORMATION OF K-TILDA FOR PLATE                                      RPLT0024
      IF(NTYPE NE 1)GO TO 160                                           RPLT0025
C   INPLANE CONTRIBUTIONS TO K-TILDA                                    RPLT0026
C   FORM C-BAR                                                          RPLT0027
      DO 10 I=1,4                                                       RPLT0028
      DO 10 J=1,4                                                       RPLT0029
 10   CB1(I,J)=0 0                                                      RPLT0030
      CB1(1,1)=1.0/(XL*XL)                                              RPLT0031
      CB1(1,2)=ANU/(XL*YL)                                              RPLT0032
      CB1(2,1)=CB1(1,2)                                                 RPLT0033
      CB1(2,2)=1 0/(YL*YL)                                              RPLT0034
      CB1(3,3)=(1 0-ANU)/(2.0*YL*YL)                                    RPLT0035
      CB1(3,4)=CB1(3,3)*YL/XL                                           RPLT0036
      CB1(4,3)=CB1(3,4)                                                 RPLT0037
      CB1(4,4)=(1 0-ANU)/(2.0*XL*XL)                                    RPLT0038
      DO 20 I=1,4                                                       RPLT0039
      DO 20 J=1,4                                                       RPLT0040
 20   CB1(I,J)=CB1(I,J)*TH/(1 0-ANU*ANU)                                RPLT0041
      NGS=2                                                             RPLT0042
      DO 30 I=1,4                                                       RPLT0043
      DO 30 J=1,8                                                       RPLT0044
```

```
 30   D1(I,J)=0.0
      CALL GAUSS(NGS,GS,GW)
      DO 35 II=1,NGS
      GS(II)=(1.0+GS(II))/2.0
 35   GW(II)=GW(II)/2.0
C   LOOP OVER GAUSS STATIONS
      DO 70 NY=1,NGS
      DO 70 NX=1,NGS
      D1(1,2)=1.0
      D1(1,4)=GS(NY)
      D1(2,7)=1 0
      D1(2,8)=GS(NX)
      D1(3,3)=1 0
      D1(3,4)=GS(NX)
      D1(4,6)=1.0
      D1(4,8)=GS(NY)
      DO 40 I=1,4
      DO 40 J=1,8
      STOR1(I,J)=0.0
      DO 40 K=1,4
 40   STOR1(I,J)=STOR1(I,J)+CB1(I,K)*D1(K,J)
      DO 50 I=1,8
      DO 50 J=1,8
      STOR2(I,J)=0 0
      DO 50 K=1,4
 50   STOR2(I,J)=STOR2(I,J)+D1(K,I)*STOR1(K,J)
      DO 60 I=1,8
      DO 60 J=1,8
 60   KT(I,J)=KT(I,J)+STOR2(I,J)*GW(NX)*GW(NY)
 70   CONTINUE
C   BENDING CONTRIBUTIONS TO K-TILDA
      DO 80 I=1,3
      DO 80 J=1,3
 80   CB1(I,J)=0 0
      CB1(1,1)=1 0/XL**4
      CB1(1,2)=ANU/(XL*XL*YL*YL)
      CB1(2,1)=CB1(1,2)
      CB1(2,2)=1 0/YL**4
      CB1(3,3)=(1.0-ANU)/(2.0*XL*XL*YL*YL)
      DO 90 I=1,3
      DO 90 J=1,3
 90   CB1(I,J)=CB1(I,J)*TH**3/(12.0*(1.0-ANU*ANU))
      NGS=4
      DO 100 I=1,3
      DO 100 J=1,13
 100  D1(I,J)=0 0
      CALL GAUSS(NGS,GS,GW)
      DO 105 II=1,NGS
      GS(II)=(1.0-GS(II))/2.0
 105  GW(II)=GW(II)/2.0
C   LOOP OVER GAUSS STATIONS
      DO 140 NY=1,NGS
      DO 140 NX=1,NGS
      D1(1,2)=2 0
      D1(1,4)=2 0*GS(NY)
      D1(1,6)=2 0*GS(NY)*GS(NY)
      D1(1,7)=6 0*GS(NX)
      D1(1,9)=D1(1,7)*GS(NY)
      D1(1,11)=D1(1,9)*GS(NY)
```

```
      D1(1,12)=D1(1,6)*GS(NY)
      D1(1,13)=D1(1,11)*GS(NY)
      D1(2,3)=2.0
      D1(2,5)=2.0*GS(NX)
      D1(2,6)=D1(2,5)*GS(NX)
      D1(2,8)=6.0*GS(NY)
      D1(2,10)=D1(2,8)*GS(NX)
      D1(2,11)=D1(2,6)*GS(NX)
      D1(2,12)=D1(2,10)*GS(NX)
      D1(2,13)=D1(2,12)*GS(NX)
      D1(3,1)=2.0
      D1(3,4)=4.0*GS(NX)
      D1(3,5)=4.0*GS(NY)
      D1(3,6)=8.0*GS(NX)*GS(NY)
      D1(3,9)=6.0*GS(NX)*GS(NX)
      D1(3,10)=6.0*GS(NY)*GS(NY)
      D1(3,11)=D1(3,9)*2.0*GS(NY)
      D1(3,12)=D1(3,10)*2.0*GS(NX)
      D1(3,13)=D1(3,9)*D1(3,10)/2.0
      DO 110 I=1,3
      DO 110 J=1,13
      STOR1(I,J)=0.0
      DO 110 K=1,3
  110 STOR1(I,J)=STOR1(I,J)+CB1(I,K)*D1(K,J)
      DO 120 I=1,13
      DO 120 J=1,13
      STOR2(I,J)=0.0
      DO 120 K=1,3
  120 STOR2(I,J)=STOR2(I,J)+D1(K,I)*STOR1(K,J)
      DO 130 I=1,13
      DO 130 J=1,13
  130 KT(I+11,J+11)=KT(I+11,J+11)+STOR2(I,J)*GW(NX)*GW(NY)
  140 CONTINUE
      DO 150 I=1,24
      DO 150 J=1,24
  150 KT(I,J)=KT(I,J)*XL*YL
C    THIS COMPLETES FORMATION OF K-TILDA FOR PLATE
      GO TO 270
  160 CONTINUE
C    FORM BASIC BLOCK (10 BY 10) FOR K-TILDA OF STIFFENERS
      DO 170 I=1,10
      DO 170 J=1,10
  170 STOR2(I,J)=0.0
      STOR2(1,1)=1.0
      STOR2(1,2)=SL
      STOR2(1,3)=2.0
      STOR2(1,4)=2.0*SL
      STOR2(1,5)=2.0*SL*SL
      STOR2(1,6)=3.0
      STOR2(1,7)=3.0*SL
      STOR2(1,8)=3.0*SL*SL
      STOR2(1,9)=STOR2(1,5)*SL
      STOR2(1,10)=3.0*SL**3
      DO 180 J=1,10
      STOR2(2,J)=STOR2(1,J)*SL
      STOR2(3,J)=STOR2(1,J)*2.0
      STOR2(4,J)=STOR2(3,J)*SL
  180 STOR2(5,J)=STOR2(4,J)*SL
      STOR2(6,6)=12.0
```

```
      STOR2(6,7)=12.0*SL                                                RPLT0163
      STOR2(6,8)=12.0*SL*SL                                             RPLT0164
      STOR2(6,9)=6.0*SL**3                                              RPLT0165
      STOR2(6,10)=2.0*STOR2(6,9)                                        RPLT0166
      DO 190 J=7,10                                                     RPLT0167
      STOR2(7,J)=STOR2(6,J)*SL                                          RPLT0168
      STOR2(8,J)=STOR2(7,J)*SL                                          RPLT0169
  190 STOR2(9,J)=STOR2(5,J)*SL                                          RPLT0170
      STOR2(10,10)=STOR2(8,10)*SL                                       RPLT0171
      DO 200 I=1,10                                                     RPLT0172
      DO 200 J=I,10                                                     RPLT0173
  200 STOR2(J,I)=STOR2(I,J)                                             RPLT0174
      IF(NTYPE.EQ.2)DL=XL                                               RPLT0175
      IF(NTYPE.EQ.3)DL=YL                                               RPLT0176
      C1=CSA/DL                                                         RPLT0177
      C2=C1*SLAM/DL                                                     RPLT0178
      C3=SMOI/(DL**3)                                                   RPLT0179
      DO 210 I=1,2                                                      RPLT0180
      DO 210 J=1,2                                                      RPLT0181
  210 STOR2(I,J)=STOR2(I,J)*C1                                          RPLT0182
      DO 220 I=1,2                                                      RPLT0183
      DO 220 J=3,10                                                     RPLT0184
      STOR2(I,J)=STOR2(I,J)*C2                                          RPLT0185
  220 STOR2(J,I)=STOR2(I,J)                                             RPLT0186
      DO 230 I=3,10                                                     RPLT0187
      DO 230 J=3,10                                                     RPLT0188
  230 STOR2(I,J)=STOR2(I,J)*C3                                          RPLT0189
C     EXPAND TO K-TILDA FOR X-DIRECTION STIFFENERS                      RPLT0190
      IF(NTYPE.EQ.3)GO TO 250                                           RPLT0191
      DO 240 I=1,10                                                     RPLT0192
      DO 240 J=1,10                                                     RPLT0193
      NI=KTLXS(I)                                                       RPLT0194
      NJ=KTLXS(J)                                                       RPLT0195
  240 KT(NI,NJ)=STOR2(I,J)                                              RPLT0196
      GO TO 270                                                         RPLT0197
C     THIS COMPLETES FORMATION OF K-TILDA FOR X-DIR. STIFFENER.         RPLT0198
  250 CONTINUE                                                          RPLT0199
C     EXPAND TO K-TILDA FOR Y-DIRECTION STIFFENERS                      RPLT0200
      DO 260 I=1,10                                                     RPLT0201
      DO 260 J=1,10                                                     RPLT0202
      NI=KTLYS(I)                                                       RPLT0203
      NJ=KTLYS(J)                                                       RPLT0204
  260 KT(NI,NJ)=STOR2(I,J)                                              RPLT0205
C     THIS COMPLETES FORMATION OF K-TILDA FOR Y-DIR. STIFFENER          RPLT0206
  270 CONTINUE                                                          RPLT0207
C     FORM K-BAR FOR ELEMENT                                            RPLT0208
      DO 280 I=1,24                                                     RPLT0209
      DO 280 J=1,24                                                     RPLT0210
      STOR2(I,J)=0.0                                                    RPLT0211
      DO 280 K=1,24                                                     RPLT0212
  280 STOR2(I,J)=STOR2(I,J)+KT(I,K)*GBI(K,J)                            RPLT0213
      DO 290 I=1,24                                                     RPLT0214
      DO 290 J=1,24                                                     RPLT0215
      KT(I,J)=0.0                                                       RPLT0216
      DO 290 K=1,24                                                     RPLT0217
  290 KT(I,J)=KT(I,J)+GBI(K,I)*STOR2(K,J)                               RPLT0218
C     FORM SCALING VECTOR FOR NODAL DOF                                 RPLT0219
      T(1)=1.0                                                          RPLT0220
      T(2)=1.0                                                          RPLT0221
```

```
      T(3)=1.0
      T(4)=XL
      T(5)=YL
      T(6)=XL*YL
      DO 300 I=1,6
      T(I+6)=T(I)
      T(I+12)=T(I)
  300 T(I+18)=T(I)
C   SCALE STIFFNESS MATRIX
      DO 310 I=1,24
      DO 310 J=1,24
  310 KT(I,J)=KT(I,J)*T(I)*T(J)*YMOD
      RETURN
      END
      SUBROUTINE SOLV(STIFM,G,SOL,NCOL,KROW,NDEX)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ASRL TR 154-13... . .ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
      DIMENSION STIFM(1),G(1),KROW(1),NDEX(1),SOL(1),NCOL(1)
      COMMON/BAS/NROWS,NET,MN,NBD,NIRREG,MNC
C
C THIS SUBROUTINE PERFORMS THE FORWARD AND BACKWARD SUBSTITUTION
C OPERATIONS REQUIRED TO COMPLETE THE SOLUTION OF A SYSTEM OF LINEAR
C SIMULTANEOUS EQUATIONS BY THE TRIPLE-FACTORING METHOD. THIS
C SUBROUTINE MUST BE USED IN CONJUNCTION WITH SUBROUTINE FACT.
C
C     SOLVE (LL*)(SOL)=(FORCE) FOR DISPLACEMENTS (SOL)
C     INTERMEDIATE SOLUTION USING THE LOWER TRIANGLE
  100 INDEX=0
      SOL(1)=G(1)
      DO 104 I=2,NROWS
      IM1=I-1
      SUM=0.0
      K=NCOL(I)
      INDEX=INDEX+I-K
      IF (IM1-K) 103,101,101
  101 DO 102 J=K,IM1
      IJ=INDEX+J
      SU=SOL(J)
  102 SUM=SUM+STIFM(IJ)*SU
  103 II=INDEX+I
  104 SOL(I)=G(I)-SUM
C     SOL CONTAINS THE INTERMEDIATE SOLUTION
C     COMPLETE THE SOLUTION USING THE UPPER TRIANGLE
      SOL(NROWS)=SOL(NROWS)/STIFM(II)
      INDEX=INDEX-NROWS+NCOL(NROWS)
      IF(NIRREG.GT.0) GO TO 111
      DO 109 KK=2,NROWS
      I=NROWS+1-KK
      IP1=I+1
      SUM=0.0
      JNDEX=INDEX+I
      DO 107 J=IP1,NROWS
      K=NCOL(J)
      IF (I-K) 108,106,106
```

```
  106 JNDEX=JNDEX+J-K                                                 SOLV0045
      SU=SOL(J)                                                       SOLV0046
  107 SUM=SUM+STIFM(JNDEX)*SU                                         SOLV0047
  108 II=INDEX+I                                                      SOLV0048
      SOL(I)=SOL(I)/STIFM(II)-SUM                                     SOLV0049
  109 INDEX=INDEX-I+NCOL(I)                                           SOLV0050
      RETURN                                                          SOLV0051
  111 IF(NIRREG-NROWS/2) 116,116,112                                  SOLV0052
C     TOO MANY IRREGULAR ROWS FOR ACCELERATED SOLUTION                SOLV0053
  112 DO 115 KK=2,NROWS                                               SOLV0054
      I=NROWS+1-KK                                                    SOLV0055
      IP1=I+1                                                         SOLV0056
      JNDEX=INDEX+I                                                   SOLV0057
      SUM=0.0                                                         SOLV0058
      JNDEX=INDEX+I                                                   SOLV0059
      DO 114 J=IP1,NROWS                                              SOLV0060
      K=NCOL(J)                                                       SOLV0061
      JNDEX=JNDEX+J-K                                                 SOLV0062
      IF (I-K) 114,113,113                                            SOLV0063
  113 SU=SOL(J)                                                       SOLV0064
      SUM=SUM+STIFM(JNDEX)*SU                                         SOLV0065
  114 CONTINUE                                                        SOLV0066
      II=INDEX+I                                                      SOLV0067
      SOL(I)=SOL(I)/STIFM(II)-SUM                                     SOLV0068
  115 INDEX=INDEX-I+NCOL(I)                                           SOLV0069
      PETURN                                                          SOLV0070
C     ACCELERATED SOLUTION FOR CASE WITH IRREGULAR ROWS               SOLV0071
  116 DO 125 KK=2,NROWS                                               SOLV0072
      I=NROWS+1-KK                                                    SOLV0073
      IP1=I+1                                                         SOLV0074
      SUM=0 0                                                         SOLV0075
      NCK=0                                                           SOLV0076
      JNDEX=INDEX+I                                                   SOLV0077
  117 DO 119 J=IP1,NROWS                                              SOLV0078
      K=NCOL(J)                                                       SOLV0079
      IF (I-K) 120,118,118                                            SOLV0080
  118 JNDEX=JNDEX+J-K                                                 SOLV0081
      SU=SOL(J)                                                       SOLV0082
  119 SUM=SUM+STIFM(JNDEX)*SU                                         SOLV0083
      GO TO 124                                                       SOLV0084
  120 NCK=NCK+1                                                       SOLV0085
      IF(NIRREG-NCK) 124,121,121                                      SOLV0086
  121 IP1=KROW(NCK)                                                   SOLV0087
      IF (I-NCOL(IP1)) 120,122,122                                    SOLV0088
  122 IF (IP1-J) 120,123,123                                          SOLV0089
  123 JNDEX=NDEX(NCK)+I                                               SOLV0090
      GO TO 117                                                       SOLV0091
  124 II=INDEX+I                                                      SOLV0092
      SOL(I)=SOL(I)/STIFM(II)-SUM                                     SOLV0093
  125 INDEX=INDEX-I+NCOL(I)                                           SOLV0094
      RETURN                                                          SOLV0095
      END                                                             SOLV0096
      SUBROUTINE SPRING(EK,SS,ISIDE,XL,YL)                            SPNG0000
      IMPLICIT REAL*8(A-H,O-Z)                                        SPNG0001
C                                                                     SPNG0002
C    ASRL TR 154-14 . . ..ORIGINAL REPORT VERSION OF PROGRAM          SPNG0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980          SPNG0004
C                                                                     SPNG0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.SPNG0006
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C
      DIMENSION EK(24,24),SS(5),SK(12,12),LIST(12,4),T(24)
      DATA LIST/1,2,3,4,5,6,7,8,9,10,11,12,7,8,9,10,11,12,13,14,15,16,
     B17,18,19,20,21,22,23,24,13,14,15,16,17,18,1,2,3,4,5,6,19,20,21,
     C22,23,24/
C    SUBROUTINE TO CALCULATE THE EFFECTIVE ELEMENT STIFFNESS MATRIX,
C    EK(24,24), CORRESPONDING TO LINE TRANSLATIONAL AND TORSIONAL LINEAR
C    RESTORING SPRINGS APPLIED ALONG THE SIDE (ISIDE) OF THE RECTANGULAR
C    PLATE ELEMENT, RPLATE.
C    THE SPRING CONSTANTS, SS(I),I=1,5 CORRESPOND, RESPECTIVELY, TO
C    TRANSLATIONAL SPRINGS (LB/IN) IN THE U, V, AND W DISPLACEMENT
C    DIRECTIONS AND ROTATIONAL SPRINGS (IN-LB/IN) IN THE W,X AND W,Y
C    ROTATION DIRECTIONS
C    SIDE NOTATION ,
C         ISIDE=1 ALONG SIDE Y=0.
C         ISIDE=2 ALONG SIDE X=XL.
C         ISIDE=3 ALONG SIDE Y=YL.
C         ISIDE=4 ALONG SIDE X=0.
C
C
      DO 10 I=1,12
      DO 10 J=1,12
   10 SK(I,J)=0.0
      DO 20 I=1,24
      DO 20 J=1,24
   20 EK(I,J)=0.0
C    ASSIGN TERMS WHICH ARE COMMON TO ALL SIDES.
      SK(1,1)=SS(1)/3 0
      SK(2,2)=SS(2)/3 0
      SK(7,1)-SS(1)/6 0
      SK(7,7)=SS(1)/3 0
      SK(8,2)=SS(2)/6.0
      SK(8,8)=SS(2)/3.0
C    ASSIGN TERMS FOR SIDES 1 OR 3.
      IF(ISIDE.EQ 2 OR ISIDE EQ 4)GO TO 30
      SL=XL
      SK(3,3)=13 0*SS(3)/35.0+6.0*SS(4)/(5 0*XL*XL)
      SK(4,3)=11 0*SS(3)/210.0+SS(4)/(10.0*XL*XL)
      SK(4,4)=SS(3)/105 0+2.0*SS(4)/(15.0*XL*XL)
      SK(5,5)=13 0*SS(5)/(35.0*YL*YL)
      SK(6,5)=11 0*SS(5)/(210.0*YL*YL)
      SK(6,6)=SS(5)/(105.0*YL*YL)
      SK(9,3)=SS(3)/70 0-6.0*SS(4)/(5 0*XL*XL)
      SK(9,4)=13 0*SS(3)/420.0-SS(4)/(10.0*XL*XL)
      SK(9,9)=SK(3,3)
      SK(10,3)=-SK(9,4)
      SK(10,4)=-SS(3)/140 0-SS(4)/(30.0*XL*XL)
      SK(10,9)=-SK(4,3)
      SK(10,10)=SK(4,4)
      SK(11,5)=SS(5)/(70 0*YL*YL)
      SK(11,6)=13 0*SS(5)/(420 0*YL*YL)
      SK(11,11)=SK(5,5)
      SK(12,5)=-SK(11,6)
      SK(12,6)=-SK(11,5)/2.0
      SK(12,11)=-SK(6,5)
      SK(12,12)=SK(6,6)
      GO TO 40
C    ASSIGN TERMS FOR SIDE 2 OR 4.
   30 SL=YL
```

```
      SK(3,3)=13.0*SS(3)/35.0+6.0*SS(5)/(5.0*YL*YL)                     SPNG0066
      SK(4,4)=13.0*SS(4)/(35.0*XL*XL)                                   SPNG0067
      SK(5,3)=11.0*SS(3)/210.0+SS(5)/(10.0*YL*YL)                       SPNG0068
      SK(5,5)=SS(3)/105.0+2.0*SS(5)/(15.0*YL*YL)                        SPNG0069
      SK(6,4)=11.0*SS(4)/(210.0*XL*XL)                                  SPNG0070
      SK(6,6)=SS(4)/(105.0*XL*XL)                                       SPNG0071
      SK(9,3)=SS(3)/70.0-6.0*SS(5)/(5.0*YL*YL)                          SPNG0072
      SK(9,5)=13.0*SS(3)/420.0-SS(5)/(10.0*YL*YL)                       SPNG0073
      SK(9,9)=SK(3,3)                                                   SPNG0074
      SK(10,4)=SS(4)/(70.0*XL*XL)                                       SPNG0075
      SK(10,6)=13.0*SS(4)/(420.0*XL*XL)                                 SPNG0076
      SK(10,10)=SK(4,4)                                                 SPNG0077
      SK(11,3)=-SK(9,5)                                                 SPNG0078
      SK(11,5)=-SS(3)/140.0-SS(5)/(30.0*YL*YL)                          SPNG0079
      SK(11,9)=-SK(5,3)                                                 SPNG0080
      SK(11,11)=SK(5,5)                                                 SPNG0081
      SK(12,4)=-SK(10,6)                                                SPNG0082
      SK(12,6)=-SS(4)/(140.0*XL*XL)                                     SPNG0083
      SK(12,10)=-SK(6,4)                                                SPNG0084
      SK(12,12)=SK(6,6)                                                 SPNG0085
   40 CONTINUE                                                          SPNG0086
C     FORM UPPER TRIANGLE AND MULTIPLY BY SIDE LENGTH.                  SPNG0087
      DO 50 I=1,12                                                      SPNG0088
      DO 50 J=1,I                                                       SPNG0089
      SK(I,J)=SK(I,J)*SL                                                SPNG0090
   50 SK(J,I)=SK(I,J)                                                   SPNG0091
C     ASSEMBLE INTO APPROPRIATE LOCATIONS IN 24 BY 24 STIFFNESS MATRIX. SPNG0092
      DO 60 I=1,12                                                      SPNG0093
      IR=LIST(I,ISIDE)                                                  SPNG0094
      DO 60 J=1,12                                                      SPNG0095
      IC=LIST(J,ISIDE)                                                  SPNG0096
   60 EK(IR,IC)=SK(I,J)                                                 SPNG0097
C     FORM TRANSFORMATION VECTOR.                                       SPNG0098
      DO 70 I=1,24                                                      SPNG0099
   70 T(I)=1.0                                                          SPNG0100
      DO 80 I=4,22,6                                                    SPNG0101
      T(I)=XL                                                           SPNG0102
      T(I+1)=YL                                                         SPNG0103
   80 T(I+2)=XL*YL                                                      SPNG0104
C     SCALE STIFFNESS MATRIX.                                           SPNG0105
      DO 90 I=1,24                                                      SPNG0106
      DO 90 J=1,24                                                      SPNG0107
   90 EK(I,J)=EK(I,J)*T(I)*T(J)                                         SPNG0108
      RETURN                                                            SPNG0109
      END                                                               SPNG0110
```

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      SUBROUTINE SFDM(SL,EL,XL,YL,GBI,STD)                              SFDM0000
      IMPLICIT REAL*8(A-H,O-Z)                                          SFDM0001
C                                                                       SFDM0002
C     ASRL TR 154-14... .. ORIGINAL REPORT VERSION OF PROGRAM           SFDM0003
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980           SFDM0004
C                                                                       SFDM0005
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.SFDM0006
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER             SFDM0007
      DIMENSION D(9,24),STD(9,24),GBI(24,24)                            SFDM0008
C    SUBROUTINE TO CALCULATE MATRIX WHICH DETERMINES STRAIN             SFDM0009
C    COMPONENTS FROM NODAL DISPL..  ORDER IS U,X  V,Y   U,Y+V,X   W,XX   SFDM0010
C    W,YY  2*W,XY   W,X     W,Y    0.5*(U,Y-V,X)                        SFDM0011
      DO 10 I=1,9                                                       SFDM0012
      DO 10 J=1,24                                                      SFDM0013
 10   D(I,J)=0.0                                                        SFDM0014
      D(1,2)=1.0/XL                                                     SFDM0015
      D(1,4)=EL/XL                                                      SFDM0016
      D(2,7)=1.0/YL                                                     SFDM0017
      D(2,8)=SL/YL                                                      SFDM0018
      D(3,3)=1.0/YL                                                     SFDM0019
      D(3,4)=SL/YL                                                      SFDM0020
      D(3,6)=1.0/XL                                                     SFDM0021
      D(3,8)=EL/XL                                                      SFDM0022
      D(4,13)=2.0                                                       SFDM0023
      D(4,15)=2.0*EL                                                    SFDM0024
      D(4,17)=D(4,15)*EL                                                SFDM0025
      D(4,18)=6.0*SL                                                    SFDM0026
      D(4,20)=D(4,18)*EL                                                SFDM0027
      D(4,22)=D(4,20)*EL                                                SFDM0028
      D(4,23)=D(4,17)*EL                                                SFDM0029
      D(4,24)=D(4,22)*EL                                                SFDM0030
      D(5,14)=2.0                                                       SFDM0031
      D(5,16)=2.0*SL                                                    SFDM0032
      D(5,17)=D(5,16)*SL                                                SFDM0033
      D(5,19)=6.0*EL                                                    SFDM0034
      D(5,21)=D(5,19)*SL                                                SFDM0035
      D(5,22)=D(5,17)*SL                                                SFDM0036
      D(5,23)=D(5,21)*SL                                                SFDM0037
      D(5,24)=D(5,23)*SL                                                SFDM0038
      D(6,12)=2.0                                                       SFDM0039
      D(6,15)=4.0*SL                                                    SFDM0040
      D(6,16)=4.0*EL                                                    SFDM0041
      D(6,17)=8.0*SL*EL                                                 SFDM0042
      D(6,20)=6.0*SL*SL                                                 SFDM0043
      D(6,21)=6.0*EL*EL                                                 SFDM0044
      D(6,22)=12.0*SL*SL*EL                                             SFDM0045
      D(6,23)=12.0*SL*EL*EL                                             SFDM0046
      D(6,24)=18.0*SL*SL*EL*EL                                          SFDM0047
      D(7,10)=1.0                                                       SFDM0048
      D(7,12)=EL                                                        SFDM0049
      D(7,13)=2.0*SL                                                    SFDM0050
      D(7,15)=D(7,13)*EL                                                SFDM0051
      D(7,16)=EL*EL                                                     SFDM0052
      D(7,17)=D(7,15)*EL                                                SFDM0053
      D(7,18)=3.0*SL*SL                                                 SFDM0054
      D(7,20)=D(7,18)*EL                                                SFDM0055
      D(7,21)=D(7,16)*EL                                                SFDM0056
      D(7,22)=D(7,20)*EL                                                SFDM0057
      D(7,23)=D(7,17)*EL                                                SFDM0058
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      D(7,24)=D(7,22)*EL
      D(8,11)=1 0
      D(8,12)=SL
      D(8,14)=2 0*EL
      D(8,15)=SL*SL
      D(8,16)=D(8,14)*SL
      D(8,17)=D(8,16)*SL
      D(8,19)=3.0*EL*EL
      D(8,20)=D(8,15)*SL
      D(8,21)=D(8,19)*SL
      D(8,22)=D(8,16)*SL*SL
      D(8,23)=D(8,21)*SL
      D(8,24)=D(8,23)*SL
      D(9,3)=1.0/(2 0*YL)
      D(9,4)=SL/(2.0*YL)
      D(9,6)=-1 0/(2.0*XL)
      D(9,8)=-EL/(2 0*XL)
      DO 20 J=10,24
      D(7,J)=D(7,J)/XL
      D(8,J)=D(8,J)/YL
      D(6,J)=D(6,J)/(XL*YL)
      D(4,J)=D(4,J)/(XL*XL)
   20 D(5,J)=D(5,J)/(YL*YL)
      DO 30 I=1,9
      DO 30 J=1,24
      STD(I,J)=0.0
      DO 30 K=1,24
   30 STD(I,J)=STD(I,J)+D(I,K)*GBI(K,J)
      DO 40 J=4,22,3
      DO 40 I=1,9
      STD(I,J)=STD(I,J)*XL
      STD(I,J+1)=STD(I,J+1)*YL
   40 STD(I,J+2)=STD(I,J+2)*XL*YL
      RETURN
      END
      SUBROUTINE STRESA(DISP,DELD,FLVA,ITT,NODE,MAXEL,MAX2,
     2TSGEE,PLASTW,XGI,YGI,NP,STDEA,GBI,NMSUB,DSR,PSR,SUBW,SNOM,NXST,
     3XSPROP,LNXS,MATXS,NZS)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.     . ORIGINAL REPORT VERSION OF PROGRAM
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C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C
      DIMENSION DISP(1),DELD(1),FLVA(1),SNL(9),EQNL(36),EP(9),DEP(9),
     2ELFP(36),NODE(1),              TSGEE(MAX2,NZS,3)
      DIMENSION XGI(1),YGI(1),NP(4,1),STDEA(9,24),GBI(24,24)
      DIMENSION NMSUB(1),DSR(1),PSR(1),SUBW(5,1),SNOM(5,1),ZGAUS(6),
     2ZGAUW(6),AGAUS(6),AGAUW(6),RFL(4),RZETA(4),RWEGH(20),SNO(5),
     3XSPROP(7,1),LNXS(1),MATXS(1)
      COMMON/PLATE/XDIST,YDIST,TH,SLE,ELE,NEAD,NECD,DENSP,EPAN,ANUP,
     2THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
C
C THIS SUBROUTINE CALCULATES THE EFFECTIVE NODAL FORCE VECTOR FOR AN
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C X-DIRECTION STIFFENER ELEMENT CORRESPONDING TO NONLINEAR GEOMETRIC
C AND MATERIAL EFFECTS  THE EXPRESSION FOR THIS VECTOR IS GIVEN
C BY EQUATION A.97
C
      NA=3
      NZ=4
      CALL GAUSS(NA,AGAUS,AGAUW)
      CALL GAUSS(NZ,ZGAUS,ZGAUW)
      DO 50 NST=1,NXST
      RH=XSPROP(1,NST)
      RW=XSPROP(2,NST)
      YOUNG=XSPROP(3,NST)
      HNU=XSPROP(4,NST)
      EL=XSPROP(6,NST)
      XF=XSPROP(7,NST)
      LNUMR=LNXS(NST)
      LNUM=NST
      MAT=MATXS(NST)
      D=DSR(MAT)
      P=PSR(MAT)
      NSUBL=NMSUB(MAT)
      DO 10 LZ=1,NZ
      RFL(LZ)=0.5*RH*ZGAUW(LZ)
   10 RZETA(LZ)=0.5*RH*ZGAUS(LZ)-XF
      DO 11 LZ=1,NZ
      DO 11 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
   11 RWEGH(JZ)=RFL(LZ)*SUBW(MAT,LS)
      DO 12 LS=1,NSUBL
   12 SNO(LS)=SNOM(MAT,LS)
      CC5=0.0
      CC6=0.0
      IF(D.EQ.0.0)GO TO 13
      CC5=1.0/P
      CC6=1.0/D/DELTAT
   13 CONTINUE
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      SLE=(XGI(NNN2)-XGI(NNN1))/2.0
      ELE=(YGI(NNN4)-YGI(NNN1))/2.0
      XL=2.0*SLE
      YL=2.0*ELE
       DO 99 J=1,24
   99  ELFP(J)=0.0
      PLAST2=0.0
       DO 100 KS=1,1
       SW=RW
       DO 100 KE=1,NA
      SL=(AGAUS(KE)+1.0)/2.0
      CALL SFDM(SL,EL,XL,YL,GBI,STDEA)
       EW=SLE*AGAUW(KE)
       SWEW=SW*EW
       JA=KE+(KS-1)*NA
       DO 60 J=1,9
       EP(J)=0.0
       DEP(J)=0.0
       DO 60 K=1,24
      NNN=(LNUMR-1)*24+K
```

```
      INDEX=NODE(NNN)
       EP(J)=EP(J)+STDEA(    J,K)*DISP(INDEX)
       DEP(J)=DEP(J)+STDEA(    J,K)*DELD(INDEX)
   60  CONTINUE
       EPEEM=EP(1)+EP(7)**2/2.
       DEPEEM=DEP(1)+EP(7)*DEP(7)-DEP(7)**2/2.
       BPNEE=0.0
       BPMEE=0.0
      PLAST1=0 0
       DO 150 LZ=1,NZ
       EPEEZ=EPEEM-RZETA(LZ)*EP(4)
       DEPEEZ=DEPEEM-RZETA(LZ)*DEP(4)
       GPEE=YOUNG*EPEEZ
       PTEE=0 0
       FACTOR=1.
       IF(D EQ 0.0) GO TO 152
      FACTOR=1.+(CC6*DABS(DEPEEZ))**CC5
  152  DO 155 LS=1,NSUBL
       JZ=LZ+(LS-1)*NZ
       SNY=SNO(LS)*FACTOR
       TAEE=TSGEE(LNUM ,JZ,JA)
       DSNEE=YOUNG*DEPEEZ
       TMEE=TAEE+DSNEE
       IF(TMEE-SNY) 20,45,21
   21  TAEE=SNY
      PSS1=(TMEE-SNY)/YOUNG
      PLAST1=PLAST1+PSS1*TAEE*RWEGH(JZ)
       GO TO 156
   20  IF(TMEE+SNY) 22,45,45
   22  TAEE=-SNY
      PSS1=(TMEE+SNY)/YOUNG
      PLAST1=PLAST1+PSS1*TAEE*RWEGH(JZ)
       GO TO 156
   45  CONTINUE
       TAEE=TMEE
  156  CONTINUE
       TSGEE(LNUM ,JZ,JA)=TAEE
       PLAEE=GPEE-TSGEE(LNUM ,JZ,JA)
       PTEE=PTEE+PLAEE*RWEGH(JZ)
  155  CONTINUE
       BPNEE=BPNEE+PTEE
       BPMEE=BPMEE-PTEE*RZETA(LZ)
  150  CONTINUE
       DO 163 J=1,9
  163  SNL(J)=0 0
       SNL(1)=YOUNG*RH*EP(7)**2/2.-BPNEE
       SNL(4)=YOUNG*RH*XF*EP(7)**2/2.-BPMEE
       SNL(7)=(YOUNG*RH*(EPEEM+XF*EP(4))-BPNEE)*EP(7)
       DO 164 J=1,9
  164  SNL(J)=SNL(J)*SWEW
       DO 165 J=1,24
       EQNL(J)=0.0
       DO 165 K=1,9
  165  EQNL(J)=EQNL(J)+STDEA(    K,J)*SNL(K)
       DO 166 J=1,24
  166  ELFP(J)=ELFP(J)+EQNL(J)
      PLAST2=PLAST2+PLAST1*SWEW
  100  CONTINUE
       DO 201 J=1,24
```

```
      NNN=(LNUMR-1)*24+J
      M=NODE(NNN)
 201   FLVA(M)=FLVA(M)-ELFP(J)
      PLASTW=PLASTW+PLAST2
 50    CONTINUE
       RETURN
      END
      SUBROUTINE STRESC(DISP,DELD,FLVA,ITT,NODE,MAXEL,MAX2,
     2TSGEE,PLASTW,XGI,YGI,NP,STDEC,GBI,NMSUB,DSR,PSR,SUBW,SNOM,NYST,
     3YSPROP,LNYS,MATYS,NZS)
      IMPLICIT REAL*8(A-H,O-Z)
C
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C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C
      DIMENSION DISP(1),DELD(1),FLVA(1),SNL(9),EQNL(36),EP(9),DEP(9),
     2ELFP(36),NODE(1),              TSGEE(MAX2,NZS,3)
      DIMENSION XGI(1),YGI(1),NP(4,1),STDEC(9,24),GBI(24,24)
      DIMENSION NMSUB(1),DSR(1),PSR(1),SUBW(5,1),SNOM(5 1),ZGAUS(6),
     2ZGAUW(6),AGAUS(6),AGAUW(6),RFL(4),RZETA(4),RWEGH(20),SNO(5),
     3YSPROP(7,1),LNYS(1),MATYS(1)
      COMMON/PLATE/XDIST,YDIST,TH,SLE,ELE,NEAD,NECD,DENSP,EPAN,ANUP,
     2THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
C
C THIS SUBROUTINE CALCULATES THE EFFECTIVE NODAL FORCE VECTOR FOR A
C Y-DIRECTION STIFFENER ELEMENT CORRESPONDING TO NONLINEAR GEOMETRIC
C AND MATERIAL EFFECTS. THE EXPRESSION FOR THIS VECTOR IS GIVEN
C BY EQUATION A.98.
C
      NA=3
      NZ=4
      CALL GAUSS(NA,AGAUS,AGAUW)
      CALL GAUSS(NZ,ZGAUS,ZGAUW)
      DO 50 NST=1,NYST
      RH=YSPROP(1,NST)
      RW=YSPROP(2,NST)
      YOUNG=YSPROP(3 NST)
      HNU=YSPROP(4,NST)
      SL=YSPROP(6,NST)
      XF=YSPROP(7,NST)
      LNUMR=LNYS(NST)
      LNUM=NST
      MAT=MATYS(NST)
      D=DSR(MAT)
      P=PSR(MAT)
      NSUBL=NMSUB(MAT)
      DO 10 LZ=1,NZ
      RFL(LZ)=0 5*RH*ZGAUW(LZ)
 10   RZETA(LZ)=0.5*RH*ZGAUS(LZ)-XF
      DO 11 LZ=1,NZ
      DO 11 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
 11   RWEGH(JZ)=RFL(LZ)*SUBW(MAT,LS)
```

```
      DO 12 LS=1,NSUBL
   12 SNO(LS)=SNOM(MAT,LS)
      CC5=0.0
      CC6=0.0
      IF(D.EQ.0.0)GO TO 13
      CC5=1.0/P
      CC6=1.0/D/DELTAT
   13 CONTINUE
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      SLE=(XGI(NNN2)-XGI(NNN1))/2.0
      ELE=(YGI(NNN4)-YGI(NNN1))/2.0
      XL=2.0*SLE
      YL=2.0*ELE
       DO 99 J=1,24
   99  ELFP(J)=0.0
      PLAST2=0.0
       DO 100 KS=1,1
       SW=RW
       DO 100 KE=1,NA
      EL=(AGAUS(KE)+1.0)/2.0
      CALL SFDM(SL,EL,XL,YL,GBI,STDEC)
       EW=ELE*AGAUW(KE)
       SWEW=SW*EW
       JA=KE+(KS-1)*NA
       DO 60 J=1,9
       EP(J)=0.0
       DEP(J)=0.0
       DO 60 K=1,24
      NNN=(LNUMR-1)*24+K
      INDEX=NODE(NNN)
       EP(J)=EP(J)+STDEC(    J,K)*DISP(INDEX)
       DEP(J)=DEP(J)+STDEC(    J,K)*DELD(INDEX)
   60  CONTINUE
       EPEEM=EP(2)+EP(8)**2/2.
       DEPEEM=DEP(2)+EP(8)*DEP(8)-DEP(8)**2/2.
       BPNEE=0.0
       BPMEE=0.0
      PLAST1=0.0
       DO 150 LZ=1,NZ
       EPEEZ=EPEEM-RZETA(LZ)*EP(5)
       DEPEEZ=DEPEEM-RZETA(LZ)*DEP(5)
       GPEE=YOUNG*EPEEZ
       PTEE=0.0
       FACTOR=1.
       IF(D.EQ.0.0) GO TO 152
      FACTOR=1.+(CC6*DABS(DEPEEZ))**CC5
  152  DO 155 LS=1,NSUBL
       JZ=LZ+(LS-1)*NZ
       SNY=SNO(LS)*FACTOR
       TAEE=TSGEE(LNUM ,JZ,JA)
       DSNEE=YOUNG*DEPEEZ
       TMEE=TAEE+DSNEE
       IF(TMEE-SNY) 20,45,21
   21  TAEE=SNY
      PSS1=(TMEE-SNY)/YOUNG
      PLAST1=PLAST1+PSS1*TAEE*RWEGH(JZ)
       GO TO 156
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```
20    IF(TMEE+SNY) 22,45,45
22    TAEE=-SNY
     PSS1=(TMEE+SNY)/YOUNG
     PLAST1=PLAST1+PSS1*TAEE*RWEGH(JZ)
      GO TO 156
45    CONTINUE
      TAEE=TMEE
156   CONTINUE
      TSGEE(LNUM ,JZ,JA)=TAEE
      PLAEE=GPEE-TSGEE(LNUM ,JZ,JA)
      PTEE=PTEE+PLAEE*RWEGH(JZ)
155   CONTINUE
      BPNEE=BPNEE+PTEE
      BPMEE=BPMEE-PTEE*RZETA(LZ)
150   CONTINUE
      DO 163 J=1,9
163   SNL(J)=0.0
      SNL(2)=YOUNG*RH*EP(8)**2/2.-BPNEE
      SNL(5)=YOUNG*RH*XF*EP(8)**2/2.-BPMEE
      SNL(8)=(YOUNG*RH*(EPEEM+XF*EP(5))-BPNEE)*EP(8)
      DO 164 J=1,9
164   SNL(J)=SNL(J)*SWEW
      DO 165 J=1,24
      EQNL(J)=0.0
      DO 165 K=1,9
165   EQNL(J)=EQNL(J)+STDEC(    K,J)*SNL(K)
      DO 166 J=1,24
166   ELFP(J)=ELFP(J)+EQNL(J)
     PLAST2=PLAST2+PLAST1*SWEW
100   CONTINUE
      DO 201 J=1,24
     NNN=(LNUMR-1)*24+J
     M=NODE(NNN)
201   FLVA(M)=FLVA(M)-ELFP(J)
     PLASTW=PLASTW+PLAST2
50    CONTINUE
      RETURN
     END
```

```
      SUBROUTINE STRESP(DISP,DELD,FLVA,ITT,NODE,MAXEL,TAUSS,TAUEE,TAUSE,
     2EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,PLASTW,XGI,YGI,NP,STDEP,GBI,
     3NMSUB,DSR,PSR,SUBW,SNOM,NZS)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14...... ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C
      DIMENSION DISP(1),DELD(1),FLVA(1),SNL(9),EQNL(36),EP(9),DEP(9),
     2ELFP(36),NODE(1),TAUSS(MAXEL,NZS,9),TAUEE(MAXEL,NZS,9),TAUSE(MAXEL
     3,NZS,9),EPSSI(9,1),EPSSO(9,1),EPEEI(9,1),EPEEO(9,1),EPSEI(9,1),
     4EPSEO(9,1)
      DIMENSION XGI(1),YGI(1),NP(4,1),STDEP(9,24),GBI(24,24)
      DIMENSION NMSUB(1),DSR(1),PSR(1),SUBW(5,1),SNOM(5,1),ZGAUS(6),
     2ZGAUW(6),GFL(4),GZETA(4),GWEGH(20),SNO(5),AGAUS(6),AGAUW(6)
      COMMON/PLATE/XDIST,YDIST,TH,SLE,ELE,NEAD,NECD,DENS,YOUNG,HNU,THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C THIS SUBROUTINE CALCULATES THE EFFECTIVE NODAL FORCE VECTOR FOR A
C PLATE ELEMENT CORRESPONDING TO NONLINEAR GEOMETRIC AND MATERIAL
C EFFECTS  THE THEORETICAL BASIS FOR THESE CALCULATIONS IS GIVEN IN
C SUBSECTION A.1.3.A.
C
      CON1=(1.0+HNU)/(3.0*YOUNG)
      NA=3
      NZ=4
      NSUBL=NMSUB(1)
      D=DSR(1)
      P=PSR(1)
      CALL GAUSS(NA,AGAUS,AGAUW)
      CALL GAUSS(NZ,ZGAUS,ZGAUW)
      DO 10 LZ=1,NZ
      GFL(LZ)=0.5*TH*ZGAUW(LZ)
   10 GZETA(LZ)=0.5*TH*ZGAUS(LZ)
      DO 20 LZ=1,NZ
      DO 20 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
   20 GWEGH(JZ)=GFL(LZ)*SUBW(1,LS)
      CC5=0.0
      CC6=0.0
      IF(D.EQ.0.0)GO TO 30
      CC5=1.0/P
      CC6=1.0/D/DELTAT
   30 EE1=YOUNG/(1.0-HNU*HNU)
      EE2=HNU*EE1
      EE3=YOUNG/2.0/(1.0+HNU)
      HNU1=(1.0-2.0*HNU)/3.0/(1.0-HNU)
      DO 35 LS=1,NSUBL
   35 SNO(LS)=SNOM(1,LS)
      DO 50 LNUM=1,NET
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      SLE=(XGI(NNN2)-XGI(NNN1))/2.0D0
      ELE=(YGI(NNN4)-YGI(NNN1))/2.0D0
       DO 99 J=1,24
   99  ELFP(J)=0.0
```

```
      DO 100 KS=1,NA
      SW=SLE*AGAUW(KS)
      DO 100 KE=1,NA
     SL=(AGAUS(KS)+1.0)/2.0
     EL=(AGAUS(KE)+1.0)/2.0
     XL=2.0*SLE
     YL=2.0*ELE
     CALL SFDM(SL,EL,XL,YL,GBI,STDEP)
      EW=ELE*AGAUW(KE)
      SWEW=SW*EW
     PLAST3=0.0
      JA=KE+(KS-1)*NA
      DO 60 J=1,9
      EP(J)=0.0
      DEP(J)=0.0
      DO 60 K=1,24
     NNN=(LNUM-1)*24+K
     INDEX=NODE(NNN)
      EP(J)=EP(J)+STDEP(    J,K)*DISP(INDEX)
      DEP(J)=DEP(J)+STDEP(    J,K)*DELD(INDEX)
 60   CONTINUE
      EPSSM=EP(1)+EP(7)**2/2.
    *+EP(9)**2/2.
      EPEEM=EP(2)+EP(8)**2/2.
    *+EP(9)**2/2.
      EPSEM=EP(3)+EP(7)*EP(8)
      DEPSSM=DEP(1)+EP(7)*DEP(7)-DEP(7)**2/2.
    *+EP(9)*DEP(9)-DEP(9)**2/2.
      DEPEEM=DEP(2)+EP(8)*DEP(8)-DEP(8)**2/2.
    *+EP(9)*DEP(9)-DEP(9)**2/2
      DEPSEM=DEP(3)+EP(7)*DEP(8)+DEP(7)*EP(8)-DEP(7)*DEP(8)
     LN=LNUM
      EPSSI(JA,LN)=EPSSM+THALF*EP(4)
      EPEEI(JA,LN)=EPEEM+THALF*EP(5)
      EPSEI(JA,LN)=EPSEM+THALF*EP(6)
      EPSSO(JA,LN)=EPSSM-THALF*EP(4)
      EPEEO(JA,LN)=EPEEM-THALF*EP(5)
      EPSEO(JA,LN)=EPSEM-THALF*EP(6)
 61   BPNSS=0.0
      BPNEE=0.0
      BPNSE=0.0
      BPMSS=0.0
      BPMEE=0.0
      BPMSE=0.0
      DO 150 LZ=1,NZ
     PLAST2=0.0
      EPSSZ=EPSSM-GZETA(LZ)*EP(4)
      EPEEZ=EPEEM-GZETA(LZ)*EP(5)
      EPSEZ=EPSEM-GZETA(LZ)*EP(6)
      DEPSSZ=DEPSSM-GZETA(LZ)*DEP(4)
      DEPEEZ=DEPEEM-GZETA(LZ)*DEP(5)
      DEPSEZ=DEPSEM-GZETA(LZ)*DEP(6)
      GPSS=EE1*EPSSZ+EE2*EPEEZ
      GPEE=EE2*EPSSZ+EE1*EPEEZ
      GPSE=EE3*EPSEZ
      PTSS=0.0
      PTEE=0.0
      PTSE=0.0
      FACTOR=1.
      IF(D.EQ.0.0) GO TO 152
      SECINV=DEPSSZ**2+DEPEEZ**2-DEPSSZ*DEPEEZ+.75D0*DEPSEZ**2
     SECINV=SECINV*2.D0/3.D0
     IF(SECINV .LT. 0.0D0) WRITE(MWRITE,151) NNC,ITT
 151 FORMAT(1H0,'SECOND INVARIANT OF DEVIATORIC STRAIN RATE IS NEGATIVE
    + AT SUBINCREMENT',I3,' AT TIME STEP',I10/20X,'STOP IN SUBROUTINE S
    2TRESP.')
```

```
      IF(SECINV.LT.0.0) CALL EXIT                                       STRP0129
      FACTOR=1.+(CC6*DSQRT(SECINV))**CC5                                STRP0130
  152 DO 155 LS=1,NSUBL                                                 STRP0131
      SIGMSQ=(SNO(LS)*FACTOR)**2                                        STRP0132
      JZ=LZ+(LS-1)*NZ                                                   STRP0133
      NNN=1                                                             STRP0134
      SNN=1.                                                            STRP0135
   40 CONTINUE                                                          STRP0136
     PLAST1=0 0                                                         STRP0137
      NNC=1                                                             STRP0138
      TASS=TAUSS(LNUM ,JZ,JA)                                           STRP0139
      TAEE=TAUEE(LNUM ,JZ,JA)                                           STRP0140
      TASE=TAUSE(LNUM ,JZ,JA)                                           STRP0141
      DSNSS=(EE1*DEPSSZ+EE2*DEPEEZ)/SNN                                 STRP0142
      DSNEE=(EE2*DEPSSZ+EE1*DEPEEZ)/SNN                                 STRP0143
      DSNSE=EE3*DEPSEZ/SNN                                              STRP0144
   41 CONTINUE                                                          STRP0145
      TMSS=TASS+DSNSS                                                   STRP0146
      TMEE=TAEE+DSNEE                                                   STRP0147
      TMSE=TASE+DSNSE                                                   STRP0148
      CZ=TMSS**2+TMEE**2-TMSS*TMEE+3.*TMSE**2-SIGMSQ                    STRP0149
      IF(CZ  LE. 0.0) GO TO 45                                          STRP0150
      TAUM=TASS+TAEE                                                    STRP0151
      TCSS=TASS-HNU1*TAUM                                               STRP0152
      TCEE=TAEE-HNU1*TAUM                                               STRP0153
      TCSE=TASE                                                         STRP0154
  159 AZ=TCSS**2-TCSS*TCEE+TCEE**2+3.*TCSE**2                           STRP0155
C                                                                       STRP0156
      BZ=TMSS*TCSS+TMEE*TCEE-(TMSS*TCEE+TCSS*TMEE)/2.+3.*TMSE*TCSE      STRP0157
      DISCR=BZ*BZ-AZ*CZ                                                 STRP0158
      IF(DISCR LT 0 0) GO TO 157                                        STRP0159
      HLAMD2=BZ+DSQRT(DISCR)                                            STRP0160
      IF(HLAMD2 .LE  0.0) GO TO 157                                     STRP0161
      HLAMDA=CZ/HLAMD2                                                  STRP0162
     PSS1=HLAMDA*CON1*(2 0*TASS-TAEE)                                   STRP0163
     PSS2=HLAMDA*CON1*(2.0*TAEE-TASS)                                   STRP0164
     PSS3=HLAMDA*CON1*6 0*TASE                                          STRP0165
     PLAST1=PLAST1+PSS1*TASS+PSS2*TAEE+PSS3*TASE                        STRP0166
      TASS=TMSS-HLAMDA*TCSS                                             STRP0167
      TAEE=TMEE-HLAMDA*TCEE                                             STRP0168
      TASE=TMSE-HLAMDA*TCSE                                             STRP0169
      GO TO 46                                                          STRP0170
   45 TASS=TMSS                                                         STRP0171
      TAEE=TMEE                                                         STRP0172
      TASE=TMSE                                                         STRP0173
   46 IF(NNC.EQ NNN) GO TO 156                                          STRP0174
      NNC=NNC+1                                                         STRP0175
      GO TO 41                                                          STRP0176
  157 NNN=NNN+1                                                         STRP0177
      SNN=NNN                                                           STRP0178
      IF(NNN-99) 40,40,44                                               STRP0179
   44 WRITE(MWRITE,49)ITT                                               STRP0180
   49 FORMAT('  SUBINCREMENT IN PLASTICITY  RELATION .GT.99 AT IT=',I5)STRP0181
      CALL EXIT                                                         STRP0182
  156 TAUSS(LNUM ,JZ,JA)=TASS                                           STRP0183
      TAUEE(LNUM ,JZ,JA)=TAEE                                           STRP0184
      TAUSE(LNUM ,JZ,JA)=TASE                                           STRP0185
      PLASS=GPSS-TAUSS(LNUM ,JZ,JA)                                     STRP0186
      PLAEE=GPEE-TAUEE(LNUM ,JZ,JA)                                     STRP0187
      PLASE=GPSE-TAUSE(LNUM ,JZ,JA)                                     STRP0188
      PTSS=PTSS+PLASS*GWEGH(JZ)                                         STRP0189
      PTEE=PTEE+PLAEE*GWEGH(JZ)                                         STRP0190
      PTSE=PTSE+PLASE*GWEGH(JZ)                                         STRP0191
     PLAST2=PLAST2+PLAST1*GWEGH(JZ)                                     STRP0192
  155 CONTINUE                                                          STRP0193
      BPNSS=BPNSS+PTSS                                                  STRP0194
```

```
      BPNEE=BPNEE+PTEE                                                  STRP0195
      BPNSE=BPNSE+PTSE                                                  STRP0196
      BPMSS=BPMSS-PTSS*GZETA(LZ)                                        STRP0197
      BPMEE=BPMEE-PTEE*GZETA(LZ)                                        STRP0198
      BPMSE=BPMSE-PTSE*GZETA(LZ)                                        STRP0199
     PLAST3=PLAST3+PLAST2                                               STRP0200
150   CONTINUE                                                          STRP0201
      SNL(1)=(EE1*EP(7)**2/2.+EE2*EP(8)**2/2.)*TH-BPNSS                 STRP0202
    *+((EE1+EE2)*EP(9)**2/2.)*TH                                        STRP0203
      SNL(2)=(EE2*EP(7)**2/2.+EE1*EP(8)**2/2.)*TH-BPNEE                 STRP0204
    *+((EE1+EE2)*EP(9)**2/2.)*TH                                        STRP0205
      SNL(3)=(EE3*EP(7)*EP(8))*TH-BPNSE                                 STRP0206
      SNL(4)=-BPMSS                                                     STRP0207
      SNL(5)=-BPMEE                                                     STRP0208
      SNL(6)=-BPMSE                                                     STRP0209
      SNL(7)=((EE1*EPSSM+EE2*EPEEM)*TH-BPNSS)*EP(7)                     STRP0210
    *+(EE3*EPSEM*TH-BPNSE)*EP(8)                                        STRP0211
      SNL(8)=((EE2*EPSSM+EE1*EPEEM)*TH-BPNEE)*EP(8)                     STRP0212
    *+(EE3*EPSEM*TH-BPNSE)*EP(7)                                        STRP0213
      SNL(9)=((EE1*EPSSM+EE2*EPEEM)*TH-BPNSS)*EP(9)                     STRP0214
    *+((EE2*EPSSM+EE1*EPEEM)*TH-BPNEE)*EP(9)                            STRP0215
      DO 164 J=1,9                                                      STRP0216
164   SNL(J)=SNL(J)*SWEW                                                STRP0217
      DO 165 J=1,24                                                     STRP0218
      EQNL(J)=0 0                                                       STRP0219
      DO 165 K=1,9                                                      STRP0220
165   EQNL(J)=EQNL(J)+STDEP(    K,J)*SNL(K)                             STRP0221
      DO 166 J=1,24                                                     STRP0222
166   ELFP(J)=ELFP(J)+EQNL(J)                                           STRP0223
     PLASTW=PLASTW+PLAST3*SWEW                                          STRP0224
100   CONTINUE                                                          STRP0225
      DO 201 J=1,24                                                     STRP0226
     NNNN=(LNUM-1)*24+J                                                 STRP0227
     M=NODE(NNNN)                                                       STRP0228
201   FLVA(M)=FLVA(M)+ELFP(J)                                           STRP0229
50    CONTINUE                                                          STRP0230
      RETURN                                                            STRP0231
     END                                                                STRP0232
```

```
      SUBROUTINE TSTEP(STF,AMASS,NBC,BC,ICOL,INUM,KROW,NDEX,TRIAL,SOLN)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14...... ORIGINAL REPORT VERSION OF PROGRAM
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C
C THIS SUBROUTINE IS USED BY BOTH THE PLATE AND THE CIVM PLATE PROGRAMS.
C
      DIMENSION STF(1),AMASS(1),BC(1),NBC(1),ICOL(1),INUM(1),KROW(1),
     2NDEX(1),TRIAL(1),SOLN(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/PLATE/XD,YD,TH,HX,HY,NEA,NEC,DENS,EPAN,AN,THF
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
      COMMON /MASSM/ IMASS
C
C   SUBROUTINE TO CALCULATE THE CENTRAL-DIFFERENCE TIME-STEP BOUND.
C   ITERATION IS USED TO CALCULATE THE HIGHEST NATURAL FREQUENCY OF THE
C   ASSEMBLED STRUCTURE.                 NOTE-- HOUBOLT OPERATOR
C   USED IN PRESENT PROGRAM.  CALCULATED BOUND SERVES ONLY AS A
C   REFERENCE NO
C
C   GENERATE INITIAL GUESS
      DO 20 I=1,NDT
   20 TRIAL(I)=1 0D0
C   CONSTRAIN ASSEMBLED STIFFNESS MATRIX
      CALL BCONMK(STF,BC,ICOL,INUM,NBC)
      IF(IMASS  EQ. 2) GO TO 28
      DO 25 I=1,NB
      KK=NBC(I)
   25 TRIAL(KK)=0 0D0
      DO 27 I=1,NDT
      IF(AMASS(I) EQ.0.0)TRIAL(I)=0.0D0
   27 CONTINUE
      GO TO 29
C CONSTRAIN CONSISTENT MASS MATRIX.
   28 CALL BCONMK(AMASS,BC,ICOL,INUM,NBC)
C   NORMALIZE SO TRIAL VECTOR IS A UNIT VECTOR
   29 SUM=0 0D0
      DO 30 I=1,NDT
   30 SUM=SUM+TRIAL(I)*TRIAL(I)
      SUM=DSQRT(SUM)
      DO 35 I=1,NDT
   35 TRIAL(I)=TRIAL(I)/SUM
      EPS=1 0D-7
      BOLD=1 0D0
      NNN=0
C FOR CONSISTENT MASS MATRIX, REPLACE M BY FACTORED M.
      IF(IMASS .EQ. 2) CALL FACT(AMASS,ICOL,KROW,NDEX,IDET,MWRITE,INUM)
C   BEGIN ITERATION LOOP
      DO 100 ITER=1,200
      NNN=NNN+1
C   ZERO OUT SOLUTION VECTOR
      DO 40 I=1,NDT
   10 SOLN(I)=0 0D0
C   MULTIPLY STIFFNESS TIMES TRIAL
      CALL OMULT(STF,TRIAL,SOLN,ICOL,KROW,NDEX)
      IF(IMASS EQ 2) GO TO 46
C   MULTIPLY PRODUCT BY MASS-INVERSE
      DO 45 I=1,NDT
      IF(AMASS(I).EQ 0.0)GO TO 45
      TRIAL(I)=SOLN(I)/AMASS(I)
   45 CONTINUE
      GO TO 47
```

```
 46   CALL SOLV(AMASS,SOLN,TRIAL,ICOL,KROW,NDEX)                        TSTP0063
 47   BNEW=-1 DO                                                        TSTP0064
C SEARCH FOR MAX VALUE IN TRIAL VECTOR.                                 TSTP0065
      IHOLD=0                                                           TSTP0066
      DO 48 I=1,NDT                                                     TSTP0067
      CHECK=DABS(TRIAL(I))                                              TSTP0068
      IF(BNEW .GT. CHECK) GO TO 48                                      TSTP0069
      BNEW=CHECK                                                        TSTP0070
      IHOLD=I                                                           TSTP0071
 48   CONTINUE                                                          TSTP0072
      IF(IHOLD .NE. 0) BNEW=TRIAL(IHOLD)                                TSTP0073
C   NORMALIZE SO TRIAL VECTOR IS A UNIT VECTOR                          TSTP0074
      DO 55 I=1,NDT                                                     TSTP0075
      IF(TRIAL(I) .EQ. 0.D0) GO TO 55                                   TSTP0076
      IF(DABS(10.0D0*TRIAL(I)/BNEW) .GT. 1.D-76) GO TO 55               TSTP0077
      TRIAL(I)=0.D0                                                     TSTP0078
 55   TRIAL(I)=TRIAL(I)/BNEW                                            TSTP0079
C   IF THIS IS FIRST ITERATION, SKIP CONVERGENCE CHECK                  TSTP0080
      IF(ITER EQ 1)GO TO 100                                            TSTP0081
      CRATIO=DABS(1.0D0-BNEW/BOLD)                                      TSTP0082
      IF(CRATIO.LE.EPS)GO TO 110                                        TSTP0083
C   IF NOT CONVERGED CONTINUE ITERATION                                 TSTP0084
      BOLD=BNEW                                                         TSTP0085
 100  CONTINUE                                                          TSTP0086
C   CONVERGENCE FOUND OR MORE THAN 200 ITERATIONS TAKEN                 TSTP0087
 110  BONE=BNEW                                                         TSTP0088
      BONE=DSQRT(BONE)                                                  TSTP0089
      WRITE(MWRITE,600)BONE                                             TSTP0090
C PRINT OUT FINAL VALUE OF EIGENVECTOR.                                 TSTP0091
      NNT=NDT/6                                                         TSTP0092
      WRITE(MWRITE,114)                                                 TSTP0093
      DO 112 NN=1,NNT                                                   TSTP0094
      J1=(NN-1)*6+1                                                     TSTP0095
      J2=J1+5                                                           TSTP0096
 112  WRITE(MWRITE,116) NN, (TRIAL(J),J=J1,J2)                          TSTP0097
 114  FORMAT(1H0,23X,'HIGHEST EIGENVECTOR NORMALIZED BY LARGEST VALUE'/ TSTP0098
     1 1H0,10X, 'NODE',6X,'U',12X,'V',12X,'W',11X,'PSIX',9X,'PSIY',9X,  TSTP0099
     2 'TWIST')                                                         TSTP0100
 116  FORMAT(11X,I4,6D13.5)                                             TSTP0101
      WRITE(MWRITE,610)NNN,CRATIO                                       TSTP0102
      STS=2.0D0/BONE                                                    TSTP0103
      WRITE(MWRITE,620) STS                                             TSTP0104
      STS=0 8D0*STS                                                     TSTP0105
      WRITE(MWRITE,630)STS                                              TSTP0106
 600  FORMAT('0','HIGHEST NATURAL FREQUENCY(RAD/SEC)=',D15.7)           TSTP0107
 610  FORMAT(' ','AFTER',I4,2X,'ITERATIONS, CONVERGENCE RATIO=',D15.7)  TSTP0108
 620  FORMAT(' ','CENTRAL-DIFFERENCE BOUND ON TIME-STEP(SEC) FOR LINEAR TSTP0109
     2SYSTEM=',D15.7)                                                   TSTP0110
 630  FORMAT(' ','ACTUAL TIME-STEP BOUND (SEC) WOULD BE=',D15.7)        TSTP0111
      RETURN                                                            TSTP0112
      END                                                               TSTP0113
```

## 6.2 Subprograms Unique to the PLATE Code

The following 6 subprograms (or subroutines) are unique to the PLATE program:

| | | |
|---|---|---|
| CMASSP | INCOND | MAINP |
| EXTL | MAIN | PRINT |

These specific versions of Subprograms EXTL, INCOND, and MAIN given in this subsection are principally for illustration, but are also those used for the illustrative example given in Subsection 7.2.1 for the complete plate. The user must write his own versions of these programs consistent with his problem data if he wishes to analyze any other example problem.

A FORTRAN IV listing of these 6 subprograms follows.

```
      SUBROUTINE CMASSP(NTYPE,XL,YL,TH,XF,SLOC,CSA,SMOI,EM,GBI,DENS)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.. .   ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
      DIMENSION EM(24,24),GBI(24,24),AMT(5,5),GS(6),GW(6),P(5,24),
     2STOR1(5,24),STOR2(24,24),T(24)
C
C THIS SUBROUTINE COMPUTES THE CONSISTENT MASS MATRIX FOR A
C RECTANGULAR PLATE ELEMENT, X-DIRECTION STIFFENER ELEMENT, OR
C Y-DIRECTION STIFFENER ELEMENT. THE FORMULATION CORRESPONDS TO
C EQUATIONS A 23 AND A 29 (FOR PLATE ELEMENTS), EQUATION A.91A FOR
C X-DIRECTION STIFFENERS, AND EQUATION A.91B FOR Y-DIRECTION
C STIFFENERS. A GAUSSIAN QUADRATURE IS USED TO COMPUTE THE REQUIRED
C INTEGRAL
C
      NGS=4
      CALL GAUSS(NGS,GS,GW)
      DO 5 I=1,NGS
      GS(I)=(1.0+GS(I))/2.0
    5 GW(I)=GW(I)/2.0
      NGX=NGS
      NGY=NGS
      DO 10 I=1,5
      DO 10 J=1,5
   10 AMT(I,J)=0.0
      DO 20 I=1,5
      DO 20 J=1,24
   20 P(I,J)=0.0
      DO 25 I=1,24
      DO 25 J=1,24
   25 EM(I,J)=0.0
      GO TO(30,40,40),NTYPE
C   M-TILDA FOR PLATE ELEMENT
   30 AMT(1,1)=TH
      AMT(2,2)=TH
      AMT(3,3)=TH
      AMT(4,4)=TH**3/(12.0D0*XL*XL)
      AMT(5,5)=TH**3/(12.0D0*YL*YL)
      GO TO 50
C   M-TILDA FOR STIFFENERS.
   40 AMT(1,1)=CSA
      AMT(2,2)=CSA
      AMT(3,3)=CSA
      AMT(4,4)=SMOI/(XL*XL)
      AMT(5,5)=SMOI/(YL*YL)
      AMT(4,1)=-CSA*XF/XL
      AMT(1,4)=AMT(4,1)
      AMT(5,2)=-CSA*XF/YL
      AMT(2,5)=AMT(5,2)
      IF(NTYPE.EQ.2)NGY=1
      IF(NTYPE.EQ.3)NGX=1
   50 CONTINUE
C   LOOP OVER GAUSS STATIONS.
      DO 100 NY=1,NGY
      DO 100 NX=1,NGX
```

```
      SL=GS(NX)
      EL=GS(NY)
      GWX=GW(NX)
      GWY=GW(NY)
      IF(NTYPE.EQ.2)EL=SLOC
      IF(NTYPE.EQ 2)GWY=1 0
      IF(NTYPE.EQ.3)SL=SLOC
      IF(NTYPE EQ.3)GWX=1.0
      P(1,1)=1.0
      P(1,2)=SL
      P(1,3)=EL
      P(1,4)=EL*SL
      P(2,5)=1.0
      P(2,6)=SL
      P(2,7)=EL
      P(2,8)=SL*EL
      P(3,9)=1 0
      P(3,10)=SL
      P(3,11)=EL
      P(3,12)=SL*EL
      P(3,13)=SL*SL
      P(3,14)=EL*EL
      P(3,15)=SL*SL*EL
      P(3,16)=SL*EL*EL
      P(3,17)=SL*SL*EL*EL
      P(3,18)=SL**3
      P(3,19)=EL**3
      P(3,20)=P(3,18)*EL
      P(3,21)=SL*P(3,19)
      P(3,22)=P(3,20)*EL
      P(3,23)=P(3,21)*SL
      P(3,24)=P(3,18)*P(3,19)
      P(4,10)=1.0
      P(4,12)=EL
      P(4,13)=2 0*SL
      P(4,15)=2.0*SL*EL
      P(4,16)=EL*EL
      P(4,17)=P(4,15)*EL
      P(4,18)=3.0*SL*SL
      P(4,20)=P(4,18)*EL
      P(4,21)=P(4,16)*EL
      P(4,22)=P(4,20)*EL
      P(4,23)=P(4,17)*EL
      P(4,24)=P(4,22)*EL
      P(5,11)=1 0
      P(5,12)=SL
      P(5,14)=2.0*EL
      P(5,15)=SL*SL
      P(5,16)=2 0*SL*EL
      P(5,17)=P(5,16)*SL
      P(5,19)=3.0*EL*EL
      P(5,20)=P(5,15)*SL
      P(5,21)=P(5,19)*SL
      P(5,22)=P(5,17)*SL
      P(5,23)=P(5,21)*SL
      P(5,24)=P(5,23)*SL
      DO 60 I=1,5
      DO 60 J=1,24
      STOR1(I,J)=0.0
```

```
      DO 60 K=1,5
   60 STOR1(I,J)=STOR1(I,J)+AMT(I,K)*P(K,J)
      DO 70 I=1,24
      DO 70 J=1,24
      STOR2(I,J)=0.0
      DO 70 K=1,5
   70 STOR2(I,J)=STOR2(I,J)+P(K,I)*STOR1(K,J)
      DO 80 I=1,24
      DO 80 J=1,I
   80 EM(I,J)=EM(I,J)+STOR2(I,J)*GWX*GWY*DENS
  100 CONTINUE
      DO 110 I=1,24
      DO 110 J=1,I
      EM(I,J)=EM(I,J)*XL*YL
  110 EM(J,I)=EM(I,J)
C   PRE- AND POST-MULTIPLY BY G-BAR-INVERSE MATRIX.
      DO 120 I=1,24
      DO 120 J=1,24
      STOR2(I,J)=0.0
      DO 120 K=1,24
  120 STOR2(I,J)=STOR2(I,J)+EM(I,K)*GBI(K,J)
      DO 130 I=1,24
      DO 130 J=1,24
      EM(I,J)=0 0
      DO 130 K=1,24
  130 EM(I,J)=EM(I,J)+GBI(K,I)*STOR2(K,J)
C   FORM SCALING MATRIX FOR NODAL D.O.F..
      DO 140 I=1,24
  140 T(I)=1.0
      DO 150 I=4,22,6
      T(I)=XL
      T(I+1)=YL
  150 T(I+2)=XL*YL
C   SCALE MASS MATRIX.
      DO 160 I=1,24
      DO 160 J=1,24
  160 EM(I,J)=EM(I,J)*T(I)*T(J)
      RETURN
      END
```

```
      SUBROUTINE EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)                      EXTL0000
      IMPLICIT REAL*8(A-H,O-Z)                                          EXTL0001
C                                                                       EXTL0002
C   ASRL TR 154-14. .  ..ORIGINAL REPORT VERSION OF PROGRAM             EXTL0003
C  COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980             EXTL0004
C                                                                       EXTL0005
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.             EXTL0006
C                                                                       EXTL0007
C THIS VERSION IS FOR THE EXAMPLE OF A RECTANGULAR PLATE, CLAMPED       EXTL0008
C ON THE TOP AND BOTTOM EDGES, FREE ON THE OTHER TWO EDGES  WITH AN     EXTL0009
C IMPULSIVE LOAD APPLIED SYMMETRICALLY TO ITS CENTRAL REGION.           EXTL0010
      DIMENSION ELV(1),NP(4,1),NODE(1),XGI(1),YGI(1),GBI(24,24)         EXTL0011
C                                                                       EXTL0012
C THIS USER-PREPARED SUBROUTINE IS USED TO GENERATE THE NODAL           EXTL0013
C GENERALIZED FORCES CORRESPONDING TO PRESCRIBED EXTERNALLY-APPLIED     EXTL0014
C FORCES AT TIME T SUB M (=M*DELTA T) AS DESCRIBED IN SUBSECTION 4.6.2  EXTL0015
C AND CORRESPONDING TO THE DEVELOPMENT IN SUBSECTION A.1.3.4.           EXTL0016
C                                                                       EXTL0017
C DEFINE SIZE OF VECTOR CONTAINING UNIT LOAD OVER ALL DOF IN CENTRAL    EXTL0018
C REGION OF PLATE. WIDTH OF LOADED REGION IS 1.497 IN.,LENGTH IS 1.8 IN.EXTL0019
      DIMENSION ALOAD(150),AL1(72),AL2(78)                              EXTL0020
      EQUIVALENCE (AL1(1),ALOAD(1)),(AL2(1),ALOAD(73))                  EXTL0021
C                                                                       EXTL0022
      COMMON /BAS/ NDT,NET,MN,NB,NIRREG,MNC                             EXTL0023
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME              EXTL0024
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                EXTL0025
C DEFINE TIME AT WHICH LOAD DROPS TO ZERO AND MAX VALUE OF IMPULSE.     EXTL0026
      DATA TEND/25.D-6/AMAXP/21934.4DO/                                 EXTL0027
C DEFINE FIRST AND LAST DOF IN LOADED REGION.                           EXTL0028
      DATA IDOF/121/,LDOF/270/                                          EXTL0029
C DEFINE VALUES OF UNIT LOAD AT EACH DOF IN LOADED REGION               EXTL0030
      DATA AL1/2*0.DO, 4210312D-1, .2626182D-2, .3157734D-2, .1969637D-3EXTL0031
     1          ,2*0.DO, 8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0EXTL0032
     2          ,2*0.DO,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0EXTL0033
     3          ,2*0 DO, 8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0EXTL0034
     4          ,2*0.DO, 4210312D-1,-.2626182D-2, .3157734D-2,-.1969637D-3EXTL0035
     5          ,2*0 DO,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0EXTL0036
     6          ,2*0.DO,.1684125D+0,  0000000D+0, .0000000D+0, .0000000D+0EXTL0037
     7          ,2*0.DO,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0038
     8          ,2*0 DO, 1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0039
     9          ,2*0.DO,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0EXTL0040
     A          ,2*0 DO, 8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0EXTL0041
     B        ,2*0 DO, 1684125D+0,  0000000D+0,  0000000D+0   0000000D+0/EXTL0042
      DATA AL2/2*0 DO,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0043
     1          ,2*0.DO,.1684125D+0,  0000000D+0, .0000000D+0, .0000000D+0EXTL0044
     2          ,2*0.DO, 8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0EXTL0045
     3          ,2*0.DO,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0EXTL0046
     4          ,2*0.DO,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0047
     5          ,2*0 DO,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0048
     6          ,2*0.DO,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0EXTL0049
     7          .2*0.DO, 8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0EXTL0050
     8          ,2*0.DO, 4210312D-1, .2626182D-2,-.3157734D-2,-.1969637D-3EXTL0051
     9          ,2*0 DO,.8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0EXTL0052
     A          ,2*0.DO, 8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0EXTL0053
     B          ,2*0.DO,.8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0EXTL0054
     C        ,2*0 DO, 4210312D-1,-.2626182D-2,-.3157734D-2, .1969637D-3/EXTL0055
C ZERO OUT LOAD VECTOR.                                                 EXTL0056
      DO 1 I=1,NDT                                                      EXTL0057
    1 ELV(I)=0.0D0                                                      EXTL0058
```

```
C IF THIS TIME STEP IS BEYOND LAST TIME OF IMPULSE RETURN WITH
C EXTERNAL LOAD VECTOR SET TO ZERO.
      TYM=(ITT-1)*DELTAT
      IF(TYM   GT. TEND) RETURN
C INTERPOLATE TO FIND MAGNITUDE OF IMPULSE AT THIS TIME.
      AMULT=AMAXP*(TEND-TYM )/TEND
C FILL IN VALUES OF LOAD IN CENTRAL REGION OF PLATE.
      DO 2 I=IDOF,LDOF
      J=I-IDOF+1
    2 ELV(I)=AMULT*ALOAD(J)
C OUTPUT LOADS IF THIS IS A REGULAR PRINTING CYCLE.
      III=ITIME/INCRT*INCRT-ITIME
      IF(III.NE.0) RETURN
      NNT=NDT/6
      WRITE(MWRITE,100) TYM ,(J,J=1,6)
100   FORMAT(1H0/1H0,10X,'TIME=',D16.6,' SEC.'/1H ,33(1H.),'EXTERNALLY A
     +PPLIED USER SPECIFIED LOADING',33(1H.)/1H ,6X,'NODE',6(7X,'DOF#',
     & I1,3X)/8X,3(10X,'(LBS)'),1X,3(7X,'(IN-LBS)'))
      I1=1
      DO 3 I=1,NNT
      I2=6*I
      WRITE(MWRITE,101) I,(ELV(J),J=I1,I2)
      I1=I2+1
  3   CONTINUE
  101 FORMAT(6X,I3,2X,6D15.6)
      RETURN
      END
      SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ASRL TR 154-14. ...  ORIGINAL REPORT VERSION OF PROGRAM
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
      DIMENSION DISP(1),VEL(1),NP(4,1),NODE(1),XGI(1),YGI(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
C
C THIS USER-PREPARED SUBROUTINE IS USED TO SPECIFY THE INITIAL (TIME
C ZERO) CONDITIONS ON THE PLATE NODAL DISPLACEMENT AND VELOCITIES,
C (Q ZERO)* AND (Q DOT ZERO)*, RESPECTIVELY, REQUIRED FOR THE FINITE-
C DIFFERENCE SOLUTION OF THE GOVERNING EQUATIONS OF MOTION (SEE
C SUBSECTION A 3).
C
C   DUMMY ROUTINE - SETS VELOCITY AND DISPLACEMENT TO ZERO INITIALLY.
C
      DO 10 I=1,NDT
      DISP(I)=0.0D0
 10   VEL(I)=0.0D0
      RETURN
      END
```

```
C   PLATE 1 COMPUTER CODE.
C
C     ASRL TR 154-14........ORIGINAL REPORT VERSION OF PROGRAM
C
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS IS THE SMALL STRAIN VERSION.
C
C ILLUSTRATIVE EXAMPLE FOR PLATE PROGRAM - FULL MODEL.
C
C THIS DUMMY MAIN PROGRAM FOR THE PLATE PROGRAM REQUIRES SUBROUTINES:
C        ARRT,ASSEMK,BCONMK,CMASSP,EAROW,EXTL,FACT,GAUSS,
C        INCOND,LMASSP,MAINP,MESHPA,MESHPM,OMULT,PRINT,
C        PRTOP,PSTRN,RCON,RPLATE,SFDM,SOLV,SPRING,STRESA,
C        STRESC,STRESP,TSTEP.
C
C   DUMMY MAIN PROGRAM WHICH PROVIDES DIMENSIONS FOR THOSE ARRAYS
C   (VECTORS) WHOSE DIMENSIONS ARE PROBLEM DEPENDENT.
C   THE FOLLOWING ARRAYS MUST BE DIMENSIONED BY THE USER AS SHOWN:
C     DIMENSION DELD(NDT),DIS(NDT),DISP(NDT),DISM1(NDT),DISM2(NDT),
C     FLN(NDT),FLVA(NDT),FLVM(NDT),FLVP(NDT),ELV(NDT),VEL(NDT),ICOL(NDT)
C     ,INUM(NDT),KROW(NDT),NDEX(NDT),STOR(NDT),STF(MNC),AMASS(MDIM),
C     NP(4,MAXEL),NODE(24*MAXEL),TAUSS(MAXEL,4*MNSL,9),
C     TAUSE(MAXEL,4*MNSL,9),TAUEE(MAXEL,4*MNSL,9),EPSSI(9,MAXEL),
C     EPSSO(9,MAXEL),EPEEI(9,MAXEL),EPEEO(9,MAXEL),EPSEI(9,MAXEL),
C     EPSEO(9,MAXEL),NBC(NB),BC(NB),RFM(NBE,MBWE),ILAST(NBE),UCF1(NBE),
C     UCF2(NBE),XG(NNT),YG(NNT),ZG(NNT),XGI(NNT),YGI(NNT),
C     TAGSS(MNXST,4*MNSL,3),LNXS(MNXST),XSPROP(7,MNXST),MATXS(MNXST),
C     TSGEE(MNYST,4*MNSL,3),LNYS(MNYST),YSPROP(7,MNYST),MATYS(MNYST),
C     LNRS(NRSS),ISRS(NRSS),SC(5,NRSS),NVSA(MNNSA),NCON(4,MNNSA)
C
C   WHERE,
C   NNT     = TOTAL NUMBER OF NODES IN THE ASSEMBLED FINITE-ELEMENT
C             MODEL.
C   NDT     = TOTAL NUMBER OF DEGREES OF FREEDOM IN THE ASSEMBLED
C             FINITE-ELEMENT MODEL = 6*NNT.
C   MNC     = ESTIMATED NUMBER OF WORDS OF STORAGE FOR THE ASSEMBLED
C             STIFFNESS MATRIX (LOWER TRIANGLE STORED BY ROWS FROM FIRST
C             NONZERO TERM).
C   MDIM    = DIMENSION OF ASSEMBLED MASS MATRIX.
C           = NDT IF LUMPED MASS OPTION IS CHOSEN.
C           = MNC IF CONSISTENT MASS OPTION IS CHOSEN.
C   MAXEL   = NUMBER OF ELEMENTS IN FINITE-ELEMENT MODEL.
C   MNSL    = MAXIMUM NUMBER OF MECHANICAL SUBLAYERS EMPLOYED.
C             NOTE-- MNSL MUST BE LESS THAN OR EQUAL TO 5.
C   NB      = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM PRIOR
C             TO ELIMINATION OF DUPLICATES.
C   NBE     = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM AFTER
C             ALL DUPLICATES HAVE BEEN ELIMINATED (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
C   MBWE    = ESTIMATED MAXIMUM BANDWIDTH OF ASSEMBLED STIFFNESS MATRIX
C             AT A CONSTRAINED DEGREE OF FREEDOM (IF IOP5 = 1).
C           = 1 IF IOP5 = 0
```

```
C    MNXST  = MAXIMUM NUMBER OF X-DIRECTION STIFFENERS.                  MAIN0053
C           = 1 IF NO X-DIRECTION STIFFENERS.                            MAIN0054
C    MNYST  = MAXIMUM NUMBER OF Y-DIRECTION STIFFENERS.                  MAIN0055
C           = 1 IF NO Y-DIRECTION STIFFENERS.                            MAIN0056
C    NRSS   = TOTAL NUMBER OF ELEMENT SIDES ON WHICH LINE RESTORING      MAIN0057
C             SPRINGS ARE LOCATED.                                       MAIN0058
C           = 1 IF NO RESTORING SPRINGS ARE PRESENT.                     MAIN0059
C    MNSLT4 = 4*MNSL                                                     MAIN0060
C    MNNSA  = MAXIMUM NUMBER OF NODES REQUESTED FOR STRAIN OUTPUT (IF    MAIN0061
C           IOP8=1). = 1 IF IOP8 = 0                                     MAIN0062
C                                                                        MAIN0063
      IMPLICIT REAL*8(A-H,O-Z)                                           MAIN0064
      DIMENSION DELD(390),DIS(390),DISP(390),DISM1(390),DISM2(390),      MAIN0065
     2FLN(390),FLVA(390),FLVM(390),FLVP(390),ELV(390),VEL(390),ICOL(390)MAIN0066
     3,INUM(390),KROW(390),NDEX(390),STOR(390),STF(14037),AMASS(14037), MAIN0067
     4NP(4,40),NODE(1152),TAUSS(48,12,9),TAUSE(48,12,9),TAUEE(48,12,9), MAIN0068
     5EPSSI(9,48),EPSSO(9,48),EPEEI(9,48),EPEEO(9,48),EPSEI(9,48),      MAIN0069
     6EPSEO(9,48),NBC(60),BC(60),RFM(60,60),ILAST(60),UCF1(60),UCF2(60),MAIN0070
     7XG(65),YG(65),ZG(65),XGI(65),YGI(65),TAGSS(1,12,3),LNXS(1),       MAIN0071
     8XSPROP(7,1),MATXS(1),TSGEE(1,12,3),LNYS(1),YSPROP(7,1),MATYS(1),  MAIN0072
     9LNRS(1),ISRS(1),SC(5,1),NVSA(6 ),NCON(4,6 )                       MAIN0073
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                                MAIN0074
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                 MAIN0075
C DEFINE CARD READER, LINE PRINTER AND CARD PUNCH UNIT NUMBERS.          MAIN0076
      MREAD = 5                                                          MAIN0077
      MWRITE = 6                                                         MAIN0078
      MPUNCH = 7                                                         MAIN0079
      READ(MREAD,500)MAXEL,MNSL,MNXST,MNYST,NBE,MBWE,MNC                 MAIN0080
  500 FORMAT(8I10)                                                       MAIN0081
      WRITE(MWRITE,600)                                                  MAIN0082
      WRITE(MWRITE,610) MAXEL                                            MAIN0083
      WRITE(MWRITE,620) MNSL                                             MAIN0084
      WRITE(MWRITE,630) MNXST                                            MAIN0085
      WRITE(MWRITE,640) MNYST                                            MAIN0086
      WRITE(MWRITE,650) NBE                                              MAIN0087
      WRITE(MWRITE,660) MBWE                                             MAIN0088
      WRITE(MWRITE,670) MNC                                              MAIN0089
  600 FORMAT('1 PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) :  USER INPUTMAIN0090
     + FOR ARRAY DIMENSIONS')                                            MAIN0091
  610 FORMAT(' ','MAXEL  =',I10)                                         MAIN0092
  620 FORMAT(' ','MNSL   =',I10)                                         MAIN0093
  630 FORMAT(' ','MNXST  =',I10)                                         MAIN0094
  640 FORMAT(' ','MNYST  =',I10)                                         MAIN0095
  650 FORMAT(' ','NBE    =',I10)                                         MAIN0096
  660 FORMAT(' ','MBWE   =',I10)                                         MAIN0097
  670 FORMAT(' ','MNC    =',I10)                                         MAIN0098
      MNSLT4=MNSL*4                                                      MAIN0099
      CALL       MAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,ELV,MAIN0100
     2VEL,ICOL,INUM,KROW,NDEX,STOR,STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE, MAIN0101
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,   MAIN0102
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS, MAIN0103
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON)    MAIN0104
      CALL EXIT                                                          MAIN0105
      END
```

```
      SUBROUTINE MAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,ELV,
     2VEL,ICOL,INUM,KROW,NDEX,STOR,STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS,
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14...... ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R  L. SPILKER
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM
C
      DIMENSION DELD(1),DIS(1),DISP(1),DISM1(1),DISM2(1),FLN(1),FLVA(1),
     2FLVM(1),FLVP(1),ELV(1),VEL(1),ICOL(1),INUM(1),KROW(1),NDEX(1),
     3STOR(1),STF(1),AMASS(1),NP(4,1),NODE(1),TAUSS(MAXEL,MNSLT4,9),
     4TAUSE(MAXEL,MNSLT4 ,9),TAUEE(MAXEL,MNSLT4,9),EPSSI(9,1),EPSSO(9,1)
     5,EPEEI(9,1),EPEEO(9,1),EPSEI(9,1),EPSEO(9,1),NBC(1),BC(1),RFM(NBE,
     6MBWE),ILAST(1),UCF1(1),UCF2(1),XG(1),YG(1),ZG(1),XGI(1),YGI(1),
     7TAGSS(MNXST,MNSLT4,3),LNXS(1),XSPROP(7,1),MATXS(1),TSGEE(MNYST,
     8MNSLT4,3),LNYS(1),YSPROP(7,1),MATYS(1),LNRS(1),ISRS(1),SC(5,1)
     9,NVSA(1),NCON(4,1)
C
C   THE FOLLOWING DIMENSION STATEMENTS ARE FIXED FOR ALL COMPUTER RUNS.
      DIMENSION SIG(5),EPS(5),DSR(5),PSR(5),ES(6),SUBW(5,5),SNO(5,5),
     2NMSUB(5),EK(24,24),GBI(24,24),EML(24),STDE(9,24),EM(24,24),SS(5)
      DATA GBI/1 0,-1 0,-1 0,1.0,24*0.0,1 0,-1 0,-1.0,1 0,24*0 0,1.0,
     23*0 0,-3 0,-3 0,2*0 0,9.0,2 0,2 0,2*0 0,-6.0,-6 0,4 0,9*0 0,1.0,
     32*0 0,-2 0,2*0 0,-3 0,6.0,1 0,2*0 0,2 0,-3.0,-4 0,2.0,10*0.0,1.0,
     42*0 0,-2.0,-3 0,0.0,6 0,0 0,1.0,2.0,0.0,-4.0,-3 0,2 0,11*0.0,1.0,
     52*0.0,-2 0,-2 0,4 0,2*0.0,2*1 0,-2.0,-2.0,1 0,0.0,1.0,0 0,-1 0,
     C25*0 0,1 0,0 0,-1 0,
     628*0 0,3 0,3*0.0,-9 0,-2.0,3*0 0,6.0,6 0,-4 0,12*0.0,-1 0,3*0.0,
     73 0,1.0,3*0 0,-3 0,-2.0,2.0,14*0 0,3 0,0 0,-6 0,2*0 0,-2 0,0 0,
     84 0,3.0,-2 0,14*0 0,-1.0,0 0,2 0,2*0 0,1 0,0 0,-2.0,-1.0,1.0,3*0 0
     9,1.0,27*0 0,1 0,32*0 0,9 0,4*0.0,-6 0,-6.0,4 0,16*0.0,-3 0,4*0 0,
     A3 0,2.0,-2 0,16*0 0,-3 0,4*0 0,2.0,3 0,-2.0,16*0 0,1 0,4*0 0,-1.0,
     B-1 0,1 0,2*0 0,1 0,-1.0,26*0 0,1.0,-1.0,29*0.0,3.0,2*0.0,-9 0,0.0,
     C-2 0,2*0 0,6 0,6 0,-4 0,15*0.0,3 0,-6 0,3*0 0             ,-2.0,3.0,
     D4 0,-2 0,13*0 0,-1 0,2*0.0,3 0,0 0,1.0,2*0.0,-2 0,-3 0,2.0,15*0.0,
     E-1 0,2 0,3*0.0,1 0,-1.0,-2 0,1.0/
      COMMON/PLATE/XDIST,YDIST,TH ,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,
     2THALF
      COMMON/BAS/NOT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON/PRNT/IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
      COMMON /MASSM/ IMASS
C
C   PLATE COMPUTER CODE.
C   CONTROLLING ROUTINE FOR ELASTIC-PLASTIC, LARGE DEFLECTION TRANSIENT
C   ANALYSIS OF STIFFENED AND/OR UNSTIFFENED FLAT PLATES
C
      IPASS=1
      NDPE=24
      READ(MREAD,500)IMESH,IMASS,IMCONT,IPUNCH
C
C   MESH GENERATION
      IF(IMESH EQ.0 OR.IMESH EQ.1)CALL MESHPA(NP,NODE,XG,YG,ZG,NBC,BC,
     2LNXS,LNYS,XGI,YGI,XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)
      IF(IMESH EQ.2)CALL MESHPM(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,
     2YGI,XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)
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      IF(IMASS.EQ.1)WRITE(MWRITE,700)
      IF(IMASS.EQ.2)WRITE(MWRITE,710)
  700 FORMAT('0','PRESENT RUN USES LUMPED MASS MATRIX')
  710 FORMAT('0','PRESENT RUN USES CONSISTENT MASS MATRIX')
C
C   SETUP OF STORAGE
      CALL ARRT(STF,AMASS,FLVA,ICOL,INUM,KROW,NDEX,NODE,NDPE)
      IF(IMASS.EQ.1)GO TO 8
      DO 7 I=1,MNC
    7 AMASS(I)=0.0
    8 CONTINUE
C
C   STRESS-STRAIN INFORMATION FOR EACH MATERIAL.
C   NOTE-- PLATE MATERIAL IS ASSUMED TO CORRESPOND TO FIRST MATERIAL.
      NA=3
      NZ=4
      READ(MREAD,500)NMAT
      WRITE(MWRITE,600)
      CON1=2.D0+(1.D0+ANUP)/3.D0
      CON2=2.D0+(0.5D0-ANUP)/3.D0
      DO 60 I=1,NMAT
      READ(MREAD,500)NSUB
      READ(MREAD,510)(SIG(J),J=1,NSUB)
      READ(MREAD,510)(EPS(J),J=1,NSUB)
      READ(MREAD,510)DSR(I),PSR(I)
      ES(1)=SIG(1)/EPS(1)
C DEFINE VALUE OF YOUNGS MODULUS BASED ON EPS(1) AND SIG(1) .
      IF(I.EQ.1) EPAN=ES(1)
      IF(NSUB-1)30,30,10
   10 DO 20 LS=2,NSUB
   20 ES(LS)=(SIG(LS)-SIG(LS-1))/(EPS(LS)-EPS(LS-1))
   30 ES(NSUB+1)=0.0
      CON3=ES(1)*(0.5D0-ANUP)/(1.0D0+ANUP)
      AI=0.0D0
      ASUM=0.0D0
      EI=0.0D0
      ESUM=0.0D0
      DO 40 LS=1,NSUB
      ASUM=ASUM+AI
      AI=1.D0-CON1*ES(LS+1)/(ES(1)-CON2*ES(LS+1))-ASUM
      SUBW(I,LS)=AI
      ESUM=ESUM+EI
      IF(LS.NE.NSUB) EI=(1.D0-ES(LS+1)/ES(1))*(EPS(LS+1)-EPS(LS))
      SNO(I,LS)=ES(1)*EPS(LS)+CON3*ESUM
   40 CONTINUE
      NMSUB(I)=NSUB
      WRITE(MWRITE,610)I
      WRITE(MWRITE,615)
      DO 50 J=1,NSUB
   50 WRITE(MWRITE,620)J,SIG(J),EPS(J),SNO(I,J),SUBW(I,J)
      WRITE(MWRITE,630)DSR(I),PSR(I)
   60 CONTINUE
  500 FORMAT(20I4)
  510 FORMAT(5D16.7)
  600 FORMAT('0','PROPERTIES FOR USER-SPECIFIED MATERIALS')
  610 FORMAT('0','PROPERTIES FOR MATERIAL NUMBER',I3)
  615 FORMAT(' ','SUBLAYER',5X,'STRESS POINT',5X,'STRAIN POINT',5X,
     2'YIELD STRESS',2X,'WEIGHTING FACTOR')
  620 FORMAT(' ',I5,4X,D16.7,1X,D16.7,1X,D16.7,1X,D16.7)
  630 FORMAT(' ','STRAIN RATE PARAMETERS: D=',D16.7,2X,'P=',D16.7)
C   TIME-STEP INFORMATION
      READ(MREAD,500)ITIMEF,INCRT,IOUT
      READ(MREAD,510)DELTAT,TIMEF
      WRITE(MWRITE,640)
      WRITE(MWRITE,650)DELTAT
      WRITE(MWRITE,660)TIMEF,ITIMEF
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      WRITE(MWRITE,670)INCRT                                            MANP0129
  640 FORMAT('0','TIMEWISE SOLUTION PARAMETERS')                        MANP0130
  650 FORMAT(' ','TIME-STEP SIZE=',D16.7,2X,'SEC.')                     MANP0131
  660 FORMAT(' ','RUN WILL TERMINATE AT TIME =',D16.7,2X,'SEC. OR AT CYCMANP0132
     2LE NUMBER',I8)                                                    MANP0133
  670 FORMAT(' ','REGULAR PRINTOUT WILL BE GIVEN EVERY',I5,2X,'CYCLES') MANP0134
C                                                                       MANP0135
C   ESTABLISH PRINT OPTIONS                                             MANP0136
      CALL PRTOP(NNSA,NVSA,NCON,NP)                                     MANP0137
      IF(MN .GT  MNC) WRITE(MWRITE,675) MN,MNC                          MANP0138
  675 FORMAT(1H0,10(1H*),'ASSEMBLED K REQUIRES',I7,' ENTRIES.USER HAS DIMANP0139
     *MENSIONED FOR ONLY',I7,'. CALL EXIT.')                            MANP0140
      IF(MN GT MNC)CALL EXIT                                            MANP0141
C                                                                       MANP0142
C   INITIALIZATION                                                      MANP0143
      NZSUBL=NZ*MNSL                                                    MANP0144
      NASQ=NA*NA                                                        MANP0145
      DO 200 I=1,9                                                      MANP0146
      DO 200 J=1,NET                                                    MANP0147
      EPSSI(I,J)=0.0                                                    MANP0148
      EPSSO(I,J)=0.0                                                    MANP0149
      EPEEI(I,J)=0 0                                                    MANP0150
      EPEEO(I,J)=0.0                                                    MANP0151
      EPSEI(I,J)=0.0                                                    MANP0152
  200 EPSEO(I,J)=0 0                                                    MANP0153
      DO 201 I=1,NET                                                    MANP0154
      DO 201 JZ=1,NZSUBL                                                MANP0155
      DO 201 JA=1,NASQ                                                  MANP0156
      TAUSS(I,JZ,JA)=0.0                                                MANP0157
      TAUEE(I,JZ,JA)=0.0                                                MANP0158
  201 TAUSE(I,JZ,JA)=0.0                                                MANP0159
C                                                                       MANP0160
      DO 301 I=1,MNYST                                                  MANP0161
      DO 301 JZ=1,NZSUBL                                                MANP0162
      DO 301 JA=1,NA                                                    MANP0163
  301 TSGEE(I,JZ,JA)=0.0                                                MANP0164
C                                                                       MANP0165
      DO 303 I=1,MNXST                                                  MANP0166
      DO 303 JZ=1,NZSUBL                                                MANP0167
      DO 303 JA=1,NA                                                    MANP0168
  303 TAGSS(I,JZ,JA)=0.0                                                MANP0169
C                                                                       MANP0170
      PLASTW=0 0                                                        MANP0171
      TIME=0 0                                                          MANP0172
      ITIME=0                                                           MANP0173
C                                                                       MANP0174
C   FORMATION AND ASSEMBLY OF STIFFNESS AND MASS MATRICES               MANP0175
C   FOR PLATE ELEMENTS                                                  MANP0176
  304 NTYPE=1                                                           MANP0177
      SL=0 D0                                                           MANP0178
      CSA=0 D0                                                          MANP0179
      SLAM=0 D0                                                         MANP0180
      DO 220 LNUM=1,NET                                                 MANP0181
      NNN1=NP(1,LNUM)                                                   MANP0182
      NNN2=NP(2,LNUM)                                                   MANP0183
      NNN4=NP(4,LNUM)                                                   MANP0184
      XL=XGI(NNN2)-XGI(NNN1)                                            MANP0185
      YL=YGI(NNN4)-YGI(NNN1)                                            MANP0186
C IF DURING THIS PASS ONLY THE CONSISTENT MASS MATRIX IS BEING REFORMED,MANP0187
C BYPASS OTHER UNNECESSARY CALCULATIONS.                                MANP0188
      IF(IPASS .EQ  2) GO TO 214                                        MANP0189
      CALL RPLATE(NTYPE,ANUP,XL,YL,TH,SL,CSA,SLAM,SMOI,EK,GBI,EPAN)     MANP0190
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)                           MANP0191
      GO TO(204,214),IMASS                                              MANP0192
  204 CALL LMASSP(NTYPE,XL,YL,TH,SL,CSA,SMOI,EML,DENSP)                 MANP0193
      LL=(LNUM-1)*NDPE                                                  MANP0194
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      DO 210 I=1,24
      LLI=LL+I
      NN=NODE(LLI)
  210 AMASS(NN)=AMASS(NN)+EML(I)
      GO TO 220
  214 CALL CMASSP(NTYPE,XL,YL,TH,XF,SL,CSA,SMOI,EM,GBI,DENSP)
      CALL ASSEMK(EM,AMASS,INUM,NDPE,NODE,LNUM)
  220 CONTINUE
C   FOR X-DIRECTION STIFFENERS
      IF(NXST EQ 0)GO TO 250
      NTYPE=2
      DO 240 NN=1,NXST
      SL=XSPROP(6,NN)
      XF=XSPROP(7,NN)
      ANUS=XSPROP(4,NN)
      RW=XSPROP(2,NN)
      LNUM=LNXS(NN)
      RH=XSPROP(1,NN)
      YM=XSPROP(3,NN)
      DEN=XSPROP(5,NN)
      CSA=RW*RH
      SMOI=RH*RW*XF*XF+RW*RH**3/12.0
      XL=XGI(NP(2,LNUM))-XGI(NP(1,LNUM))
      YL=YGI(NP(4,LNUM))-YGI(NP(1,LNUM))
C IF DURING THIS PASS ONLY THE CONSISTENT MASS MATRIX IS BEING REFORMED,
C BYPASS OTHER UNNECESSARY CALCULATIONS.
      IF(IPASS  EQ  2) GO TO 234
      CALL RPLATE(NTYPE,ANUS,XL,YL,RH ,SL,CSA,XF  ,SMOI,EK,GBI,YM)
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      GO TO(224,234),IMASS
  224 CALL LMASSP(NTYPE,XL,YL,RH ,SL,CSA,SMOI,EML,DEN)
      LL=(LNUM-1)*NDPE
      DO 230 I=1,24
      NNN=NODE(LL+I)
  230 AMASS(NNN)=AMASS(NNN)+EML(I)
      GO TO 240
  234 CALL CMASSP(NTYPE,XL,YL,TH,XF,SL,CSA,SMOI,EM,GBI,DEN)
      CALL ASSEMK(EM,AMASS,INUM,NDPE,NODE,LNUM)
  240 CONTINUE
  250 CONTINUE
C   FOR Y-DIRECTION STIFFENERS
      IF(NYST EQ 0)GO TO 280
      NTYPE=3
      DO 270 NN=1,NYST
      RH=YSPROP(1,NN)
      RW=YSPROP(2,NN)
      YM=YSPROP(3,NN)
      ANUS=YSPROP(4,NN)
      DEN=YSPROP(5,NN)
      SL=YSPROP(6,NN)
      XF=YSPROP(7,NN)
      CSA=RW*RH
      SMOI=RH*RW*XF*XF+RW*RH**3/12.0
      LNUM=LNYS(NN)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
C IF DURING THIS PASS ONLY THE CONSISTENT MASS MATRIX IS BEING REFORMED,
C BYPASS OTHER UNNFCESSARY CALCULATIONS.
      IF(IPASS  EQ. 2) GO TO 264
      CALL RPLATE(NTYPE,ANUS,XL,YL,RH ,SL,CSA,XF  ,SMOI,EK,GBI,YM)
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      GO TO(254,264),IMASS
  254 CALL LMASSP(NTYPE,XL,YL,RH ,SL,CSA,SMOI,EML,DEN)
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      LL=(LNUM-1)*NDPE
      DO 260 I=1,24
      NNN=NODE(LL+I)
 260  AMASS(NNN)=AMASS(NNN)+EML(I)
      GO TO 270
 264  CALL CMASSP(NTYPE,XL,YL,TH,XF,SL,CSA,SMOI,EM,GBI,DEN)
      CALL ASSEMK(EM,AMASS,INUM,NDPE,NODE,LNUM)
 270  CONTINUE
 280  CONTINUE
C IF DURING THIS PASS ONLY THE CONSISTENT MASS MATRIX IS BEING REFORMED,
C BYPASS OTHER UNNECESSARY CALCULATIONS.
      IF(IPASS .EQ  2) GO TO 920
      IF(IPASS  EQ  3 .AND. NELES .EQ. 0) GO TO 290
      IF(IPASS  EQ  3) GO TO 741
C   FORMATION AND ASSEMBLY OF EQUIVALENT STIFFNESS MATRIX FOR LINE
C   TRANSLATIONAL AND TORSIONAL LINEAR RESTORING SPRINGS APPLIED TO
C   SIDES OF ELEMENTS
      READ(MREAD,500) NELES
      IF(NELES EQ 0)GO TO 289
      WRITE(MWRITE,720)
      WRITE(MWRITE,730)
      DO 288 I=1,NELES
      READ(MREAD,500)LNUM,ISIDE
      READ(MREAD,510)(SS(J),J=1,5)
      LNRS(I)=LNUM
      ISRS(I)=ISIDE
      DO 285 K=1,5
 285  SC(K,I)=SS(K)
      WRITE(MWRITE,740)LNUM,ISIDE,(SS(J),J=1,5)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
 288  CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
 720  FORMAT('0','LOCATION AND PROPERTIES OF LINE TRANSLATIONAL AND ROTA
     2TIONAL LINEAR RESTORING SPRINGS')
 730  FORMAT('0','ELEM  NO ',2X, 'ELEM. SIDE',8X,'KX(LB/IN/IN)',8X,
     2'KY(LB/IN/IN)',8X,'KZ(LB/IN/IN)',2X,'K-THETA(IN-LB/RAD/IN)',2X,
     3'K-PSI(IN-LB/RAD/IN)')
 740  FORMAT(' ',I5,6X,I5,5X,5D20.7)
      GO TO 289
C RESTORE THE SPRING STIFNESS MATRICES.
 741  DO 743 I=1,NELES
      LNUM=LNRS(I)
      ISIDE=ISRS(I)
      DO 742 KAY=1,5
 742  SS(KAY)=SC(KAY,I)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
 743  CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
 289  CONTINUE
      IF(INCONT EQ 1)GO TO 334
      IF(IPASS GT 1) GO TO 920
C   GENERATE PLATE INITIAL CONDITIONS (POS. AND VELOCITY)
      CALL INCOND(DISP,VEL,NP,NODE,XGI,YGI)
C   CONSTRAIN INITIAL CONDITIONS.
      CALL RCON(VEL,NBC)
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      CALL RCON(DISP,NBC)
C   CALCULATE KINETIC ENERGY IMPARTED TO PLATE BY INITIAL VELOCITY
C   DISTRIBUTION.
      CINETI=0 0
      GO TO (291,293),IMASS
 291  CONTINUE
      DO 292 I=1,NDT
 292  CINETI=CINETI+AMASS(I)*VEL(I)*VEL(I)/2.0
      GO TO 296
 293  CONTINUE
      DO 294 I=1,NDT
 294  FLVM(I)=0 0
      CALL OMULT(AMASS,VEL,FLVM,ICOL,KROW,NDEX)
      DO 295 I=1,NDT
 295  CINETI=CINETI+VEL(I)*FLVM(I)/2.0
C WORK AT INITIAL TIME IS ZERO.
 296  WFORCE=0.0
 334  CONTINUE
      HDT=0 5D0*DELTAT
C   CALCULATE TIME STEP BOUND.
      IF(IPASS EQ  1)CALL TSTEP(STF,AMASS,NBC,BC,ICOL,INUM,KROW,NDEX,
     1 FLN,DELD)
      IF(IPASS EQ 3 .AND. IMCONT.EQ.1) GO TO 290
      IF(IMCONT EQ 1) GO TO 2891
C CALCULATE USER EXTERNAL LOAD VECTOR AT INITIAL TIME.
      ITT=1
      CALL EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)
C CALCULATE POWER AT INITIAL TIME STEP.
      POWMP=0 D0
      DO 8999 I=1,NDT
 8999 POWMP=POWMP+ELV(I)*VEL(I)
C SET UP EQ  A 117 FOR THE INITIAL TIME AND SOLVE FOR ACCELERATIONS.
C ALSO SET INITIAL EQUIVALENT FORCES TO ZERO.
      DO 900 I=1,NDT
      STOR(I)=0 D0
      FLN (I)=0 D0
 900  FLVM(I)=ELV(I)+FLN(I)
      CALL OMULT(STF,VEL,STOR,ICOL,KROW,NDEX)
C FINISH FORMING R H S  OF EQUATION
      DO 902 I=1,NDT
 902  FLVM(I)=FLVM(I)-STOR(I)
C SOLVE FOR ACCELERATIONS AT INITIAL TIME.
      GO TO (904,908) , IMASS
C FOR LUMPED MASS MATRIX OPTION
 904  DO 906 I=1,NDT
      IF(AMASS(I)  .EQ. 0 D0) STOR(I)=0.D0
      IF(AMASS(I)  EQ 0 D0) GO TO 906
      STOR(I)=FLVM(I)/AMASS(I)
 906  CONTINUE
      GO TO 910
C FOR CONSISTENT MASS MATRIX OPTION.
 908  CALL SOLV(AMASS,FLVM,STOR,ICOL,KROW,NDEX)
 910  CONTINUE
C   PRINTOUT FOR INITIAL VALUES
      IF(IOP5 EQ 0.OR.NB EQ.0)GO TO 320
      DO 319 I=1,NB
 319  UCF1(I)=0 0
 320  CONTINUE
C ZERO OUT VECTOR FLVM FOR USE IN PRINT.
      DO 911 I=1,NDT
911   FLVM(I)=0 D0
      CALL PRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,XGI,
     2YGI,NP,GBI,STDE ,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,NMSUB,SUBW,
     3TAUSS,TAUEE,TAUSE,MAXEL,NZSUBL,NXST,LNXS,MATXS,XSPROP,TAGSS,MNXST,
     4NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,EK,IMASS,
     5AMASS,VEL,FLVM,KROW,NDEX,WFORCE,CINETI,NNSA,NVSA,NCON)
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      IF(IMASS.EQ.1) GO TO 920                                          MANP0390
C MUST RECREATE THE CONSISTENT MASS MATRIX.                             MANP0391
      IPASS=2                                                           MANP0392
      DO 914 I=1,MN                                                     MANP0393
  914 AMASS(I)=0 DO                                                     MANP0394
      GO TO 304                                                         MANP0395
  920 CONTINUE                                                          MANP0396
      IF(IPASS EQ 3) GO TO 290                                          MANP0397
      ITT=ITT+1                                                         MANP0398
C CALCULATE USER EXTERNAL LOAD VECTOR AT SECOND TIME STEP.              MANP0399
      CALL EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)                            MANP0400
      ITT=ITT-1                                                         MANP0401
C FIND DISPLACEMENTS AT SECOND TIME STEP BY SOLVING EQ. A.116.          MANP0402
      CONA=6 DO/DELTAT                                                  MANP0403
      CONB=CONA/DELTAT                                                  MANP0404
      DO 922 I=1,NDT                                                    MANP0405
      FLVM(I)=2 DO*STOR(I)+CONA*VEL(I)+CONB*DISP(I)                     MANP0406
  922 FLVA(I)=ELV(I)+FLN(I)                                             MANP0407
      GO TO (924,930), IMASS                                            MANP0408
C LUMPED MASS                                                           MANP0409
  924 DO 926 I=1,NB                                                     MANP0410
      KK=NBC(I)                                                         MANP0411
  926 AMASS(KK)=0 DO                                                    MANP0412
      DO 928 I=1,NDT                                                    MANP0413
      L=I+INUM(I)                                                       MANP0414
      FLVA(I)=FLVA(I)+AMASS(I)*FLVM(I)                                  MANP0415
  928 STF(L)=STF(L)+CONB*AMASS(I)                                       MANP0416
      GO TO 938                                                         MANP0417
  930 CALL BCONWK(AMASS,BC,ICOL,INUM,NBC)                               MANP0418
      CALL OMULT(AMASS,FLVM,FLVA,ICOL,KROW,NDEX)                        MANP0419
      DO 932 I=1,MN                                                     MANP0420
  932 STF(I)=STF(I)+CONB*AMASS(I)                                       MANP0421
  938 CALL FACT(STF,ICOL,KROW,NDEX,IDET,MWRITE,INUM)                    MANP0422
      CALL SOLV(STF,FLVA,DIS,ICOL,KROW,NDEX)                            MANP0423
      DT2=DELTAT**2                                                     MANP0424
      CONC=0 5DO/DELTAT                                                 MANP0425
C EVALUATE LEFT SIDE OF EQS. A.115C AND OF A.118.                       MANP0426
      DO 940 I=1,NDT                                                    MANP0427
      DISM1(I)=DT2*STOR(I)+2 0DO*DISP(I)-DIS(I)                         MANP0428
  940 VEL(I)=CONC*(3 DO*DIS(I)-4 0DO*DISP(I)+DISM1(I))                  MANP0429
C CALCULATE POWER FOR SECOND TIME STEP.                                 MANP0430
      POWMP1=0.DO                                                       MANP0431
      DO 942 I=1,NDT                                                    MANP0432
  942 POWMP1=POWMP1+ELV(I)*VEL(I)                                       MANP0433
      WFORCE=WFORCE+HDT*(POWMP+POWMP1)                                  MANP0134
C GO BACK AND RECREATE ASSEMBLED M AND ASSEMBLED K.                     MANP0435
 2891 DO 2892 I=1,MNC                                                   MANP0436
 2892 STF(I)=0 0DO                                                      MANP0437
      IPASS=3                                                           MANP0438
C DEFINE NUMBER OF TERMS IN MASS MATRIX.                                MANP0439
 2893 IF(IMASS EQ. 1) IMEND=NDT                                         MANP0440
      IF(IMASS EQ 2) IMEND=MNC                                          MANP0441
      DO 2894 I=1,IMEND                                                 MANP0442
 2894 AMASS(I)=0 0DO                                                    MANP0443
      GO TO 304                                                         MANP0444
  290 CONTINUE                                                          MANP0445
      DO 333 I=1,NDT                                                    MANP0446
  333 STOR(I)=ELV(I)                                                    MANP0447
      IF(IMASS.EQ 2)GO TO 322                                           MANP0448
      DO 321 I=1,NDT                                                    MANP0149
  321 FLVM(I)=ELV(I)+2.0*AMASS(I)*(VEL(I)+DISP(I)/DELTAT)/DELTAT        MANP0450
      GO TO 325                                                         MANP0451
  322 CONTINUE                                                          MANP0452
      DO 323 I=1,NDT                                                    MANP0453
  323 FLVM(I)=2 0*(VEL(I)+DISP(I)/DELTAT)/DELTAT                        MANP0454
      CALL OMULT(AMASS,FLVM,ELV,ICOL,KROW,NDEX)                         MANP0455
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      DO 324 I=1,NDT
  324 FLVM(I)=ELV(I)
  325 CONTINUE
      IF(IOP5 EQ.0.OR.NB.EQ.0)GO TO 329
      DO 327 I=1,NB
      II=NBC(I)
  327 UCF2(I)=FLVM(II)
  329 DO 330 I=1,NDT
  330 FLVA(I)=0.0
      COND=2 D0/DT2
C   FORM LHS
      GO TO(297,310),IMASS
  297 CONTINUE
      DO 307 II=1,NDT
      L=II+INUM(II)
  307 STF(L)=STF(L)+COND*AMASS(II)
      GO TO 318
  310 CONTINUE
      DO 315 II=1,MN
  315 STF(II)=STF(II)+COND*AMASS(II)
  318 CONTINUE
      IF(IOP5.EQ.1)CALL EAROW(STF,NBC,ICOL,INUM,RFM,ILAST,NBE,MBWE)
C
C
C   CONSTRAIN LHS
      CALL BCONMK(STF,BC,ICOL,INUM,NBC)
C
C
C   FACTOR LHS
      CALL FACT(STF,ICOL,KROW,NDEX,IDET,MWRITE,INUM)
      IF(IMCONT EQ 0) GO TO 340
C   INPUT FOR CONTINUATION RUN
  335 READ(MREAD,800) ITT,CINETI,WFORCE,POWMP1
      READ (MREAD,810)(DIS(I),I=1,NDT)
      READ (MREAD,810)(DISP(I),I=1,NDT)
      READ (MREAD,810)(VEL(I),I=1,NDT)
      READ (MREAD,810)(FLVA(I),I=1,NDT)
      IF(IOP5 EQ 1 AND NB GT 0)READ (MREAD,810)(UCF2(I),I=1,NB)
      READ (MREAD,810)(STOR(I),I=1,NDT)
      READ (MREAD,810)(((TAUSS(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      READ (MREAD,810)(((TAUEE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      READ (MREAD,810)(((TAUSE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      IF(NXST EQ 0)GO TO 336
      READ (MREAD,810)(((TAGSS(I,J,K),I=1,NXST),J=1,MNSLT4),K=1,3)
  336 CONTINUE
      IF(NYST EQ 0)GO TO 337
      READ (MREAD,810)(((TSGEE(I,J,K),I=1,NYST),J=1,MNSLT4),K=1,3)
  337 NNT=NDT/6
      DO 338 I=1,NNT
      NN=(I-1)*6
      XG(I)=XGI(I)+DISP(NN+1)
      YG(I)=YGI(I)+DISP(NN+2)
  338 ZG(I)=ZG(I)+DISP(NN+3)
C   BEGIN LOOP OVER TIME STEPS
  340 ITT=ITT+1
      DO 350 J=1,NDT
      DELD(J)=DIS(J)-DISP(J)
      DISM1(J)=DISP(J)
      DISP(J)=DIS(J)
      FLN(J)=FLVA(J)
      FLVM(J)=0.0
  350 FLVA(J)=0.0
      POWM=POWMP1
      NNT=NDT/6
      DO 351 J=1,NNT
      NN=(J-1)*6
```

```
      XG(J)=XG(J)+DELD(NN+1)
      YG(J)=YG(J)+DELD(NN+2)
  351 ZG(J)=ZG(J)+DELD(NN+3)
      IF(IOP5.EQ 0.OR.NB.EQ.0)GO TO 353
      DO 352 I=1,NB
  352 UCF1(I)=UCF2(I)
  353 CONTINUE
C   CALCULATE WORK OF EXTERNAL FORCES.
      POWMP1=0 D0
      DO 355 I=1,NDT
  355 POWMP1=POWMP1+STOR(I)*VEL(I)
      WFORCE=WFORCE+HDT*(POWM+POWMP1)
C   CALCULATE EQUIVALENT LOADS
C   FOR PLATE ELEMENT
C   CALCULATE EQUIVALENT LOADS FOR PLATE ELEMENT
      CALL STRESP(DISP,DELD,FLVA,ITT,NODE,MAXEL,TAUSS,TAUEE,TAUSE,
     2EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,PLASTW,XGI,YGI,NP,STDE,GBI,
     3NMSUB,DSR,PSR,SUBW,SNO ,NZSUBL)
C   FOR X-DIRECTION STIFFENERS
      IF(NXST EQ 0)GO TO 360
      CALL STRESA(DISP,DELD,FLVA,ITT,NODE,MAXEL,MNXST,TAGSS,PLASTW,XGI,
     2YGI,NP,STDE,GBI,NMSUB,DSR,PSR,SUBW,SNO ,NXST,XSPROP,LNXS,MATXS,
     3NZSUBL)
  360 CONTINUE
C   FOR Y-DIRECTION STIFFENERS
      IF(NYST.EQ 0)GO TO 370
      CALL STRESC(DISP,DELD,FLVA,ITT,NODE,MAXEL,MNYST,TSGEE,PLASTW,XGI,
     2YGI,NP,STDE,GBI,NMSUB,DSR,PSR,SUBW,SNO ,NYST,YSPROP,LNYS,MATYS,
     3NZSUBL)
  370 CONTINUE
C   PRINT OUTPUT
      ITIME=ITT-1
      TIME=ITIME*DELTAT
      CALL PRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,XGI,
     2YGI,NP,GBI,STDE ,NODE,RFM,NBE,MOVE,ILAST,UCF1,NBC,ICOL,NMSUB,SUBW,
     3TAUSS,TAUEE,TAUSE,MAXEL,NZSUBL,NXST,LNXS,MATXS,XSPROP,TAGSS,MNXST,
     4NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,EK,IMASS,
     5AMASS,VEL,FLVM,KROW,NDEX,WFORCE,CINETI,NNSA,NVSA,NCON)
C   CALC  EXTERNAL LOADING VECTOR
      CALL EXTL(ELV,ITT+1,NP,NODE,XGI,YGI,GBI)
      DO 375 I=1,NDT
  375 STOR(I)=ELV(I)
C   FORM RHS
      IF(IMASS EQ 2)GO TO 382
      DO 380 I=1,NDT
  380 FLVM(I)=-(2.0*FLVA(I)-FLN(I))+ELV(I)+2.0*AMASS(I)*(VEL(I)+DISP(I)
     2/DELTAT)/DELTAT
      GO TO 387
  382 CONTINUE
      DO 384 I=1,NDT
      FLVM(I)=2 0*(VEL(I)+DISP(I)/DELTAT)/DELTAT
  384 ELV(I)=ELV(I)-(2.0*FLVA(I)-FLN(I))
      CALL DMULT(AMASS,FLVM,ELV,ICOL,KROW,NDEX)
      DO 386 I=1,NDT
  386 FLVM(I)=ELV(I)
  387 CONTINUE
      IF(IOP5 EQ 0 OR.NB.EQ.0)GO TO 389
      DO 388 I=1,NB
      II=NBC(I)
  388 UCF2(I)=FLVM(II)
C   CONSTRAIN RHS
  389 CALL RCON(FLVM,NBC)
C   SOLVE FOR DISPLACEMENTS
      CALL SOLV(STF,FLVM,DIS,ICOL,KROW,NDEX)
C   CALCULATE VELOCITY
      DO 390 I=1,NDT
```

```
 390  VEL(I)=(3 0*DIS(I)-4.0*DISP(I)+DISM1(I))/(2.0*DELTAT)                 MANP0588
C   CHECK FOR CONTINUATION                                                 MANP0589
      IF(ITIME LT ITIMEF.AND.TIME.LT.TIMEF)GO TO 340                       MANP0590
      IF(IPUNCH EQ.0)GO TO 420                                             MANP0591
      WRITE(MPUNCH,800) ITT,CINETI,WFORCE,POWMP1                           MANP0592
      WRITE(MPUNCH,810)(DIS(I),I=1,NDT)                                    MANP0593
      WRITE(MPUNCH,810)(DISP(I),I=1,NDT)                                   MANP0594
      WRITE(MPUNCH,810)(VEL(I),I=1,NDT)                                    MANP0595
      WRITE(MPUNCH,810)(FLVA(I),I=1,NDT)                                   MANP0596
      IF(IOP5.EQ 1 AND NB.GT 0)WRITE(MPUNCH,810)(UCF2(I),I=1,NB)           MANP0597
      WRITE(MPUNCH,810)(STOR(I),I=1,NDT)                                   MANP0598
      WRITE(MPUNCH,810)(((TAUSS(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)         MANP0599
      WRITE(MPUNCH,810)(((TAUEE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)         MANP0600
      WRITE(MPUNCH,810)(((TAUSE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)         MANP0601
      IF(NXST EQ 0)GO TO 400                                               MANP0602
      WRITE(MPUNCH,810)(((TAGSS(I,J,K),I=1,NXST),J=1,MNSLT4),K=1,3)        MANP0603
 400  CONTINUE                                                             MANP0604
      IF(NYST EQ.0)GO TO 410                                               MANP0605
      WRITE(MPUNCH,810)(((TSGEE(I,J,K),I=1,NYST),J=1,MNSLT4),K=1,3)        MANP0606
 410  WRITE(MWRITE,820)ITT                                                 MANP0607
 420  RETURN                                                               MANP0608
 800  FORMAT(I6,3D15.7)                                                    MANP0609
 810  FORMAT(5D15.7)                                                       MANP0610
 820  FORMAT('0','CONTINUATION CARDS HAVE BEEN PUNCHED AT ITT=',I7)        MANP0611
      END                                                                  MANP0612
      SUBROUTINE PRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,        PRNT0000
     2EPSEO,XGI,YGI,NP,GBI,STDEP,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,    PRNT0001
     3NMSUB,SUBW,TAUSS,TAUEE,TAUSE,MAXEL,NZS,NXST,LNXS,MATXS,XSPROP,       PRNT0002
     4TAGSS,MNXST,NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,   PRNT0003
     5EK,IMASS,AMASS,VEL,FLVM,KROW,NDEX,WFORCE,CINETI,NNSA,NVSA,NCON)      PRNT0004
C                                                                          PRNT0005
C    ASRL TR 154-14...  ..ORIGINAL REPORT VERSION OF PROGRAM               PRNT0006
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980               PRNT0007
C                                                                          PRNT0008
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER                PRNT0009
C                                                                          PRNT0010
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.                PRNT0011
C                                                                          PRNT0012
      IMPLICIT REAL*8(A-H,O-Z)                                             PRNT0013
C                                                                          PRNT0014
      DIMENSION DISP(1),XG(1),YG(1),ZG(1),EPSSI(9,1),EPSSO(9,1),EPEEI(9    PRNT0015
     2,1),EPEEO(9,1),EPSEI(9,1),EPSEO(9,1),XGI(1),YGI(1),NP(4,1),GBI(24    PRNT0016
     3,24),STDEP(9,24),NODE(1),EP(9),ELONG(2),RFM(NBE,MBWE),ILAST(1),      PRNT0017
     4UCF1(1),NBC(1),ICOL(1),RF(6),NMSUB(1),AGAUS(6),AGAUW(6),ZGAUS(6),    PRNT0018
     5ZGAUW(6) SUBW(5,1) TAUSS(MAXEL,NZS,9),TAUEE(MAXEL NZS 9) TAUSE       PRNT0019
     6(MAXEL,NZS,9),LNXS(1),MATXS(1),XSPROP(7,1),TAGSS(MNXST,NZS,3),       PRNT0020
     7LNYS(1),MATYS(1),YSPROP(7,1),TSGEE(MNYST,NZS,3),LNRS(1),ISRS(1),     PRNT0021
     8SC(5,1),SS(5),EK(24,24),EQ(24),STOR(24),AMASS(1),VEL(1),FLVM(1),     PRNT0022
     9KROW(1),NDEX(1),NVSA(1),NCON(4,1)                                    PRNT0023
      COMMON/PLATE/XDIS,YDIS,TH,HX,HY,NEAD,NECD,DSP,EPAN,ANUP,THALF        PRNT0024
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                                  PRNT0025
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME                 PRNT0026
      COMMON/ASP/SLASP(50),ELASP(50),EDIR(2,50),NASP,LNASP(50)             PRNT0027
      COMMON/PRNT/IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8                  PRNT0028
      COMMON/GSOUT/NEGS,LNGS(200)                                          PRNT0029
      COMMON/SSOUT/NXSS,NYSS,INXSS(200),INYSS(200)                         PRNT0030
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                   PRNT0031
C                                                                          PRNT0032
C THIS SUBROUTINE CONTROLS THE CALCULATION (AS NECESSARY) AND OUTPUT       PRNT0033
C OF ALL QUANTITIES (DISPLACEMENTS, STRAINS, ENERGIES, ETC.) REQUESTED     PRNT0034
C BY THE USER AT REGULAR INTERVALS DURING THE TIMEWISE SOLUTION FOR        PRNT0035
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C THE PLATE PROGRAM.
C
      IF(ITIME LE.1)GO TO 10
      III=ITIME/INCRT*INCRT-ITIME
      IF(III NE.0)GO TO 500
 10   CONTINUE
      WRITE(MWRITE,600)ITIME,TIME
C   NODAL DISPLACEMENTS AND LOCATION.
      IF(IOP1.EQ.0)GO TO 30
      WRITE(MWRITE,610)
      NNT=NDT/6
      DO 20 NN=1,NNT
      JJ=(NN-1)*6
      J1=JJ+1
      J2=JJ+6
 20   WRITE(MWRITE,620)NN,(DISP(J),J=J1,J2),XG(NN),YG(NN),ZG(NN)
 30   CONTINUE
C   REACTION FORCES AT CONSTRAINED NODES.
      IF(IOP5 EQ 0.OR NB.EQ.0)GO TO 39
      WRITE(MWRITE,550)
      WRITE(MWRITE,560)
C   DETERMINE NODE NO. OF FIRST CONSTRAINED D.O.F..
      NC=NBC(1)
      NN1=(NC-1)/6+1
      DO 31 I=1,6
 31   RF(I)=0 0
C   LOOP OVER CONSTRAINED D.O.F..
      DO 38 INB=1,NB
C   DETERMINE NODE NO. OF PRESENT CONSTRAINED D.O.F..
      NC=NBC(INB)
      NN2=(NC-1)/6+1
C   IF NN2 NE.NN1, CALC. OF RF FOR NODE NN1 IS COMPLETE--PRINT OUT
C   VALUES
      IF(NN2 EQ.NN1)GO TO 33
      WRITE(MWRITE,570)NN1,(RF(J),J=1,6)
      NN1=NN2
      DO 32 I=1,6
 32   RF(I)=0.0
 33   CC=-UCF1(INB)
      NN=0
      IB=ICOL(NC)
      IE=ILAST(INB)
      DO 34 I=IB,IE
      NN=NN+1
 34   CC=CC+RFM(INB,NN)*DISP(I)
      NDF=NC-(NN1-1)*6
      RF(NDF)=CC
 38   CONTINUE
      WRITE(MWRITE,570)NN1,(RF(J),J=1,6)
 39   CONTINUE
C   CENTROIDAL STRAIN INFORMATION.
      IF(IOP2 EQ 0)GO TO 50
      WRITE(MWRITE,630)
      WRITE(MWRITE,640)
      WRITE(MWRITE,644)
      DO 40 LNUM=1,NET
      G11=EPSSI(5,LNUM)
      G22=EPEEI(5,LNUM)
      G12=EPSEI(5,LNUM)
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      CALL PSTRN(G11,G22,G12,SI,DIRI)                                    PRNT0095
      G11=EPSSO(5,LNUM)                                                  PRNT0096
      G22=EPEEO(5,LNUM)                                                  PRNT0097
      G12=EPSEO(5,LNUM)                                                  PRNT0098
      CALL PSTRN(G11,G22,G12,SO,DIRO)                                    PRNT0099
 40   WRITE(MWRITE,650)LNUM,EPSSI(5,LNUM),EPSSO(5,LNUM),EPEEI(5,LNUM),   PRNT0100
     2EPEEO(5,LNUM),EPSEI(5,LNUM),EPSEO(5,LNUM),SI,SO,DIRI,DIRO          PRNT0101
 50   CONTINUE                                                           PRNT0102
C    NODAL AVERAGE STRAINS.                                              PRNT0103
      IF(IOP8.EQ.0)GO TO 55                                              PRNT0104
      WRITE(MWRITE,900)                                                  PRNT0105
      WRITE(MWRITE,640)                                                  PRNT0106
      WRITE(MWRITE,920)                                                  PRNT0107
      DO 53 II=1,NNSA                                                    PRNT0108
      G11U=0.0                                                           PRNT0109
      G11L=0.0                                                           PRNT0110
      G22U=0.0                                                           PRNT0111
      G22L=0.0                                                           PRNT0112
      G12U=0.0                                                           PRNT0113
      G12L=0.0                                                           PRNT0114
      NNUM=NVSA(II)                                                      PRNT0115
      DENOM=0.0D0                                                        PRNT0116
      DO 52 JJ=1,4                                                       PRNT0117
      IF(NCON(JJ,II).EQ.0)GO TO 52                                       PRNT0118
      DENOM=DENOM+1.0D0                                                  PRNT0119
      LNUM=NCON(JJ,II)                                                   PRNT0120
      NNN1=NP(1,LNUM)                                                    PRNT0121
      NNN2=NP(2,LNUM)                                                    PRNT0122
      NNN4=NP(4,LNUM)                                                    PRNT0123
      XL=XGI(NNN2)-XGI(NNN1)                                             PRNT0124
      YL=YGI(NNN4)-YGI(NNN1)                                             PRNT0125
      SL=0.0                                                             PRNT0126
      EL=0.0                                                             PRNT0127
      IF(JJ.EQ.2.OR.JJ.EQ.3)SL=1.0                                       PRNT0128
      IF(JJ.EQ.3.OR.JJ.EQ.4)EL=1.0                                       PRNT0129
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)                                   PRNT0130
      DO 51 J=1,9                                                        PRNT0131
      EP(J)=0.0                                                          PRNT0132
      DO 51 K=1,24                                                       PRNT0133
      NNN=(LNUM-1)*24+K                                                  PRNT0134
      NN=NODE(NNN)                                                       PRNT0135
 51   EP(J)=EP(J)+STDEP(J,K)*DISP(NN)                                    PRNT0136
      EPSSM=EP(1)+EP(7)*EP(7)/2.0+EP(9)*EP(9)/2.0                        PRNT0137
      EPEEM=EP(2)+EP(8)*EP(8)/2.0+EP(9)*EP(9)/2.0                        PRNT0138
      EPSEM=EP(3)+EP(7)*EP(8)                                            PRNT0139
      G11L=G11L+EPSSM+THALF*EP(4)                                        PRNT0140
      G11U=G11U+EPSSM-THALF*EP(4)                                        PRNT0141
      G22L=G22L+EPEEM+THALF*EP(5)                                        PRNT0142
      G22U=G22U+EPEEM-THALF*EP(5)                                        PRNT0143
      G12L=G12L+EPSEM+THALF*EP(6)                                        PRNT0144
      G12U=G12U+EPSEM-THALF*EP(6)                                        PRNT0145
 52   CONTINUE                                                           PRNT0146
      G11L=G11L/DENOM                                                    PRNT0147
      G11U=G11U/DENOM                                                    PRNT0148
      G22L=G22L/DENOM                                                    PRNT0149
      G22U=G22U/DENOM                                                    PRNT0150
      G12L=G12L/DENOM                                                    PRNT0151
      G12U=G12U/DENOM                                                    PRNT0152
      CALL PSTRN(G11L,G22L,G12L,SI,DIRI)                                 PRNT0153
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      CALL PSTRN(G11U,G22U,G12U,SO,DIRO)
   53 WRITE(MWRITE,650)NNUM,G11L,G11U,G22L,G22U,G12L,G12U,SI,SO,DIRI,
     + DIRO
   55 CONTINUE
C   GAUSS STATION STRAIN PRINTOUT.
      IF(IOP3.EQ.0)GO TO 70
      WRITE(MWRITE,660)
      DO 60 NN=1,NEGS
      LNUM=LNGS(NN)
      WRITE(MWRITE,670) LNUM
      WRITE(MWRITE,640)
      WRITE(MWRITE,645)
      DO 60 NG=1,9
      G11=EPSSI(NG,LNUM)
      G22=EPEEI(NG,LNUM)
      G12=EPSEI(NG,LNUM)
      CALL PSTRN(G11,G22,G12,SI,DIRI)
      G11=EPSSO(NG,LNUM)
      G22=EPEEO(NG,LNUM)
      G12=EPSEO(NG,LNUM)
      CALL PSTRN(G11,G22,G12,SO,DIRO)
   60 WRITE(MWRITE,650)NG,EPSSI(NG,LNUM),EPSSO(NG,LNUM),EPEEI(NG,LNUM),
     2EPEEO(NG,LNUM),EPSEI(NG,LNUM),EPSEO(NG,LNUM),SI,SO,DIRI,DIRO
   70 CONTINUE
C   ADDITIONAL STRAIN POINTS.
      IF(IOP4.EQ 0)GO TO 110
      WRITE(MWRITE,690)
      WRITE(MWRITE,695)
      DO 100 KK=1,NASP
      LNUM=LNASP(KK)
      SL=SLASP(KK)
      EL=ELASP(KK)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)
      DO 80 J=1,9
      EP(J)=0 0
      DO 80 K=1,24
      NNN=(LNUM-1)*24+K
      NN=NODE(NNN)
   80 EP(J)=EP(J)+STDEP(J,K)*DISP(NN)
      EPSSM=EP(1)+EP(7)*EP(7)/2.0+EP(9)*EP(9)/2.0
      EPEEM=EP(2)+EP(8)*EP(8)/2.0+EP(9)*EP(9)/2.0
      EPSEM=EP(3)+EP(7)*EP(8)
      DO 100 II=1,2
      FCT=1 0D0
      IF(II.EQ 2)FCT=-1 0D0
      G11=EPSSM+FCT*THALF*EP(4)
      G22=EPEEM+FCT*THALF*EP(5)
      G12=EPSEM+FCT*THALF*EP(6)
      CALL PSTRN(G11,G22,G12,SI,DIRI)
      DO 90 I=1,2
      THET=EDIR(I,KK)*.1745329251994330D-01
      CT=DCOS(THET)
      ST=DSIN(THET)
      ELONG(I)=G11*CT*CT+G22*ST*ST+G12*CT*ST
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 90   ELONG(I)=DSQRT(1.0+2.0*ELONG(I))-1.0                                  PRNT0213
      IF(II.EQ.1)WRITE(MWRITE,700)KK,G11,G22,G12,ELONG(1),ELONG(2),SI,DIPRNT0214
     +RI                                                                    PRNT0215
      IF(II.EQ.2)WRITE(MWRITE,710)KK,G11,G22,G12,ELONG(1),ELONG(2),SI,DIPRNT0216
     +RI                                                                    PRNT0217
 100  CONTINUE                                                              PRNT0218
 110  CONTINUE                                                              PRNT0219
C   CALCULATION AND PRINTOUT OF GAUSS STATION STRAINS ON STIFFENERS.        PRNT0220
      IF(IOP6.EQ.0)GO TO 190                                                PRNT0221
      NA=3                                                                  PRNT0222
      CALL GAUSS(NA,AGAUS,AGAUW)                                            PRNT0223
C   FOR X-DIRECTION STIFFENERS                                              PRNT0224
      IF(NXSS.EQ.0)GO TO 150                                                PRNT0225
      WRITE(MWRITE,780)                                                     PRNT0226
      WRITE(MWRITE,785)                                                     PRNT0227
      WRITE(MWRITE,790)                                                     PRNT0228
      DO 140 NX=1,NXSS                                                      PRNT0229
      NST=INXSS(NX)                                                         PRNT0230
      LNUM=LNXS(NST)                                                        PRNT0231
      NNN1=NP(1,LNUM)                                                       PRNT0232
      NNN2=NP(2,LNUM)                                                       PRNT0233
      NNN4=NP(4,LNUM)                                                       PRNT0234
      XL=XGI(NNN2)-XGI(NNN1)                                                PRNT0235
      YL=YGI(NNN4)-YGI(NNN1)                                                PRNT0236
      EL=XSPROP(6,NST)                                                      PRNT0237
      DO 130 NG=1,NA                                                        PRNT0238
      SL=(1.0+AGAUS(NG))/2.0                                                PRNT0239
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)                                      PRNT0240
      EP(1)=0.0                                                             PRNT0241
      EP(4)=0.0                                                             PRNT0242
      EP(7)=0.0                                                             PRNT0243
      DO 120 K=1,24                                                         PRNT0244
      NNN=(LNUM-1)*24+K                                                     PRNT0245
      NN=NODE(NNN)                                                          PRNT0246
      EP(1)=EP(1)+STDEP(1,K)*DISP(NN)                                       PRNT0247
      EP(4)=EP(4)+STDEP(4,K)*DISP(NN)                                       PRNT0248
 120  EP(7)=EP(7)+STDEP(7,K)*DISP(NN)                                       PRNT0249
      NN=(NG-1)*2+1                                                         PRNT0250
      STOR(NN)=EP(1)+EP(7)*EP(7)/2.0+(0.5*XSPROP(1,NST)+XSPROP(7,NST))      PRNT0251
     2*EP(4)                                                                PRNT0252
      STOR(NN+1)=EP(1)+EP(7)*EP(7)/2.0+(XSPROP(7,NST)-0.5*XSPROP(1,NST))    PRNT0253
     2*EP(4)                                                                PRNT0254
 130  CONTINUE                                                              PRNT0255
 140  WRITE(MWRITE,800)NST,(STOR(J),J=1,6)                                  PRNT0256
 150  CONTINUE                                                              PRNT0257
C   FOR Y-DIRECTION STIFFENERS.                                             PRNT0258
      IF(NYSS.EQ.0)GO TO 190                                                PRNT0259
      WRITE(MWRITE,810)                                                     PRNT0260
      WRITE(MWRITE,785)                                                     PRNT0261
      WRITE(MWRITE,790)                                                     PRNT0262
      DO 180 NY=1,NYSS                                                      PRNT0263
      NST=INYSS(NY)                                                         PRNT0264
      LNUM=LNYS(NST)                                                        PRNT0265
      NNN1=NP(1,LNUM)                                                       PRNT0266
      NNN2=NP(2,LNUM)                                                       PRNT0267
      NNN4=NP(4,LNUM)                                                       PRNT0268
      XL=XGI(NNN2)-XGI(NNN1)                                                PRNT0269
      YL=YGI(NNN4)-YGI(NNN1)                                                PRNT0270
      SL=YSPROP(6,NST)                                                      PRNT0271
```

```
      DO 170 NG=1,NA
      EL=(1.0+AGAUS(NG))/2.0
      CALL SFDM(SL,EL,XL,YL,CBI,STDEP)
      EP(2)=0.0
      EP(5)=0.0
      EP(8)=0.0
      DO 160 K=1,24
      NNN=(LNUM-1)*24+K
      NN=NODE(NNN)
      EP(2)=EP(2)+STDEP(2,K)*DISP(NN)
      EP(5)=EP(5)+STDEP(5,K)*DISP(NN)
  160 EP(8)=EP(8)+STDEP(8,K)*DISP(NN)
      NN=(NG-1)*2+1
      STOR(NN)=EP(2)+EP(8)*EP(8)/2.0+(0.5*YSPROP(1,NST)+YSPROP(7,NST))
     2*EP(5)
      STOR(NN+1)=EP(2)+EP(8)*EP(8)/2.0+(YSPROP(7,NST)-0.5*YSPROP(1,NST))
     2*EP(5)
  170 CONTINUE
  180 WRITE(MWRITE,800)NST,(STOR(J),J=1,6)
  190 CONTINUE
C   CALCULATION AND PRINTOUT OF SYSTEM ENERGIES.
      IF(IOP7.EQ.0)GO TO 440
C   ELASTIC STRAIN ENERGY STORED IN STRUCTURE.
C   FOR PLATE ELEMENTS.
      NA=3
      NZ=4
      NSUBL=NMSUB(1)
      CALL GAUSS(NA,AGAUS,AGAUW)
      CALL GAUSS(NZ,ZGAUS,ZGAUW)
      ELAST=0.0
      S1=1.0/EPAN
      S2=-ANUP*S1
      S3=2.0*(1.0+ANUP)/EPAN
      DO 240 LNUM=1,NET
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      HXL=(XGI(NNN2)-XGI(NNN1))/2.0
      HYL=(YGI(NNN4)-YGI(NNN1))/2.0
      DO 240 KS=1,NA
      SW=HXL*AGAUW(KS)
      DO 240 KE=1,NA
      EW=HYL*AGAUW(KE)
      JA=KE+(KS-1)*NA
      STOR1=0.0
      DO 230 LZ=1,NZ
      ST1=0.0
      ST2=0.0
      ST3=0.0
      DO 220 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
      ST1=ST1+SUBW(1,LS)*TAUSS(LNUM,JZ,JA)
      ST2=ST2+SUBW(1,LS)*TAUEE(LNUM,JZ,JA)
  220 ST3=ST3+SUBW(1,LS)*TAUSE(LNUM,JZ,JA)
      PROD=S1*(ST1*ST1+ST2*ST2)+2.0*S2*ST1*ST2+S3*ST3*ST3
  230 STOR1=STOR1+PROD*0.5*TH*ZGAUW(LZ)
  240 ELAST=ELAST+STOR1*SW*EW/2.0
C   ELASTIC ENERGY IN X-DIRECTION STIFFENERS.
      IF(NXST.EQ.0)GO TO 280
```

```
      DO 270 NST=1,NXST
      LNUM=LNXS(NST)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      HXL=(XGI(NNN2)-XGI(NNN1))/2.0
      MAT=MATXS(NST)
      NSUBL=NMSUB(MAT)
      S1=1 0/XSPROP(3,NST)
      EW=XSPROP(2,NST)
      DO 270 KE=1,NA
      SW=HXL*AGAUW(KE)
      STOR1=0 0
      DO 260 LZ=1,NZ
      ST1=0.0
      DO 250 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
  250 ST1=ST1+SUBW(MAT,LS)*TAGSS(NST,JZ,KE)
      PROD=S1*ST1*ST1
  260 STOR1=STOR1+PROD*0.5*XSPROP(1,NST)*ZGAUW(LZ)
  270 ELAST=ELAST+STOR1*EW*SW/2.0
  280 CONTINUE
C   ELASTIC ENERGY STORED IN Y-DIRECTION STIFFENERS.
      IF(NYST.EQ 0)GO TO 320
      DO 310 NST=1,NYST
      LNUM=LNYS(NST)
      NNN1=NP(1,LNUM)
      NNN4=NP(4,LNUM)
      HYL=(YGI(NNN4)-YGI(NNN1))/2.0
      MAT=MATYS(NST)
      NSUBL=NMSUB(MAT)
      S1=1 0/YSPROP(3,NST)
      SW=YSPROP(2,NST)
      DO 310 KS=1,NA
      EW=HYL*AGAUW(KS)
      STOR1=0 0
      DO 300 LZ=1,NZ
      ST1=0 0
      DO 290 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
  290 ST1=ST1+SUBW(MAT,LS)*TSGEE(NST,JZ,KS)
      PROD=S1*ST1*ST1
  300 STOR1=STOR1+PROD*0.5*YSPROP(1,NST)*ZGAUW(LZ)
  310 ELAST=ELAST+STOR1*EW*SW/2.0
  320 CONTINUE
C   CALCULATION OF ENERGY STORED IN ELASTIC RESTORING SPRINGS.
      SPREN=0.0
      IF(NELES EQ.0)GO TO 380
      DO 370 NS=1,NELES
      LNUM=LNRS(NS)
      ISIDE=ISRS(NS)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      DO 330 I=1,5
  330 SS(I)=SC(I,NS)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
      L=(LNUM-1)*24
```

```
      DO 340 I=1,24                                                     PRNT0390
      LL=NODE(L+I)                                                      PRNT0391
  340 EQ(I)=DISP(LL)                                                    PRNT0392
      DO 350 I=1,24                                                     PRNT0393
      STOR(I)=0 0                                                       PRNT0394
      DO 350 K=1,24                                                     PRNT0395
  350 STOR(I)=STOR(I)+EK(I,K)*EQ(K)                                     PRNT0396
      STOR1=0.0                                                         PRNT0397
      DO 360 I=1,24                                                     PRNT0398
  360 STOR1=STOR1+EQ(I)*STOR(I)                                         PRNT0399
  370 SPREN=SPREN+STOR1/2.0                                             PRNT0400
  380 CONTINUE                                                          PRNT0401
C   CALCULATE KINETIC ENERGY OF THE STRUCTURE.                          PRNT0402
      CINET=0 0                                                         PRNT0403
      GO TO(390,410),IMASS                                              PRNT0404
  390 CONTINUE                                                          PRNT0405
      DO 400 I=1,NDT                                                    PRNT0406
  400 CINET=CINET+AMASS(I)*VEL(I)*VEL(I)/2.0                            PRNT0407
      GO TO 430                                                         PRNT0408
  410 CALL OMULT(AMASS,VEL,FLVM,ICOL,KROW,NDEX)                         PRNT0409
      DO 420 I=1,NDT                                                    PRNT0410
  420 CINET=CINET+VEL(I)*FLVM(I)/2.0                                    PRNT0411
C   CALCULATE WORK INPUT TO STRUCTURE AS SUM OF CURRENT WORK OF EXTER-  PRNT0412
C   NAL FORCES PLUS KINETIC ENERGY OF INITIAL VELOCITY DISTRIBUTION.    PRNT0413
  430 EWORK=WFORCE+CINETI                                               PRNT0414
C   CALCULATE PLASTIC WORK BY SUBTRACTION.                              PRNT0415
      PLASTW=EWORK-ELAST-SPREN-CINET                                    PRNT0416
C   PRINTOUT ENERGIES                                                   PRNT0417
      WRITE(MWRITE,720)                                                 PRNT0418
      WRITE(MWRITE,730) EWORK                                           PRNT0419
      WRITE(MWRITE,740) CINET                                           PRNT0420
      WRITE(MWRITE,750) ELAST                                           PRNT0421
      WRITE(MWRITE,760) PLASTW                                          PRNT0422
      WRITE(MWRITE,770) SPREN                                           PRNT0423
  440 CONTINUE                                                          PRNT0424
  500 CONTINUE                                                          PRNT0425
  550 FORMAT('0','REACTION FORCES AT CONSTRAINED NODES')                PRNT0426
  560 FORMAT('0',2X,'NODE',4X,'RX(LBS)',8X,'RY(LBS)',8X,'RZ(LBS)',7X,   PRNT0427
     2'MX(LBS-IN)',5X,'MY(LBS-IN)',5X,'MXY(LBS-IN-IN)')                 PRNT0428
  570 FORMAT(' ',I6,6D15 5)                                             PRNT0429
  600 FORMAT('0','***** INCR. NO =',I5,2X,'TIME=',D12.4,1X,'SEC.')      PRNT0430
  610 FORMAT('0','NODE',6X,'U',12X,'V',12X,'W',11X,'PSIX',9X,'PSIY',9X, PRNT0431
     2'TWIST',7X,'Y-POS ',7X,'Y-POS.',7X,'Z-POS.')                      PRNT0432
  620 FORMAT(' ',I4,9D13 5)                                             PRNT0433
  630 FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECPRNT0434
     2TION AT CENTROID OF EACH ELEMENT')                                PRNT0435
  640 FORMAT('0',14X,'EPS-X STRAIN',12X,'EPS-Y STRAIN',12X,'SHEAR STRAINPRNT0436
     2',12X,'PRINC. STRAIN(T)',10X,'DIRECTION(DEG.)')                   PRNT0437
  644 FORMAT(' ','ELEM.',5(7X,'INNER',7X,'OUTER'))                      PRNT0438
  645 FORMAT(' ','STA. ',5(7X,'INNER',7X,'OUTER'))                      PRNT0439
  650 FORMAT(' ',I4,1X,10D12 4)                                         PRNT0440
  660 FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECPRNT0441
     2TION AT GAUSS STATIONS FOR SELECTED ELEMENTS')                    PRNT0442
  670 FORMAT('0','RESULTS FOR ELEMENT NUMBER',I5)                       PRNT0443
  690 FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECPRNT0444
     2TION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS')            PRNT0445
  695 FORMAT('0','POINT NO.',2X,'SURFACE',2X,'EPS-X STRAIN',2X,'EPS-Y STPRNT0446
     2RAIN',2X,'SHEAR STRAIN',1X,'ELONG.(DIR.1)',1X,'ELONG.(DIR.2)',1X, PRNT0447
     3'PRINC. STRN(T)',1X,'DIRECTION(DEG.)')                            PRNT0448
```

```
700  FORMAT(' ',I7,5X,'INNER',1X,7D14.5)                                  PRNT0449
710  FORMAT(' ',I7,5X,'OUTER',1X,7D14.5)                                  PRNT0450
720  FORMAT('0','SYSTEM ENERGIES(IN-LB)')                                 PRNT0451
730  FORMAT(' ',5X,'WORK INPUT TO STRUCTURE            =',D15.7)          PRNT0452
740  FORMAT(' ',5X,'STRUCTURE KINETIC ENERGY           =',D15 7)          PRNT0453
750  FORMAT(' ',5X,'STRUCTURE ELASTIC ENERGY           =',D15.7)          PRNT0454
760  FORMAT(' ',5X,'STRUCTURE PLASTIC ENERGY           =',D15.7)          PRNT0455
770  FORMAT(' ',5X,'ENERGY STORED IN ELASTIC RESTRAINTS=',D15.7)          PRNT0456
780  FORMAT('0','STRAIN, EPS-XX, AT GAUSSIAN STATIONS ON SPECIFIED X-DIPRNT0457
    2RECTION STIFFENERS')                                                 PRNT0458
785  FORMAT('0','STIFFENER',14X,'STATION 1',15X,'STATION 2',15X,          PRNT0459
    2'STATION 3')                                                         PRNT0460
790  FORMAT(' ',' NUMBER',5X,3(7X,'INNER',7X ,'OUTER'))                   PRNT0461
800  FORMAT(' ',I5,7X,6D12.4)                                             PRNT0462
810  FORMAT('0','STRAIN, EPS-YY, AT GAUSSIAN STATIONS ON SPECIFIED Y-DIPRNT0463
    2RECTION STIFFENERS')                                                 PRNT0464
900  FORMAT('0','STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIREPRNT0465
    2CTION AT USER-SPECIFIED NODES')                                      PRNT0466
920  FORMAT(' ','NODE ',5(7X,'INNER',7X,'OUTER'))                         PRNT0467
     RETURN                                                               PRNT0468
     END                                                                  PRNT0469
```

## 6.3 Subprograms Unique to the CIVM-PLATE Code

The following 9 subprograms (or subroutines) are unique to the CIVM-PLATE program:

| | | |
|---|---|---|
| COLLSN | IMAINP | IMPSS |
| IFRAG | IMPACT | INTRAC |
| IMAIN | IMPDS | IPRINT |

This version of IMAIN is provided principally for illustration, but also is appropriate for the illustrative example given in Subsection 8.2.1 for the complete plate. The user must write his own version consistent with his particular problem data if he wishes to analyze any other example problem.

A FORTRAN IV listing of these 9 subprograms follows.

```
      SUBROUTINE COLLSN(IOPT,XG,YG,ZG,PEN,RF,THALF,NNSR,XF,YF,ZF,XN,YN,
     2ZN,T,NN1,NN2,NN3)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ASRL TR 154-14.... ..ORIGINAL REPORT VERSION OF PROGRAM
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.
C
      DIMENSION XG(1),YG(1),ZG(1),PEN(1),XN(1),YN(1),ZN(1),T(3,3)
C
C    THIS ROUTINE PERFORMS DETAILED COLLISION INSPECTION OVER A
C    TRIANGULAR SUBREGION OF THE PLATE, CALCULATING THE PENETRATION
C    DISTANCE, PEN(I), AND LOCATION, XN(I), YN(I), ZN(I), OF THE
C    CONTACT POINT, ON THE SURFACE, IF COLLISION OCCURS WITHIN THE
C    TRIANGULAR SUBREGION
C    IF IOPT=1, THE ENTIRE CHECK IS FOLLOWED.
C    IF IOPT=2, THE PROCESS IS TERMINATED AFTER CALCULATION OF THE
C               COORDINATE TRANSFORMATION MATRIX FOR THE TRIANGULAR
C               SUBREGION.
C
C    FOR PRESENT, UNIFORM THICKNESS IS ASSUMED IN THE TRIANGULAR REGION.
C
C DEFINE COMPONENTS OF A,B,C OF VECTOR IN Z-BAR DIRECTION
      X21=XG(NN2)-XG(NN1)
      Y21=YG(NN2)-YG(NN1)
      Z21=ZG(NN2)-ZG(NN1)
      X31=XG(NN3)-XG(NN1)
      Y31=YG(NN3)-YG(NN1)
      Z31=ZG(NN3)-ZG(NN1)
      A=Y21*Z31-Y31*Z21
      B=X31*Z21-X21*Z31
      C=X21*Y31-X31*Y21
      AL3=DSQRT(A*A+B*B+C*C)
C    CALCULATE DISTANCE BETWEEN FRAGMENT CENTROID AND PLATE MIDSURFACE.
C    IF DFP IS NEGATIVE FRAGMENT CENTROID IS BELOW MIDSURFACE.
      DFP=(A*(XF-XG(NN1))+B*(YF-YG(NN1))+C*(ZF-ZG(NN1)))/AL3
C    CALC. PENETRATION DISTANCE (ASSUMING UNIFORM THICKNESS AND
C    PENETRATION IN DIRECTION OF NORMAL FROM FRAGMENT CENTROID TO
C    REFERENCE SURFACE)
      PEN(NNSR)=RF+THALF+DFP
C    IF PENETRATION DIST. IS LESS THAN OR EQUAL TO ZERO, RETURN TO
C    CALLING ROUTINE
      IF(PEN(NNSR).LE.0.0)RETURN
C    FIND COORDS  OF INTERSECTION OF NORMAL (FROM FRAG TO SURFACE) WITH
C    THE SURFACE.
      XN(NNSR)=XF-A*DFP/AL3
      YN(NNSR)=YF-B*DFP/AL3
      ZN(NNSR)=ZF-C*DFP/AL3
C    ESTABLISH TRANSFORMATION MATRIX ; (BAR)=T*(GLOBAL).
C    Z-BAR IS NORMAL TO THE PLANE AND DIRECTED TOWARD THE OUTER SURFACE
C    OF THE PLATE  X-BAR IS IN THE DIRECTION FROM TRIANGLE NODE 1 TO
C    TRIANGLE NODE 2
      AL1=DSQRT(X21*X21+Y21*Y21+Z21*Z21)
      T(1,1)=X21/AL1
      T(1,2)=Y21/AL1
      T(1,3)=Z21/AL1
      T(2,1)=(B*Z21-C*Y21)/(AL1*AL3)
      T(2,2)=(C*X21-A*Z21)/(AL1*AL3)
```

```
      T(2,3)=(A*Y21-B*X21)/(AL1*AL3)                                    CLSN0059
      T(3,1)=A/AL3                                                      CLSN0060
      T(3,2)=B/AL3                                                      CLSN0061
      T(3,3)=C/AL3                                                      CLSN0062
C   IF THIS ROUTINE HAS BEEN CALLED ONLY TO CALCULATE T, RETURN         CLSN0063
      IF(IOPT EQ.2)RETURN                                               CLSN0064
C   FIND X-Y COORDS  OF NODES AND INTERSECTION POINT IN BARRED SYSTEM   CLSN0065
      XB1=XG(NN1)*T(1,1)+YG(NN1)*T(1,2)+ZG(NN1)*T(1,3)                  CLSN0066
      YB1=XG(NN1)*T(2,1)+YG(NN1)*T(2,2)+ZG(NN1)*T(2,3)                  CLSN0067
      XB2=XG(NN2)*T(1,1)+YG(NN2)*T(1,2)+ZG(NN2)*T(1,3)                  CLSN0068
      YB2=XG(NN2)*T(2,1)+YG(NN2)*T(2,2)+ZG(NN2)*T(2,3)                  CLSN0069
      XB3=XG(NN3)*T(1,1)+YG(NN3)*T(1,2)+ZG(NN3)*T(1,3)                  CLSN0070
      YB3=XG(NN3)*T(2,1)+YG(NN3)*T(2,2)+ZG(NN3)*T(2,3)                  CLSN0071
      XBN=XN(NNSR)*T(1,1)+YN(NNSR)*T(1,2)+ZN(NNSR)*T(1,3)               CLSN0072
      YBN=XN(NNSR)*T(2,1)+YN(NNSR)*T(2,2)+ZN(NNSR)*T(2,3)               CLSN0073
C   DEFINE TRIANGULAR (AREA) COORDINATE PARAMETERS FOR THIS SUBREGION.  CLSN0074
      AREA2=XB2*YB3+XB1*YB2+XB3*YB1-XB2*YB1-XB3*YB2-XB1*YB3             CLSN0075
      A1=XB2*YB3-XB3*YB2                                                CLSN0076
      A2=XB3*YB1-XB1*YB3                                                CLSN0077
      A3=XB1*YB2-XB2*YB1                                                CLSN0078
      B1=YB2-YB3                                                        CLSN0079
      B2=YB3-YB1                                                        CLSN0080
      B3=YB1-YB2                                                        CLSN0081
      C1=XB3-XB2                                                        CLSN0082
      C2=XB1-XB3                                                        CLSN0083
      C3=XB2-XB1                                                        CLSN0084
C   EVALUATE TRIANGULAR COORDS (L1, L2, L3) AT FRAGMENT/NORMAL          CLSN0085
C   INTERSECTION POINT (POINT OF CONTACT)                               CLSN0086
      CL1=(A1+B1*XBN+C1*YBN)/AREA2                                      CLSN0087
      CL2=(A2+B2*XBN+C2*YBN)/AREA2                                      CLSN0088
      CL3=(A3+B3*XBN+C3*YBN)/AREA2                                      CLSN0089
C   FOR CONTACT POINT TO LIE IN TRIANGULAR SUBREGION, CL1, CL2, AND CL3 CLSN0090
C   MUST ALL BE BETWEEN 0 AND 1 (INCLUSIVE).  IF NOT, SET PENETRATION   CLSN0091
C   DISTANCE TO A LARGE NEGATIVE NUMBER.                                CLSN0092
C USE A NEGATIVE EPSILON AND 1  - EPSILON INSTEAD OF AN EXACT 0. AND 1. CLSN0093
C TO AVOID INACCURACIES WITH COMPUTER ROUNDOFF.                         CLSN0094
      ZIP=-1.D-10                                                       CLSN0095
      ONE-1 D0-ZIP                                                      CLSN0096
      IF(CL1.LT.ZIP.OR CL2.LT.ZIP.OR.CL3 LT.ZIP.OR.CL1.GT.ONE.OR.CL2.   CLSN0097
     2GT.ONE.OR.CL3 GT ONE)PEN(NNSR)=-1.0D10                            CLSN0098
      RETURN                                                            CLSN0099
      END                                                               CLSN0100
      SUBROUTINE IFRAG(PMASS,AMASS)                                     IFRG0000
      IMPLICIT REAL*8(A-H,O-Z)                                          IFRG0001
C                                                                       IFRG0002
C    ASRL TR 154-14...  ..ORIGINAL REPORT VERSION OF PROGRAM            IFRG0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            IFRG0004
C                                                                       IFRG0005
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.        IFRG0006
C                                                                       IFRG0007
      DIMENSION PMASS(1),AMASS(1)                                       IFRG0008
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RF,FMASS,   IFRG0009
     2FMOI,FKEO,FKEC                                                    IFRG0010
      COMMON /PROPC/ COEFR,FRNC,USEF,NSYM,IPSS,IUSEF                    IFRG0011
      COMMON/PLATE/XO,YO,THP,HNL,HML,NEAD,NECD,DENSP,EPAN,ANUP,THALF     IFRG0012
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC                               IFRG0013
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH                                IFRG0014
C                                                                       IFRG0015
C IN THIS SUBROUTINE THE INITIAL FRAGMENT POSITION AND VELOCITY,        IFRG0016
```

```
C FRAGMENT GEOMETRIC PROPERTIES, CONTACT SURFACE PROPERTIES               IFRG0017
C AND ANY SUMMETRY OPTIONS ARE INPUT AND PRINTED FOR VERIFICATION.        IFRG0018
C THE FRAGMENT INITIAL KINETIC ENERGY IS CALCULATED AND THE MASS MATRIX  IFRG0019
C USED IN IMPACT INTERACTION CALCULATION IS FORMED.                       IFRG0020
C                                                                         IFRG0021
      NNT=NDT/6                                                           IFRG0022
C   READ IN FRAGMENT INITIAL LOCATION, VELOCITIES, AND PROPERTIES         IFRG0023
      READ(MREAD,500)XF,YF,ZF                                             IFRG0024
      READ(MREAD,500)(VF(I),I=1,3)                                        IFRG0025
      READ(MREAD,500)(OMEGF(I),I=1,3)                                     IFRG0026
      READ(MREAD,500)RF,FMASS,FMOI                                        IFRG0027
C   READ IN CONTACT SURFACE PROPERTIES                                    IFRG0028
      READ(MREAD,500)COEFR,FRNC                                           IFRG0029
      READ(MREAD,510)NSYM,IPSS,ICUT,IUSEF                                 IFRG0030
      IF(NSYM EQ 2)FMASS=FMASS/4 0D0                                      IFRG0031
      IF(NSYM.EQ.1) FMASS=FMASS/2.0D0                                     IFRG0032
      IF(NSYM EQ.1) FMOI=FMOI/2.0D0                                       IFRG0033
      CUTR=0.0D0                                                          IFRG0034
      IF(ICUT EQ.1)READ(MREAD,500)CUTR                                    IFRG0035
      IF(IUSEF.EQ.1) READ(MREAD,500) USEF                                 IFRG0036
C   FRAGMENT INITIAL KINETIC ENERGY                                       IFRG0037
      FKEO=0.0                                                            IFRG0038
      DO 15 I=1,3                                                         IFRG0039
 15   FKEO=FKEO+0 5*(FMASS*VF(I)*VF(I)+FMOI*OMEGF(I)*OMEGF(I))            IFRG0040
      FKEC=FKEO*CUTR                                                      IFRG0041
C   PRINT OUT THIS INFORMATION                                            IFRG0042
      WRITE(MWRITE,600)                                                   IFRG0043
      WRITE(MWRITE,601) XF                                                IFRG0044
      WRITE(MWRITE,602) YF                                                IFRG0045
      WRITE(MWRITE,603) ZF                                                IFRG0046
      WRITE(MWRITE,604) VF(1)                                             IFRG0047
      WRITE(MWRITE,605) VF(2)                                             IFRG0048
      WRITE(MWRITE,606) VF(3)                                             IFRG0049
      WRITE(MWRITE,607) OMEGF(1)                                          IFRG0050
      WRITE(MWRITE,608) OMEGF(2)                                          IFRG0051
      WRITE(MWRITE,609) OMEGF(3)                                          IFRG0052
      WRITE(MWRITE,610) RF                                                IFRG0053
      WRITE(MWRITE,611) FMASS                                             IFRG0054
      WRITE(MWRITE,612) FMOI                                              IFRG0055
      WRITE(MWRITE,630) FKEO                                              IFRG0056
      WRITE(MWRITE,613)                                                   IFRG0057
      WRITE(MWRITE,614) COEFR                                             IFRG0058
      WRITE(MWRITE,615) FRNC                                              IFRG0059
      WRITE(MWRITE,616) NSYM                                              IFRG0060
      IF(NSYM EQ.1 AND IPSS EQ 1) WRITE(MWRITE,617)                       IFRG0061
      IF(NSYM EQ.1 AND.IPSS EQ.4) WRITE(MWRITE,618)                       IFRG0062
      IF(NSYM.EQ 1) WRITE(MWRITE,619)                                     IFRG0063
      IF(NSYM EQ 2) WRITE(MWRITE,620)                                     IFRG0064
      IF(ICUT EQ 0)GO TO 18                                               IFRG0065
      WRITE(MWRITE,640)                                                   IFRG0066
      WRITE(MWRITE,650) CUTR                                              IFRG0067
      WRITE(MWRITE,660) FKEC                                              IFRG0068
      WRITE(MWRITE,670)                                                   IFRG0069
 18   CONTINUE                                                            IFRG0070
      IF(IUSEF EQ.0) WRITE(MWRITE,700)                                    IFRG0071
      IF(IUSEF.EQ 1) WRITE(MWRITE,710) USEF                               IFRG0072
 500  FORMAT(5D16 7)                                                      IFRG0073
 510  FORMAT(20I4)                                                        IFRG0074
 600  FORMAT('0','FRAGMENT LOCATION, VELOCITY, AND PROPERTIES AT TIME=0.IFRG0075
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     20 SEC.')                                                           IFRG0076
 601  FORMAT(' ',5X,'X-LOCATION(IN)                 =',D15.7)            IFRG0077
 602  FORMAT(' ',5X,'Y-LOCATION(IN)                 =',D15.7)            IFRG0078
 603  FORMAT(' ',5X,'Z-LOCATION(IN)                 =',D15.7)            IFRG0079
 604  FORMAT(' ',5X,'X-VELOCITY(IN/SEC)             =',D15.7)            IFRG0080
 605  FORMAT(' ',5X,'Y-VELOCITY(IN/SEC)             =',D15.7)            IFRG0081
 606  FORMAT(' ',5X,'Z-VELOCITY(IN/SEC)             =',D15.7)            IFRG0082
 607  FORMAT(' ',5X,'OMEGA-X(RAD/SEC)               =',D15.7)            IFRG0083
 608  FORMAT(' ',5X,'OMEGA-Y(RAD/SEC)               =',D15.7)            IFRG0084
 609  FORMAT(' ',5X,'OMEGA-Z(RAD/SEC)               =',D15.7)            IFRG0085
 610  FORMAT(' ',5X,'FRAGMENT RADIUS(IN)            =',D15.7)            IFRG0086
 611  FORMAT(' ',5X,'FRAGMENT MASS(LB-SEC*SEC/IN)   =',D15.7)            IFRG0087
 612  FORMAT(' ',5X,'FRAG. MOM INERT(IN-LB-SEC*SEC) =',D15.7)            IFRG0088
 613  FORMAT('0','CONTACT SURFACE PROPERTIES')                           IFRG0089
 614  FORMAT(' ',5X,'COEF. OF RESTITUTION=',D15.7)                       IFRG0090
 615  FORMAT(' ',5X,'FRICTION COEFFICIENT=',D15.7)                       IFRG0091
 616  FORMAT(' ',5X,'NO. OF SYMMETRIES   =',I4)                          IFRG0092
 617  FORMAT(' ',5X,'THE GLOBAL X AXIS IS THE LINE OF SYMMETRY')         IFRG0093
 618  FORMAT(' ',5X,'THE GLOBAL Y AXIS IS THE LINE OF SYMMETRY')         IFRG0094
 619  FORMAT(' ',5X,'(FRAGMENT MASS, MASS MOMENT OF INERTIA AND KINETIC IFRG0095
     1ENERGY ARE ONE-HALF THEIR ACTUAL VALUES BECAUSE OF SINGLE SYMMETRYIFRG0096
     1)')                                                                IFRG0097
 620  FORMAT(' ',5X,'(FRAGMENT MASS AND KINETIC ENERGY ARE ONE-QUARTER OIFRG0098
     1F THEIR ACTUAL VALUES BECAUSE OF DOUBLE SYMMETRY)')                IFRG0099
 630  FORMAT(' ',5X,'KINETIC ENERGY(IN-LB)          =',D15.7)            IFRG0100
 640  FORMAT('0','IMPACT INSPECTION/CORRECTION WILL BE TERMINATED WHEN')IFRG0101
 650  FORMAT(' ',5X,'THE CURRENT FRAGMENT KINETIC ENERGY IS LESS THAN', IFRG0102
     2D16.7,2X,'TIMES THE INITIAL FRAGMENT KINETIC ENERGY.')             IFRG0103
 660  FORMAT(' ',5X,'THIS CUT-OFF POINT CORRESPONDS TO A FRAGMENT KINETIIFRG0104
     2C ENERGY OF',D16.7,2X,'IN-LB.')                                    IFRG0105
 670  FORMAT(' ',5X,'NOTE THAT STRUCTURAL RESPONSE CALCULATIONS WILL CONIFRG0106
     2TINUE BEYOND TERMINATION OF IMPACT INSPECTION/CORRECTION.')        IFRG0107
 700  FORMAT('0','EFFECTIVE LENGTH WILL BE CALCULATED INTERNALLY')       IFRG0108
 710  FORMAT('0','USER SPECIFIED EFFECTIVE LENGTH=',D15.7)               IFRG0109
C    CALCULATE NODAL POINT MASSES FOR IMPACT/INTERACTION USE.  THIS      IFRG0110
C    EXTRACTED FROM SYSTEM LUMPED MASS VECTOR BY NEGLECTING ROTARY       IFRG0111
C    INERTIA CONTRIBUTIONS.  EFFECTS OF POSSIBLE NONUNIFORM MESH         IFRG0112
C    AND STIFFENERS INCLUDED AUTOMATICALLY.  FOR PLATE ANALYSIS ONLY     IFRG0113
      NDPN=6                                                             IFRG0114
      DO 20 I=1,NNT                                                      IFRG0115
 20   PMASS(I)=0.0                                                       IFRG0116
      DO 30 I=1,NNT                                                      IFRG0117
      INC=(I-1)*NDPN+1                                                   IFRG0118
 30   PMASS(I)=AMASS(INC)                                                IFRG0119
      RETURN                                                             IFRG0120
      END                                                                IFRG0121
```

```
      SUBROUTINE IMPACT(XG,YG,ZG,DELD,VEL ,NP,VN,VNB,PMASS,NBC,SI,NEFF,
     2ALPHA,IMPCT1,NIAN)
      IMPLICIT REAL*8(A-H,O-Z)
C
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C
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C
      DIMENSION XG(1),YG(1),ZG(1),DELD(1),VEL(1) ,NP(4,1),LNC(4),PEN(8),
     2XN(8),YN(8),ZN(8),T(3,3),SI(1),NEFF(1),ALPHA(1),VN(3,1),VNB(3,1),
     3OMEGFB(3),VFB(3),PMASS(1),PTILDA(3),NBC(1),LRP(8)
      COMMON/PLATE/XD,YD,THP,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /PROPC/ COEFR,FRNC,USEF,NSYM,IPSS,IUSEF
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RF,FMASS,
     2FMOI,FKEO,FKEC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C    CONTROLLING ROUTINE FOR COLLISION INSPECTION/IMPACT INTERACTION
C    ANALYSIS INCLUDING FRICTION.  IF IMPACT HAS OCCURED, THE NODAL
C    VELOCITIES, VEL, AND FRAGMENT VELOCITIES, VF AND OMEGF, ARE UPDATED
C    TO THEIR POST-IMPACT VALUE. NODAL AND FRAGMENT DISPLACEMENTS AND
C    LOCATIONS ARE NOT MODIFIED IN THIS SUBROUTINE.
C    PRESENT ANALYSIS ASSUMES UNIFORM THICKNESS PLATE, AND SINGLE
C    FRAGMENT.  EFFECTS OF MAXIMUM PENETRATION ONLY ARE INCLUDED.
C
C    UPDATE GLOBAL COORDINATES TO TRIAL VALUES
      IMPCT1=0
      NDPN=6
      NNT=NDT/6
      DO 1 I=1,NNT
      NNI=(I-1)*NDPN
      XG(I)= XG(I)+DELD(NNI+1)
      YG(I)= YG(I)+DELD(NNI+2)
    1 ZG(I)= ZG(I)+DELD(NNI+3)
C    UPDATE FRAGMENT POSITION ASSUMING SAME INCREMENTAL QUANT. AS PREV.
      XF=XF+VF(1)*DELTAT
      YF=YF+VF(2)*DELTAT
      ZF=ZF+VF(3)*DELTAT
      DO 7 I=1,3
      DTHETF(I)=DELTAT*OMEGF(I)
    7 DELF(I)=DELTAT*VF(I)
C    FIND MINIMUM DISTANCE SQUARED FROM FRAGMENT TO NODE
    9 DMIN=1.0D10
      DO 10 I=1,NNT
      DIST=(XF-XG(I))**2+(YF-YG(I))**2+(ZF-ZG(I))**2
      IF(DIST.LT.DMIN)NC=I
      IF(DIST.LT.DMIN)DMIN=DIST
   10 CONTINUE
C    SEARCH NODAL CONNECTIONS FOR ELEMENTS CONNECTED TO NODE NC
      NEC=0
      DO 20 I=1,NET
      DO 20 J=1,4
      IF(NP(J,I).EQ.NC)NEC=NEC+1
      IF(NP(J,I).EQ.NC)LNC(NEC)=I
      IF(NEC.EQ.4)GO TO 30
   20 CONTINUE
   30 CONTINUE
C    CHECK FOR PENETRATION IN EACH TRIANGULAR SUBREGION OF THE NEC ELEMS.
      NRP=0
```

```
      AA=0.0
      BA=0 0
      CA=0.0
      XA=0.0
      YA=0.0
      ZA=0.0
      DO 50 I=1,NEC
      LNUM=LNC(I)
      DO 50 NSR=1,2
      NNSR=(I-1)*2+NSR
      NN1=NP(1,LNUM)
      NN2=NP(NSR+1,LNUM)
      NN3=NP(NSR+2,LNUM)
   49 CALL COLLSN(1,XG,YG,ZG,PEN,RF,THALF,NNSR,XF,YF,ZF,XN,YN,ZN,T,
     2NN1,NN2,NN3)
      IF(PEN(NNSR) LE.0.0) GO TO 50
      NRP=NRP+1
      LRP(NRP)=NNSR
      LSP=NNSR
      PENM=PEN(NNSR)
      LNMP=LNUM
      AA=AA+T(3,1)
      BA=BA+T(3,2)
      CA=CA+T(3,3)
      XA=XA+XN(NNSR)
      YA=YA+YN(NNSR)
      ZA=ZA+ZN(NNSR)
   50 CONTINUE
C   SKIP OUT IF NO PENETRATION
      IF(NRP EQ 0) GO TO 300
C   CALC  RADIUS OF REGION OF INFLUENCE, EFLN
C   FOR PRESENT ASSUME DELTAT-STAR=DELTAT
      EFLN=DSQRT(EPAN/DENSP/(1.DO-ANUP**2))*DELTAT
      IF(IUSEF.EQ.1) EFLN=USEF
C   CALCULATION OF TRANS. AND EFFECTIVE POINT OF IMPACT WHEN MORE THAN
C   ONE OVERLAP IS FOUND.
      IF(NRP EQ 1) GO TO 65
      AL3=DSQRT(AA*AA+BA*BA+CA*CA)
      DA=(AA+BA)/CA
      AL1=DSQRT(2.0D0+DA*DA)
      AL2=AL1*AL3
      T(1,1)=1 0/AL1
      T(1,2)=1 0/AL1
      T(1,3)=-DA/AL1
      T(2,1)=-(BA*DA+CA)/AL2
      T(2,2)=(CA+AA*DA)/AL2
      T(2,3)=(AA-BA)/AL2
      T(3,1)=AA/AL3
      T(3,2)=BA/AL3
      T(3,3)=CA/AL3
      LSR=LRP(1)
      XN(LSP)=XA/DFLOAT(NRP)
      YN(LSP)=YA/DFLOAT(NRP)
      ZN(LSP)=ZA/DFLOAT(NRP)
   65 CONTINUE
C   DETERMINE WHICH NODES FALL IN THIS CIRCULAR REGION
      NK=0
      DO 70 I=1,NNT
      DIST=DSQRT((XN(LSP)-XG(I))**2+(YN(LSP)-YG(I))**2+(ZN(LSP)-ZG(I))
     2**2)
      IF(DIST.GT.EFLN)GO TO 70
      NK=NK+1
      SI(NK)=DIST
      NEFF(NK)=I
      ALPHA(NK)=1.0D0-DIST/EFLN
   70 CONTINUE
```

```
C IF USER ESTIMATE OF NK IS TOO SMALL, TERMINATE RUN.
      IF(NIAN .GE. NK) GO TO 73
      WRITE(MWRITE,640) ITIME
      WRITE(MWRITE,650) NIAN
      WRITE(MWRITE,660) NK
      WRITE(MWRITE,670)
      CALL EXIT
   73 CONTINUE
C    TEMPORARY UPDATE OF EFLN IF NK=0.
      IF(NK GT 0) GO TO 74
      NN3=NP(LNMP,3)
      NN1=NP(LNMP,1)
      EFLN=DSQRT((XG(NN3)-XG(NN1))**2+(YG(NN3)-YG(NN1))**2
     2+(ZG(NN3)-ZG(NN1))**2)
      WRITE(MWRITE,730) ITIME
      WRITE(MWRITE,740) EFLN
      GO TO 65
 74   CONTINUE
C    CALC  CONSTANT, CONST, ASSUMING TOTAL IMPULSE APPLIED TO NODES IS
C    EQUAL TO TOTAL IMPARTED IMPULSE.
C    FOR PRESENT, ASSUME IMPULSE IS ZERO AT EFLN.
      CONST=0.0
      DO 80 I=1,NK
 80   CONST=CONST+ALPHA(I)
      CONST=1 0/CONST
C    CALC  ALPHA(I)=CONST*ALPHA(I)
      DO 90 I=1,NK
 90   ALPHA(I)=ALPHA(I)*CONST
C    ESTABLISH TRANSFORMATION MATRIX, T,CORRESPONDING TO TRIANGULAR
C    SUBREGION WHERE IMPACT HAS OCCURED.
C    NOTE THAT TRANS  IS IN THE FORM  XYZ-BAR=T*XYZ
C    IF MORE THAN ONE OVERLAP, SKIP (USE T ABOVE).
      IF(NRP GT 1) GO TO 95
      NN=(LSP+1)/2
      LNMP=LNC(NN)
      NSR=LSP-2*(NN-1)
      NN1=NP(1,LNMP)
      NN2=NP(NSR+1,LNMP)
      NN3=NP(NSR+2,LNMP)
      CALL COLLSN(2,XG,YG,ZG,PEN,RF,THALF,LSP,XF,YF,ZF,XN,YN,ZN,T,
     2NN1,NN2,NN3)
 95   CONTINUE
C    EXTRACT NODAL VELOCITIES IN IMPACT AFFECTED REGION.
      DO 100 I=1,NK
      NNI=(NEFF(I)-1)*NDPN
      DO 100 J=1,3
      K=NNI+J
 100  VN(J,I)=VEL(K)
C    TRANSFORM NODAL VELOCITIES INTO BAR SYSTEM
      DO 110 I=1,NK
      DO 110 J=1,3
      VNB(J,I)=0.0
      DO 110 K=1,3
 110  VNB(J,I)=VNB(J,I)+T(J,K)*VN(K,I)
C    CALC. ASSUMED PRE-IMPACT VELOCITIES OF FRAGMENT
C    TRANSFORM THESE QUANTITIES INTO BAR SYSTEM
      DO 130 I=1,3
      OMEGFB(I)=0 0
      VFB(I)=0 0
      DO 130 K=1,3
      OMEGFB(I)=OMEGFB(I)+T(I,K)*OMEGF(K)
 130  VFB(I)=VFB(I)+T(I,K)*VF(K)
C    CALC. IMPULSE INTERACTION CONSTANTS, S1,S2,A0,B1,B3
      S1=VFB(1)+RF*OMEGFB(2)
      S2=VFB(2)-RF*OMEGFB(1)
      A0=VFB(3)
```

```
      B1=1.0/FMASS+RF*RF/FMOI
      B3=1.0/FMASS
      DO 140 I=1,NK
      S1=S1-ALPHA(I)*VNB(1,I)
      S2=S2-ALPHA(I)*VNB(2,I)
      A0=A0-ALPHA(I)*VNB(3,I)
      AM=PMASS(NEFF(I))
      B1=B1+ALPHA(I)*ALPHA(I)/AM
  140 B3=B3+ALPHA(I)*ALPHA(I)/AM
C   IF A0 LE  0 0, NO COLLISION CAN OCCUR
      IF(A0 LE 0 0D0)GO TO 300
C   SET IMPACT INDEX TO INDICATE IMPACT HAS OCCURED.
      IMPCT1=1
C   CALC  POST-IMPACT MOMENTUM COORDINATES IN BAR SYSTEM
      CALL INTRAC(S1,S2,A0,B1,B3,COEFR,FRNC,PTILDA)
C   CALC  POST-IMPACT VELOCITY DIFFERENCES OF NODES IN BAR SYSTEM
C   (I E.  V-PRIME MINUS V)
      DO 180 I=1,NK
      K=NEFF(I)
      AM=PMASS(K)
      DO 180 J=1,3
  180 VNB(J,I)=ALPHA(I)*PTILDA(J)/AM
C   CALC. POST-IMPACT VELOCITY DIFFERENCES OF FRAGMENT IN BAR SYSTEM
      DO 190 I=1,3
  190 VFB(I)=-PTILDA(I)/FMASS
      OMEGFB(1)=PTILDA(2)*RF/FMOI
      OMEGFB(2)=-PTILDA(1)*RF/FMOI
      OMEGFB(3)=0 D0
C   TRANSFORM POST-IMPACT NODAL VELOCITY DIFFERENCES BACK TO XYZ SYSTEM
      DO 200 I=1,NK
      DO 200 J=1,3
      VN(J,I)=0 0
      DO 200 K=1,3
  200 VN(J,I)=VN(J,I)+T(K,J)*VNB(K,I)
C   TRANS. POST-IMPACT VELOCITY DIFFERENCES OF FRAGMENT TO GLOBAL
C   SYSTEM AND ADD TO PRE-IMPACT VELOCITIES TO GET POST-IMPACT FRAG.
C   VELOCITIES
      DO 210 I=1,3
      DO 210 K=1,3
      OMEGF(I)=OMEGF(I)+T(K,I)*OMEGFB(K)
  210 VF(I)=VF(I)+T(K,I)*VFB(K)
C   UPDATE NODAL VELOCITIES TO POST-IMPACT VALUES.
      DO 230 I=1,NK
      NNI=(NEFF(I)-1)*NDPN
      DO 230 J=1,3
      K=NNI+J
  230 VEL(K)=VEL(K)+VN(J,I)
C   CONSTRAIN VEL
      CALL RCON(VEL ,NBC)
      IF(NRP EQ.1) GO TO 250
      WRITE(MWRITE,700) ITIME
      WRITE(MWRITE,710) (LRP(I),I=1,NRP)
      WRITE(MWRITE,720) XN(LSP),YN(LSP),ZN(LSP)
      GO TO 300
  250 WRITE(MWRITE,600)ITIME,LNMP,NSR
      WRITE(MWRITE,610)XN(LSP),YN(LSP),ZN(LSP)
      WRITE(MWRITE,620)PEN(LSP)
  300 CONTINUE
C   REDUCE FRAGMENT POSITION TO PRE-IMPACT VALUE
      XF=XF-DELF(1)
      YF=YF-DELF(2)
      ZF=ZF-DELF(3)
C   REDUCE NODAL COORDS. TO PRE-IMPACT VALUES
      DO 220 I=1,NNT
      NNI=(I-1)*NDPN
      XG(I)=XG(I)-DELD(NNI+1)
```

```
      YG(I)=YG(I)-DELD(NNI+2)
  220 ZG(I)=ZG(I)-DELD(NNI+3)
  600 FORMAT('0',10X,'IMPACT CYCLE=',I10,2X,'ELEMENT=',I5,2X,'SUBREGION
     2=',I3)
  610 FORMAT(' ',10X,'GLOBAL IMPACT LOCATION (XN,YN,ZN)=',3D15.7)
  620 FORMAT(' ',10X,'PENETRATION DISTANCE(IN.)=',D15.7)
  640 FORMAT('0','***** USER ESTIMATE OF NUMBER OF IMPACT-AFFECTED NODES
     1 TOO SMALL AT CYCLE NUMBER',I7)
  650 FORMAT(' ',5X,'USER ESTIMATE (PARAMETER NIAN IN DUMMY MAIN PROGRAM
     1)=',I5)
  660 FORMAT(' ',5X,'ACTUAL NUMBER OF IMPACT AFFECTED NODES=',I7)
  670 FORMAT(' ',5X,'RUN HAS BEEN TERMINATED')
  700 FORMAT('0',10X,'IMPACT CYCLE=',I10)
  710 FORMAT('0',10X,'SUBREGIONS PENETRATED:',8I4)
  720 FORMAT('0',10X,'POINT OF EFFECTIVE IMPACT(XA,YA,ZA)=',3D15.7)
  730 FORMAT('0','** TEMPORARY UPDATE OF EFFECTIVE LENGTH AT CYCLE'
     2,I5)
  740 FORMAT(' ','EFFECTIVE LENGTH=',D15.7)
      RETURN
      END
```

```
      SUBROUTINE IMPDS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,NEFF,
     1ALPHA,IMPCT1,NIAN)
      IMPLICIT REAL*8(A-H,O-Z)
C
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C
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C
      DIMENSION XG(1),YG(1),ZG(1),DELD(1),VEL(1) ,NP(4,1),LNC(4),PEN(8),
     2XN(8),YN(8),ZN(8),T(3,3),SI(1),NEFF(1),ALPHA(1),VN(3,1),VNB(3,1),
     3OMEGFB(3),VFB(3),PMASS(1),PTILDA(3),NBC(1)
      COMMON/PLATE/XD,YD,THP,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /PROPC/ COEFR,FRNC,USEF,NSYM,IPSS,IUSEF
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RF,FMASS,
     2FMOI,FKEO,FKEC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C  CONTROLLING ROUTINE FOR COLLISION/IMPACT INTERACTIONS
C  ANALYSIS INCLUDING FRICTION, ASSUMING DOUBLE-SYMMETRY
C  OPTION IS EXERCISED.
C  IMPACT CAN OCCUR ONLY WITH ELEMENT NUMBER ONE (IMPACT IS
C  ASSUMED TO BE CENTERED AT GLOBAL NODE NUMBER 1).
C
C  UPDATE NODAL GLOBAL COORDINATES TO TRIAL VALUES.
C
      IMPCT1=0
      NDPN=6
      NNT=NDT/6
      DO 1 I=1,NNT
      NNI=(I-1)*NDPN
      XG(I)=XG(I)+DELD(NNI+1)
      YG(I)=YG(I)+DELD(NNI+2)
    1 ZG(I)=ZG(I)+DELD(NNI+3)
C
C  UPDATE FRAGEMENT POSITIONS IN Z-DIRECTION.
C
      DELF(3)=VF(3)*DELTAT
      ZF=ZF+DELF(3)
C
C  CHECK FOR PENETRATION IN EACH TRIANGULAR SUBREGION OF ELEM. #1.
C
      DO 50 NSR=1,2
      NNSR=NSR
      NN1=NP(1,1)
      NN2=NP(NSR+1,1)
      NN3=NP(NSR+2,1)
   50 CALL COLLSN(1,XG,YG,ZG,PEN,RF,THALF,NNSR,XF,YF,ZF,XN,YN,
     1ZN,T,NN1,NN2,NN3)
C
C     SEARCH FOR MAXIMUM PENETRATION
C
      PENM=0.0E0
      IF (PEN(1).GT.0 0) LSP=1
      IF (PEN(1) GT.0.0) PENM=PEN(1)
      IF (PEN(2).GT.PENM) LSP=2
      IF (PEN(2).GT.PENM) PENM=PEN(2)
C
C  SKIP OUT IF NO PENETRATION.
C
      IF(PENM.EQ.0.0D0) GO TO 300
```

```
C
C  CALCULATE RADIUS OF REGION OF INFLUENCE, EFLN.
C
      EFLN=DSQRT(EPAN/DENSP)*DELTAT
      IF(IUSEF.EQ.1) EFLN=USEF
C
C  IMPACT IS ASSUMED CENTERED AT X=0, Y=0.
C  DETERMINE WHICH NODES FALL IN THIS REGION.
C
   65 CONTINUE
      NK=0
      DO 70 I=1,NNT
      DIST=DSQRT(XG(I)*XG(I)+YG(I)*YG(I)+(ZG(1)-ZG(I))**2)
      IF (DIST.GT.EFLN) GO TO 70
      NK=NK+1
      SI(NK)=DIST
      NEFF(NK)=I
      ALPHA(NK)=1.0D0-DIST/EFLN
   70 CONTINUE
C
C  IF USER ESTIMATES OF NK IS TOO SMALL, TERMINATE PROGRAM.
C
      IF (NIAN.GE.NK) GO TO 73
      WRITE(MWRITE,640) ITIME
      WRITE(MWRITE,650) NIAN
      WRITE(MWRITE,660) NK
      WRITE(MWRITE,670)
      CALL EXIT
   73 CONTINUE
C   TEMPORARY UPDATE OF EFLN IF NK=0.
      IF(NK GT 0) GO TO 74
      NN3=NP(LNMP,3)
      NN1=NP(LNMP,1)
      EFLN=DSQRT((XG(NN3)-XG(NN1))**2+(YG(NN3)-YG(NN1))**2
     2+(ZG(NN3)-ZG(NN1))**2)
      WRITE(MWRITE,730) ITIME
      WRITE(MWRITE,740) EFLN
      GO TO 65
  74  CONTINUE
C
C  CALCULATE ALPHAS FOR WHOLE PLATE.
C
      NEAD1=NEAD+1
      NECD1=NECD+1
      DO 75 I=1,NK
      NN=NEFF(I)
      IF (NN EQ 1) GO TO 75
      NROW=(NN-1)/NEAD1
      IF (NROW EQ.0) ALPHA(I)=2.0D0*ALPHA(I)
      NCOLM=NN-NROW*NEAD1-1
      IF (NCOLM EQ 0) ALPHA(I)=2.0D0*ALPHA(I)
      IF (NROW GT.0.AND.NCOLM.GT.0) ALPHA(I)=4.0D0*ALPHA(I)
   75 CONTINUE
      CONST=0 0D0
      DO 80 I=1,NK
   80 CONST=CONST+ALPHA(I)
      CONST=1 0D0/CONST
      DO 90 I=1,NK
   90 ALPHA(I)=ALPHA(I)*CONST
C
C  EXTRACT NODAL Z-DIRECTION VELOCITES IN IMPACT-AFFECTED REGION.
C
      DO 100 I=1,NK
      NNI=(NEFF(I)-1)*NDPN
  100 VN(3,I)=VEL(NNI+3)
C
```

```
C  CALCULATE INTERACTION CONSTANTS A0 AND B3.
C
      A0=VF(3)
      B3=1.0D0/FMASS
      DO 140 I=1,NK
      A0=A0-ALPHA(I)*VN(3,I)
      J=NEFF(I)
  140 B3=B3+ALPHA(I)*ALPHA(I)/PMASS(J)
C  IF (A0.LE.0 0D0) NO COLLISION CAN OCCUR.
C
      IF (A0 LE 0.0D0) GO TO 300
C
C  SET IMPACT INDEX TO INDICATE IMPACT HAS OCCURED.
C
      IMPCT1=1
C
C  CALCULATE Z-DIRECTION IMPULSE.
C
      PTILDA(3)=(1.0D0+COEFR)*A0/B3
C
C  CALCULATE POST-IMPACT Z-DIRECTION VELOCITIES AT IMPACT
C  AFFECTED NODES.
C
      DO 150 I=1,NK
      J=NEFF(I)
      NNI=(J-1)*NDPN
  150 VEL(NNI+3)=VEL(NNI+3)+ALPHA(I)*PTILDA(3)/PMASS(J)
C  CALCULATE POST-IMPACT Z-DIRECTION VELOCITY OF FRAGMENT.
C
      VF(3)=VF(3)-PTILDA(3)/FMASS
C
C  PRINT MESSAGE.
C
      WRITE (MWRITE,600) ITIME,NSR
      WRITE(MWRITE,610)XN(1),YN(1),ZN(1)
      WRITE (MWRITE,620) PENM
      WRITE (MWRITE,630)
  300 CONTINUE
C
C  REDUCE FRAGMENT Z-POSITION TO PRE-IMPACT VALUE.
C
      ZF=ZF-DELF(3)
C
C  REDUCE NODAL COORDINATES TO PRE-IMPACT VALUES.
C
      DO 220 I=1,NNT
      NNI=(I-1)*NDPN
      XG(I)=XG(I)-DELD(NNI+1)
      YG(I)=YG(I)-DELD(NNI+2)
  220 ZG(I)=ZG(I)-DELD(NNI+3)
  600 FORMAT ('0',10X,'IMPACT IT=',I10,2X,'ELEMENT=1,
     1 SUBREGION=',I3)
 610  FORMAT(' ',10X,'GLOBAL IMPACT LOCATION (XN,YN,ZN)=',3D15.7)
 620  FORMAT(' ',10X,'PENETRATION DISTANCE(IN.)=',D15.7)
  630 FORMAT (' ',10X,'DOUBLE SYMMETRY OPTION IN EFFECT')
  640 FORMAT ('0','***** USER ESTIMATE OF NUMBER OF IMPACT-AFFECTED ',
     1'NODES TOO SMALL AT CYCLE NUMBER ',I7)
  650 FORMAT (' ', 5X,'USER ESTIMATE(PARAMETER NIAN IN DUMMY MAIN PROGRA
     1M)= ',I5)
  660 FORMAT (' ',5X,'ACTUAL NUMBER OF IMPACT AFFECTED NODES = ',I7)
  670 FORMAT (' ',5X,'RUN HAS BEEN TERMINATED')
 730  FORMAT('0','** TEMPORARY UPDATE OF EFFECTIVE LENGTH AT CYCLE'
     2,I5)
 740  FORMAT(' ','EFFECTIVE LENGTH=',D15.7)
      RETURN
      END
```

```
      SUBROUTINE IMPSS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,NEFF,    IMSS0000
     1ALPHA,IMPCT1,NIAN)                                                 IMSS0001
      IMPLICIT REAL*8(A-H,O-Z)                                           IMSS0002
C                                                                        IMSS0003
C    ASRL TR 154-14.  ... ORIGINAL REPORT VERSION OF PROGRAM             IMSS0004
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980             IMSS0005
C                                                                        IMSS0006
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.         IMSS0007
C                                                                        IMSS0008
      DIMENSION XG(1),YG(1),ZG(1),DELD(1),VEL(1)  ,NP(4,1),LNC(4),PEN(8),IMSS0009
     2XN(8),YN(8),ZN(8),T(3,3),SI(1),NEFF(1),ALPHA(1),VN(3,1),VNB(3,1),  IMSS0010
     3OMEGFB(3),VFB(3),PMASS(1),PTILDA(3),NBC(1)                         IMSS0011
      COMMON/PLATE/XD,YD,THP,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,THALF     IMSS0012
```

```
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /PROPC/ COEFP,FRNC,USEF,NSYM,IPSS,IUSEF
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RF,FMASS,
     2FMOI,FKEO,FKEC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C  CONTROLLING ROUTINE FOR COLLISION INSPECTION/IMPACT INTERACTION
C  ANALYSIS INCLUDING FRICTION, ASSUMING SINGLE-SYMMETRY OPTION IS
C  EXERCISED.  IMPACT CAN OCCUR ONLY ALONG PLATE SIDE NUMBER 1 IF
C  IPSS=1, OR PLATE SIDE NUMBER 4 IF IPSS=4
C
C  UPDATE NODAL GLOBAL COORDINATES TO TRIAL VALUES.
C
      IMPCT1=0
      NEAD1=NEAD+1
      NECD1=NECD+1
      NDPN=6
      NNT=NDT/6
      DO 1 I=1,NNT
      NNI=(I-1)*NDPN
      XG(I)=XG(I)+DELD(NNI+1)
      YG(I)=YG(I)+DELD(NNI+2)
    1 ZG(I)=ZG(I)+DELD(NNI+3)
C
C  UPDATE FRAGMENT POSITION.
C
      DO 7 I=1,3
      DTHETF(I)=OMEGF(I)*DELTAT
    7 DELF(I)=VF(I)*DELTAT
      XF=XF+DELF(1)
      YF=YF+DELF(2)
      ZF=ZF+DELF(3)
C
C  DETERMINE WHICH ELEMENTS ARE TO BE CHECKED FOR POSSIBLE IMPACT.
C
      IF (IPSS.EQ 4) GO TO 14
C
C  FOR CASE WHERE SIDE 1 (X-AXIS) IS LINE OF SYMMETRY,
C  FIND MINIMUM DISTANCE SQUARED FROM FRAGMENT TO NODE.
C
      DMIN=1 OD10
      DO 10 I=1,NEAD1
      DIST=(XF-XG(I))**2+(YF-YG(I))**2+(ZF-ZG(I))**2
      IF (DIST.LT.DMIN) NC=I
      IF (DIST LT DMIN) DMIN=DIST
   10 CONTINUE
C
C  DETERMINE ELEMENT NUMBERS CONNECTED TO NODE NC.
C
      NEC=0
      IF (NC EQ.1) GO TO 12
      NEC=NEC+1
      LNC(NEC)=NC-1
   12 CONTINUE
      IF (NC EQ NEAD1) GO TO 20
      NEC=NEC+1
      LNC(NEC)=NC
      GO TO 20
```

```
C
C  FOR CASE WHERE SIDE 4 (Y-AXIS) IS LINE OF SYMMETRY.
C
   14 CONTINUE
      DMIN=1.0D10
      DO 16 I=1,NNT,NEAD1
      DIST=(XF-XG(I))**2+(YF-YG(I))**2+(ZF-ZG(I))**2
      IF (DIST.LT.DMIN) NC=I
      IF (DIST.LT.DMIN) DMIN=DIST
   16 CONTINUE
C
C  DETERMINE ELEMENT NUMBERS CONNECTED TO NODE NC.
C
      NEC=0
      NROW=(NC-1)/NEAD1+1
      IF (NROW.EQ.1) GO TO 18
      NEC=NEC+1
      LNC(NEC)=(NROW-2)*NEAD+1
   18 CONTINUE
      IF (NROW.EQ.NECD1) GO TO 20
      NEC=NEC+1
      LNC(NEC)=(NROW-1)*NEAD+1
   20 CONTINUE
C
C  CHECK FOR PENETRATION IN EACH TRIANGULAR SUBREGION OF
C  THE NEC ELEMENTS.
C
      DO 50 I=1,NEC
      LNUM=LNC(I)
      DO 50 NSR=1,2
      NNSR=(I-1)*2+NSP
      NN1=NP(1,LNUM)
      NN2=NP(NSR+1,LNUM)
      NN3=NP(NSR+2,LNUM)
   50 CALL COLLSN(1,XG,YG,ZG,PEN,RF,THALF,NNSR,XF,YF,ZF,XN,YN,ZN,T,NN1,
     1NN2,NN3)
C
C  SEARCH FOR MAXIMUM PENETRATION.
C
      NEC2=NEC*2
      PENM=0.0D0
      DO 60 I=1,NEC2
      IF (PEN(I).GT.PENM) LSP=I
      IF (PEN(I).GT.PENM) PENM=PEN(I)
   60 CONTINUE
C
C SKIP OUT IF NO PENETRATION
C
      IF (PENM.EQ.0.0) GO TO 300
C
C  CALCULATE RADIUS OF REGION OF INFLUENCE, EFLN
C
      EFLN=DSQRT(EPAN/DENSP)*DELTAT
      IF(IUSEF.EQ.1) EFLN=USEF
C
C  POINT OF CONTACT MUST FALL ON LINE OF SYMMETRY.
C
      IF (IPSS.EQ.1) YN(LSP)=0.0D0
      IF (IPSS.EQ.4) XN(LSP)=0.0D0
```

```
C
C  DETERMINE WHICH NODES FALL IN THIS REGION OF INFLUENCE.
C
 65   CONTINUE
      NK=0
      DO 70 I=1,NNT
      DIST=DSQRT((XN(LSP)-XG(I))**2+(YN(LSP)-YG(I))**2
     1+(ZN(LSP)-ZG(I))**2)
      IF (DIST GT.EFLN) GO TO 70
      NK=NK+1
      SI(NK)=DIST
      NEFF(NK)=I
      ALPHA(NK)=1.0D0-DIST/EFLN
   70 CONTINUE
C
C  IF USER ESTIMATES OF NK IS TOO SMALL, TERMINATE PROGRAM.
C
      IF (NIAN.GE.NK) GO TO 73
      WRITE(MWRITE,650) ITIME
      WRITE(MWRITE,660) NIAN
      WRITE(MWRITE,670) NK
      WRITE(MWRITE,680)
      CALL EXIT
   73 CONTINUE
C   TEMPORARY UPDATE OF EFLN IF NK=0.
      IF(NK GT 0) GO TO 74
      NN3=NP(LNMP,3)
      NN1=NP(LNMP,1)
      EFLN=DSQRT((XG(NN3)-XG(NN1))**2+(YG(NN3)-YG(NN1))**2
     2+(ZG(NN3)-ZG(NN1))**2)
      WRITE(MWRITE,730) ITIME
      WRITE(MWRITE,740) EFLN
      GO TO 65
 74   CONTINUE
C
C  CALCULATE AND SCALE ALPHAS FOR PRESENT SINGLE SYMMETRY ANALYSIS.
C
      DO 75 I=1,NK
      NN=NEFF(I)
      NROW=(NN-1)/NEAD1
      NCOLM=NN-NROW*NEAD1-1
      IF (IPSS EQ.1 AND.NROW.EQ.0) GO TO 75
      IF (IPSS EQ.4 AND.NCOLM.EQ.0) GO TO 75
      ALPHA(I)=2.0D00*ALPHA(I)
   75 CONTINUE
      CONST=0 0D0
      DO 80 I=1,NK
   80 CONST=CONST+ALPHA(I)
      CONST=1.0D0/CONST
      DO 90 I=1,NK
   90 ALPHA(I)=ALPHA(I)*CONST
C
C  ESTABLISH COORDINATE TRANSFORMATION MATRIX, T, IN
C  FORM (XYZ-BAR)=T*(XYZ).
C
      DO 92 I=1,3
      DO 92 J=1,3
   92 T(I,J)=0.0D0
      NN=(LSP+1)/2
```

```
      LNMP=LNC(NN)
      NSR=LSP-2*(NN-1)
      IF (IPSS EQ 4) GO TO 94
C
C  TRANSFORMATION WHEN X-AXIS IS LINE OF SYMMETRY.
C
      NN1=NP(1,LNMP)
      NN2=NP(2,LNMP)
      ESL=DSQRT((XG(NN2)-XG(NN1))**2+(ZG(NN2)-ZG(NN1))**2)
      SINB=(ZG(NN2)-ZG(NN1))/ESL
      COSB=(XG(NN2)-XG(NN1))/ESL
      T(1,1)=COSB
      T(1,3)=SINB
      T(2,2)=1.0D0
      T(3,1)=-SINB
      T(3,3)=COSB
      CO TO 96
   94 CONTINUE
C
C  TRANSFORMATION WHEN Y-AXIS IS LINE OF SYMMETRY.
C
      NN1=NP(1,LNMP)
      NN2=NP(4,LNMP)
      ESL=DSQRT((YG(NN2)-YG(NN1))**2+(ZG(NN2)-ZG(NN1))**2)
      SINB=(ZG(NN2)-ZG(NN1))/ESL
      COSB=(YG(NN2)-YG(NN1))/ESL
      T(1,1)=1.0D0
      T(2,2)=COSB
      T(2,3)=SINB
      T(3,2)=-SINB
      T(3,3)=COSB
   96 CONTINUE
C
C  EXTRACT NODAL VELOCITIES IN IMPACT-AFFECTED REGION.
C
      DO 100 I=1,NK
      NNI=(NEFF(I)-1)*NDPN
      DO 100 J=1,3
      K=NNI+J
  100 VN(J,I)=VEL(K)
C  TRANSFORM NODAL VELOCITIES INTO BAR SYSTEM.
C
      DO 110 I=1,NK
      DO 110 J=1,3
      VNB(J,I)=0.0D0
      DO 110 K=1,3
  110 VNB(J,I)=VNB(J,I)+T(J,K)*VN(K,I)
C
C  TRANSFORM FRACMENT VELOCITIES INTO BAR SYSTEM
C
      DO 130 I=1,3
      OMEGFB(I)=0.0D0
      VFB(I)=0.0D0
      DO 130 K=1,3
      OMEGFB(I)=OMEGFB(I)+T(I,K)*OMEGF(K)
  130 VFB(I)=VFB(I)+T(I,K)*VF(K)
C
C  CALCULATE IMPULSE INTERACTION CONSTANTS.
C
```

```
      S1=VFB(1)+RF*OMEGFB(2)
      S2=VFB(2)-RF*OMEGFB(1)
      A0=VFB(3)
      B3=1.0D0/FMASS
      DO 140 I=1,NK
      S1=S1-ALPHA(I)*VNB(1,I)
      S2=S2-ALPHA(I)*VNB(2,I)
      A0=A0-ALPHA(I)*VNB(3,I)
      J=NEFF(I)
  140 B3=B3+ALPHA(I)*ALPHA(I)/PMASS(J)
      B1=B3+RF*RF/FMOI
C
C  IF A0 IS NON-POSITIVE, THEN NO COLLISIONS CAN OCCUR.
C
      IF (A0.LE.0.0) GO TO 300
C
C  SET IMPACT INDEX TO INDICATE THAT IMPACT HAS OCCURED.
C
      IMPCT1=1
C
C  MODIFY S1 OR S2 FOR SINGLE SYMMETRY IMPACT.
C
      IF (IPSS.EQ.1) S2=0.0D0
      IF (IPSS.EQ.4) S1=0.0D0
C
C  CALCULATE POST-IMPACT MOMENTUM COORDINATES IN BAR SYSTEM.
C
      CALL INTRAC(S1,S2,A0,B1,B3,COEFR,FRNC,PTILDA)
C
C  CALCULATE POST-IMPACT VELOCITY DIFFERENCE OF NODES AND FRAGMENT
C  IN BAR SYSTEM (I.E.  V-PRIME MINUS V).
      DO 180 I=1,NK
      K=NEFF(I)
      AM=PMASS(K)
      DO 180 J=1,3
  180 VNB(J,I)=ALPHA(I)*PTILDA(J)/AM
      DO 190 I=1,3
  190 VFB(I)=-PTILDA(I)/FMASS
      OMEGFB(1)=PTILDA(2)*RF/FMOI
      OMEGFB(2)=-PTILDA(1)*RF/FMOI
      OMEGFB(3)=0.D0
C  TRANSFORM POST-IMPACT NODAL VELOCITY DIFFERENCE BACK TO XYZ SYSTEM.
C
      DO 200 I=1,NK
      DO 200 J=1,3
      VN(J,I)=0.0D0
      DO 200 K=1,3
  200 VN(J,I)=VN(J,I)+T(K,J)*VNB(K,I)
C
C  TRANSFORM POST-IMPACT VELOCITY DIFFERENCE OF FRAGMENT TO GLOBAL
C  SYSTEM AND ADD TO PRE-IMPACT VELOCITIES TO GET POST-IMPACT
C  FRAGMENT VELOCITIES.
C
      DO 210 I=1,3
      DO 210 K=1,3
      OMEGF(I)=OMEGF(I)+T(K,I)*OMEGFB(K)
  210 VF(I)=VF(I)+T(K,I)*VFB(K)
C
C  UPDATE NODAL VELOCITIES TO POST-IMPACT VALUES.
```

```
C                                                                       IMSS0308
      DO 230 I=1,NK                                                     IMSS0309
      NNI=(NEFF(I)-1)*NDPN                                              IMSS0310
      DO 230 J=1,3                                                      IMSS0311
      K=NNI+J                                                           IMSS0312
  230 VEL(K)=VEL(K)+VN(J,I)                                             IMSS0313
C  CONSTRAIN VELOCITY                                                   IMSS0314
C                                                                       IMSS0315
      CALL RCON(VEL,NBC)                                                IMSS0316
C                                                                       IMSS0317
C  PRINT MESSAGES.                                                      IMSS0318
C                                                                       IMSS0319
      WRITE(MWRITE,600)ITIME,LNMP,NSR                                   IMSS0320
      WRITE(MWRITE,610)XN(LSP),YN(LSP),ZN(LSP)                          IMSS0321
      WRITE(MWRITE,620)PEN(LSP)                                         IMSS0322
      IF (IPSS EQ.1) WRITE(MWRITE,630)                                  IMSS0323
      IF (IPSS.EQ.4) WRITE(MWRITE,640)                                  IMSS0324
  300 CONTINUE                                                          IMSS0325
C                                                                       IMSS0326
C  REDUCE FRAGMENT POSITION TO PRE-IMPACT VALUE.                        IMSS0327
C                                                                       IMSS0328
      XF=XF-DELF(1)                                                     IMSS0329
      YF=YF-DELF(2)                                                     IMSS0330
      ZF=ZF-DELF(3)                                                     IMSS0331
C                                                                       IMSS0332
C  REDUCE NODAL COORDINATES TO PRE-IMPACT VALUES.                       IMSS0333
C                                                                       IMSS0334
      DO 220 I=1,NNT                                                    IMSS0335
      NNI=(I-1)*NDPN                                                    IMSS0336
      XG(I)=XG(I)-DELD(NNI+1)                                           IMSS0337
      YG(I)=YG(I)-DELD(NNI+2)                                           IMSS0338
  220 ZG(I)=ZG(I)-DELD(NNI+3)                                           IMSS0339
  600 FORMAT('0',10X,'IMPACT IT=',I10,2X,'ELEMENT=',I5,2X,'SUBREGION=', IMSS0340
     1I3)                                                               IMSS0341
  610 FORMAT(' ',10X,'GLOBAL IMPACT LOCATION(XN,YN,ZN)=',3D15.7)        IMSS0342
 620  FORMAT(' ',10X,'PENETRATION DISTANCE(IN.)=',D15.7)                IMSS0343
  630 FORMAT(' ',10X,'SINGLE SYMMETRY OPTION IN EFFECT ALONG X-AXIS')   IMSS0344
  640 FORMAT(' ',10X,'SINGLE SYMMETRY OPTION IN EFFECT ALONG Y-AXIS')   IMSS0345
  650 FORMAT ('0','***** USER ESTIMATE OF NUMBER OF IMPACT-AFFECTED ',  IMSS0346
     1'NODES TOO SMALL AT CYCLE NUMBER ',I7)                            IMSS0347
  660 FORMAT (' ', 5X,'USER ESTIMATE(PARAMETER NIAN IN DUMMY MAIN PROGRAIMSS0348
     1M)= ',I5)                                                         IMSS0349
  670 FORMAT (' ',5X,'ACTUAL NUMBER OF IMPACT AFFECTED NODES = ',I7)    IMSS0350
  680 FORMAT (' ',5X,'RUN HAS BEEN TERMINATED')                         IMSS0351
 730  FORMAT('0','** TEMPORARY UPDATE OF EFFECTIVE LENGTH AT CYCLE'     IMSS0352
     2,I5)                                                              IMSS0353
 740  FORMAT(' ','EFFECTIVE LENGTH=',D15.7)                             IMSS0354
      RETURN                                                            IMSS0355
      END                                                               IMSS0356
      SUBROUTINE INTRAC(S1,S2,A0,B1,B3,COEFR,FRNC,PTILDA)               INTC0000
      IMPLICIT REAL*8(A-H,O-Z)                                          INTC0001
C                                                                       INTC0002
C    ASRL TR 154-14 . . ..ORIGINAL REPORT VERSION OF PROGRAM            INTC0003
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980            INTC0004
C                                                                       INTC0005
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.        INTC0006
C                                                                       INTC0007
      DIMENSION PTILDA(3)                                               INTC0008
C                                                                       INTC0009
```

```
C     THIS SUBROUTINE CALCULATES THE POST-IMPACT MOMENTUM COORDINATES
C     IN 3-D MOMENTUM SPACE   A GRAPHICAL SOLUTION TECHNIQUE WAS USED TO
C     DETERMINE THE ANALYTIC EXPRESSIONS USED IN THIS ROUTINE.   THE
C     PARAMETERS S1, S2, AND B1 DEFINE THE LINE OF NO SLIDING, AND A0
C     AND B3 DEFINE THE PLANE OF MAXIMUM APPROACH.   COEFR IS THE
C     COEFFICIENT OF RESTITUTION AND FRNC IS THE COEFFICIENT OF FRICTION
C     BETWEEN THE CONTACTING SURFACES.  PTILDA(I) IS THE VECTOR OF
C     POST-IMPACT MOMENTUM COORDINATES (OUTPUT) IN THE X-BAR, Y-BAR,
C     AND Z-BAR (NORMAL) DIRECTIONS RESPECTIVELY.
C
C     CALC. TANL=TANGENT(LAMBDA)
        CONST1=DSQRT(S1*S1+S2*S2)
        TANL=B3*CONST1/(A0*B1)
C     CONSIDER PERFECTLY SMOOTH CONTACT SURFACES
        IF(FRNC.GT 0.0)GO TO 10
        PTILDA(1)=0.0D0
        PTILDA(2)=0.0D0
        PTILDA(3)=(1.0D0+COEFR)*A0/B3
        RETURN
  10    CONTINUE
        IF(FRNC.GE.TANL)GO TO 20
C     CASE WHERE FRNC LESS THAN TANL
        COSTB=S1/CONST1
        SINTB=S2/CONST1
        PN1=A0/B3
        PTILDA(1)=COSTB*FRNC*(1.0D0+COEFR)*PN1
        PTILDA(2)=SINTB*FRNC*(1.0D0+COEFR)*PN1
        PTILDA(3)=(1.0D0+COEFR)*PN1
        RETURN
  20    CONTINUE
C     CASE WHERE FRNC GREATER THAN OR EQUAL TO TANL
C     ALSO CASE OF PERFECTLY ROUGH CONTACT SURFACES
        PTILDA(3)=(1.0D0+COEFR)*A0/B3
        PTILDA(1)=S1/B1
        PTILDA(2)=S2/B1
        RETURN
        END
```

```
      SUBROUTINE IMAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,
     2VEL,ICOL,INUM,KROW,NDEX,      STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS,
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON,
     6PMASS,VN,VNB,SI,NEFF,ALPHA,NIAN)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14 ..... ORIGINAL REPORT VERSION OF PROGRAM
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C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C
      DIMENSION DELD(1),DIS(1),DISP(1),DISM1(1),DISM2(1),FLN(1),FLVA(1),
     2FLVM(1),FLVP(1),       VEL(1),ICOL(1),INUM(1),KROW(1),NDEX(1),
     3        STF(1),AMASS(1),NP(4,1),NODE(1),TAUSS(MAXEL,MNSLT4,9),
     4TAUSE(MAXEL,MNSLT4 ,9),TAUEE(MAXEL,MNSLT4,9),EPSSI(9,1),EPSSO(9,1)
     5,EPEEI(9,1),EPEEO(9,1),EPSEI(9,1),EPSEO(9,1),NBC(1),BC(1),RFM(NBE,
     6MBWE),ILAST(1),UCF1(1),UCF2(1),XG(1),YG(1),ZG(1),XGI(1),YGI(1),
     7TAGSS(MNXST,MNSLT4,3),LNXS(1),XSPROP(7,1),MATXS(1),TSGEE(MNYST,
     8MNSLT4,3),LNYS(1),YSPROP(7,1),MATYS(1),LNRS(1),ISRS(1),SC(5,1)
     9,NVSA(1),NCON(4,1),PMASS(1),VN(3,1),VNB(3,1),SI(1),NEFF(1),
     1ALPHA(1)
C
C    THE FOLLOWING DIMENSION STATEMENTS ARE FIXED FOR ALL COMPUTER RUNS.
      DIMENSION SIG(5),EPS(5),DSR(5),PSR(5),ES(6),SUBW(5,5),SNO(5,5),
     2NMSUB(5),EK(24,24),GBI(24,24),EML(24),STDE(9,24),SS(5)
      DATA GBI/1 0,-1 0,-1 0,1 0,24*0.0,1.0,-1 0,-1 0,1 0,24*0.0,1.0,
     23*0.0,-3 0,-3.0,2*0 0,9 0,2 0,2 0,2*0 0,-6.0,-6.0,4 0,9*0 0,1 0,
     32*0 0,-2 0,2*0 0,-3 0,6 0,1 0,2*0 0,2 0,-3 0,-4 0,2 0,10*0.0,1.0,
     42*0.0,-2 0,-3 0,0 0,6 0,0 0,1 0,2 0,0 0,-4.0,-3.0,2.0,11*0.0,1.0,
     52*0 0,-2 0,-2 0,4 0,2*0 0,2*1 0,-2 0,-2.0,1 0,0.0,1 0,0 0,-1.0,
     C25*0 0,1 0,0.0,-1 0,
     628*0 0,3 0,3*0 0,-9 0,-2 0,3*0.0,6.0,6 0,-4.0,12*0 0,-1 0,3*0 0,
     73 0,1 0,3*0.0,-3.0,-2 0,2 0,14*0.0,3 0,0 0,-6 0,2*0.0,-2 0,0.0,
     84 0,3.0,-2 0,14*0 0,-1 0,0 0,2.0,2*0 0,1 0,0 0,-2 0,-1.0,1 0,3*0.0
     9,1 0,27*0 0,1.0,32*0.0,9.0,4*0.0,-6 0,-6 0,4 0,16*0.0,-3 0,4*0.0,
     A3 0,2 0,-2 0,16*0 0,-3.0,4*0 0,2.0,3 0,-2 0,16*0 0,1 0,4*0 0,-1.0,
     B-1 0,1 0,2*0 0,1 0,-1 0,26*0 0,1 0,-1 0,29*0 0,3 0,2*0.0,-9.0,0.0,
     C-2 0,2*0 0,6.0,6 0,-4 0,15*0 0,3 0,-6 0,3*0 0          ,-2.0,3.0,
     D4 0,-2 0,13*0 0,-1.0,2*0 0,3 0,0.0,1.0,2*0.0,-2 0,-3.0,2.0,15*0.0,
     E-1 0,2 0,3*0 0,1 0,-1 0,-2 0,1 0/
      COMMON/PLATE/XDIST,YDIST,TH ,HXL,HYL,NEAD,NECD,DENSP,EPAN,ANUP,
     2THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON/PRNT/IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RF,FMASS,
     2FMOI,FKEO,FKEC
      COMMON /PROPC/ COEFR,FRNC,USEF,NSYM,IPSS,IUSEF
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
      COMMON /MASSM/ IMASS
C    CIVM-PLATE 1 COMPUTER CODE
C    CONTROLLING ROUTINE FOR ELASTIC-PLASTIC, LARGE DEFLECTION TRANSIENT
C    ANALYSIS OF STIFFENED AND/OR UNSTIFFENED FLAT PLATES SUBJECTED TO
C    IMPACT BY A SINGLE FRAGMENT.
C
      IPASS=1
      IMASS=1
      NDPE=24
      READ(MREAD,500)IMESH,IMCONT,IPUNCH
```

```
C
C   MESH GENERATION
      IF(IMESH EQ 0.OR.IMESH.EQ.1)CALL MESHPA(NP,NODE,XG,YG,ZG,NBC,BC,
     2LNXS,LNYS,XGI,YGI,XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)
      IF(IMESH.EQ 2)CALL MESHPM(NP,NODE,XG,YG,ZG,NBC,BC,LNXS,LNYS,XGI,
     2YGI,XSPROP,YSPROP,NXST,NYST,IMESH,MATXS,MATYS)
C
C   SETUP OF STORAGE
      CALL ARRT(STF,AMASS,FLVA,ICOL,INUM,KROW,NDEX,NODE,NDPE)
C
C   STRESS-STRAIN INFORMATION FOR EACH MATERIAL.
C   NOTE-- PLATE MATERIAL IS ASSUMED TO CORRESPOND TO FIRST MATERIAL.
      NA=3
      NZ=4
      READ(MREAD,500)NMAT
      WRITE(MWRITE,600)
      CON1=2 D0*(1 D0+ANUP)/3.D0
      CON2=2.D0*(0 5D0-ANUP)/3.D0
      DO 60 I=1,NMAT
      READ(MREAD,500)NSUB
      READ(MREAD,510)(SIG(J),J=1,NSUB)
      READ(MREAD,510)(EPS(J),J=1,NSUB)
      READ(MREAD,510)DSR(I),PSR(I)
      ES(1)=SIG(1)/EPS(1)
C DEFINE VALUE OF YOUNGS MODULUS BASED ON EPS(1) AND SIG(1) .
      IF(I EQ 1) EPAN=ES(1)
      IF(NSUB-1)30,30,10
   10 DO 20 LS=2,NSUB
   20 ES(LS)=(SIG(LS)-SIG(LS-1))/(EPS(LS)-EPS(LS-1))
   30 ES(NSUB+1)=0.0
      CON3=ES(1)*(0.5D0-ANUP)/(1.0D0+ANUP)
      AI=0 0D0
      ASUM=0.0D0
      EI=0 0D0
      ESUM=0 0D0
      DO 40 LS=1,NSUB
      ASUM=ASUM+AI
      AI=1 D0-CON1*ES(LS+1)/(ES(1)-CON2*ES(LS+1))-ASUM
      SUBW(I,LS)=AI
      ESUM=ESUM+EI
      IF(LS  NE. NSUB) EI=(1.D0-ES(LS+1)/ES(1))*(EPS(LS+1)-EPS(LS))
      SNO(I,LS)=ES(1)*EPS(LS)+CON3*ESUM
   40 CONTINUE
      NMSUB(I)=NSUB
      WRITE(MWRITE,610) I
      WRITE(MWRITE,615)
      DO 50 J=1,NSUB
   50 WRITE(MWRITE,620)J,SIG(J),EPS(J),SNO(I,J),SUBW(I,J)
      WRITE(MWRITE,630)DSR(I),PSR(I)
   60 CONTINUE
  500 FORMAT(20I4)
  510 FORMAT(5D16 7)
  600 FORMAT('0','PROPERTIES FOR USER-SPECIFIED MATERIALS')
  610 FORMAT('0','PROPERTIES FOR MATERIAL NUMBER',I3)
  615 FORMAT(' ','SUBLAYER',5X,'STRESS POINT',5X,'STRAIN POINT',5X,
     2'YIELD STRESS',2X,'WEIGHTING FACTOR')
  620 FORMAT(' ',I5,4X,D16 7,1X,D16.7,1X,D16.7,1X,D16.7)
  630 FORMAT(' ','STRAIN RATE PARAMETERS: D=',D16.7,2X,'P=',D16.7)
C   TIME-STEP INFORMATION
      READ(MREAD,500)ITIMEF,INCRT,IOUT
      READ(MREAD,510)DELTAT,TIMEF
      WRITE(MWRITE,640)
      WRITE(MWRITE,650) DELTAT
      WRITE(MWRITE,660) TIMEF,ITIMEF
      WRITE(MWRITE,670) INCRT
  640 FORMAT('0','TIMEWISE SOLUTION PARAMETERS')
```

```
  650 FORMAT(' ','TIME-STEP SIZE=',D16.7,2X,'SEC.')
  660 FORMAT(' ','RUN WILL TERMINATE AT TIME =',D16.7,2X,'SEC. OR AT CYC
     2LE NUMBER',I8)
  670 FORMAT(' ','REGULAR PRINTOUT WILL BE GIVEN EVERY',I5,2X,'CYCLES')
C
C     ESTABLISH PRINT OPTIONS
      CALL PRTOP(NNSA,NVSA,NCON,NP)
      IF(MN .GT. MNC) WRITE(MWRITE,675) MN,MNC
  675 FORMAT(1H0,10(1H*),'ASSEMBLED K REQUIRES',I7,' ENTRIES.USER HAS DI
     *MENSIONED FOR ONLY',I7,'. CALL EXIT.')
      IF(MN GT MNC)CALL EXIT
C
C     INITIALIZATION
      NZSUBL=NZ*MNSL
      NASQ=NA*NA
      DO 200 I=1,9
      DO 200 J=1,NET
      EPSSI(I,J)=0.0
      EPSSO(I,J)=0.0
      EPEEI(I,J)=0.0
      EPEEO(I,J)=0 0
      EPSEI(I,J)=0.0
  200 EPSEO(I,J)=0 0
      DO 201 I=1,NET
      DO 201 JZ=1,NZSUBL
      DO 201 JA=1,NASQ
      TAUSS(I,JZ,JA)=0.0
      TAUEE(I,JZ,JA)=0.0
  201 TAUSE(I,JZ,JA)=0 0
C
      DO 301 I=1,MNYST
      DO 301 JZ=1,NZSUBL
      DO 301 JA=1,NA
  301 TSGEE(I,JZ,JA)=0.0
C
      DO 303 I=1,MNXST
      DO 303 JZ=1,NZSUBL
      DO 303 JA=1,NA
  303 TAGSS(I,JZ,JA)=0.0
C
      PLASTW=0.0
      TIME=0.0
      ITIME=0
C
C     FORMATION AND ASSEMBLY OF STIFFNESS AND MASS MATRICES
C     FOR PLATE ELEMENTS
  304 NTYPE=1
      DO 220 LNUM=1,NET
      NN1=NP(1,LNUM)
      NN2=NP(2,LNUM)
      NN4=NP(4,LNUM)
      XL=XGI(NN2)-XGI(NN1)
      YL=YGI(NN1)-YGI(NN1)
      CALL RPLATE(NTYPE,ANUP,XL,YL,TH,0 D0,0.D0,0.D0,0.D0,EK,GBI,EPAN)
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      CALL LMASSP(NTYPE,XL,YL,TH,SL,CSA,SMOI,EML,DENSP)
      LL=(LNUM-1)*NDPE
      DO 210 I=1,24
      NN=NODE(LL+I)
  210 AMASS(NN)=AMASS(NN)+EML(I)
  220 CONTINUE
C     FOR X-DIRECTION STIFFENERS
      IF(NXST EQ 0)GO TO 250
      NTYPE=2
      DO 240 NN=1,NXST
      SL=XSPROP(6,NN)
```

```
      XXF=XSPROP(7,NN)
      ANUS=XSPROP(4,NN)
      RW=XSPROP(2,NN)
      LNUM=LNXS(NN)
      RH=XSPROP(1,NN)
      YM=XSPROP(3,NN)
      DEN=XSPROP(5,NN)
      CSA=RW*RH
      SMOI=RH*RW*XXF*XXF+RW*RH**3/12.0
      NN1=NP(1,LNUM)
      NN2=NP(2,LNUM)
      NN4=NP(4,LNUM)
      XL=XGI(NN2)-XGI(NN1)
      YL=YGI(NN4)-YGI(NN1)
      CALL RPLATE(NTYPE,ANUS,XL,YL,RH ,SL,CSA,XXF ,SMOI,EK,GBI,YM)
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      CALL LMASSP(NTYPE,XL,YL,RH ,SL,CSA,SMOI,EML,DEN)
      LL=(LNUM-1)*NDPE
      DO 230 I=1,24
      J=LL+I
      NNN=NODE(J)
  230 AMASS(NNN)=AMASS(NNN)+EML(I)
  240 CONTINUE
  250 CONTINUE
C   FOR Y-DIRECTION STIFFENERS
      IF(NYST EQ.0)GO TO 280
      NTYPE=3
      DO 270 NN=1,NYST
      RH=YSPROP(1,NN)
      RW=YSPROP(2,NN)
      YM=YSPROP(3,NN)
      ANUS=YSPROP(4,NN)
      DEN=YSPROP(5,NN)
      SL=YSPROP(6,NN)
      XXF=YSPROP(7,NN)
      CSA=RW*RH
      SMOI=RH*RW*XXF*XXF+RW*RH**3/12.0D0
      LNUM=LNYS(NN)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL RPLATE(NTYPE,ANUS,XL,YL,RH ,SL,CSA,XXF ,SMOI,EK,GBI,YM)
      CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      CALL LMASSP(NTYPE,XL,YL,RH ,SL,CSA,SMOI,EML,DEN)
      LL=(LNUM-1)*NDPE
      DO 260 I=1,24
      J=LL+I
      NNN=NODE(J)
  260 AMASS(NNN)=AMASS(NNN)+EML(I)
  270 CONTINUE
  280 CONTINUE
      IF(IPASS .EQ. 3  AND. NELES .EQ. 0) GO TO 290
      IF(IPASS  EQ. 2) GO TO 741
C   FORMATION AND ASSEMBLY OF EQUIVALENT STIFFNESS MATRIX FOR LINE
C   TRANSLATIONAL AND TORSIONAL LINEAR RESTORING SPRINGS APPLIED TO
C   SIDES OF ELEMENTS
      READ(MREAD,500)NELES
      IF(NELES EQ 0)GO TO 289
      WRITE(MWRITE,720)
      WRITE(MWRITE,730)
      DO 288 I=1,NELES
      READ(MREAD,500)LNUM,ISIDE
      READ(MREAD,510)(SS(J),J=1,5)
      LNRS(I)=LNUM
```

```
      ISRS(I)=ISIDE
      DO 285 K=1,5
  285 SC(K,I)=SS(K)
      WRITE(MWRITE,740)LNUM,ISIDE,(SS(J),J=1,5)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
  288 CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
  720 FORMAT('0','LOCATION AND PROPERTIES OF LINE TRANSLATIONAL AND ROTA
     2TIONAL LINEAR RESTORING SPRINGS')
  730 FORMAT('0','ELEM. NO ',2X, 'ELEM. SIDE',8X,'KX(LB/IN/IN)',8X,
     2'KY(LB/IN/IN)',8X,'KZ(LB/IN/IN)',2X,'K-THETA(IN-LB/RAD/IN)',2X,
     3'K-PSI(IN-LB/RAD/IN)')
  740 FORMAT(' ',I5,6X,I5,5X,5D20.7)
      GO TO 289
  741 DO 743 I=1,NELES
      LNUM=LNRS(I)
      ISIDE=ISRS(I)
      DO 742 KAY=1,5
  742 SS(KAY)=SC(KAY,I)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
  743 CALL ASSEMK(EK,STF,INUM,NDPE,NODE,LNUM)
      GO TO 290
  289 CONTINUE
C   CALCULATE TIME STEP BOUND (FOR REFERENCE ONLY).
      CALL TSTEP(STF,AMASS,NBC,BC,ICOL,INUM,KROW,NDEX,DISP,DELD)
      IF(IOP5  EQ  0 .OR. NB .EQ. 0) GO TO 290
      IPASS=2
 2891 DO 2892 I=1,MNC
 2892 STF(I)=0 0D0
      DO 2894 I=1,NDT
 2894 AMASS(I)=0 0D0
      GO TO 304
  290 CONTINUE
C   GENERATE INITIAL CONDITIONS ON FRAGMENT.
      CALL IFRAG(PMASS,AMASS)
      NIMPCT=0
      ITT=0
      DO 291 I=1,NDT
      DIS(I)=0.0
      DISP(I)=0 0
      FLVA(I)=0.0
  291 VEL(I)=0 0
C   FORM LHS
      DO 307 II=1,NDT
      L=II+INUM(II)
  307 STF(L)=STF(L)+2.0D0*     AMASS(II)/(DELTAT*DELTAT)
      IF(IOP5 EQ 1)CALL EAROW(STF,NBC,ICOL,INUM,RFM,ILAST,NBE,MBWE)
C   CONSTRAIN LHS
      CALL BCONMK(STF,BC,ICOL,INUM,NBC)
C   FACTOR LHS
      CALL FACT(STF,ICOL,KROW,NDEX,IDET,MWRITE,INUM)
      IF(IMCONT EQ 1)GO TO 335
C   PRINTOUT FOR INITIAL VALUES
      IF(IOP5 EQ 0.OR.NB.EQ.0)GO TO 320
      DO 319 I=1,NB
  319 UCF1(I)=0.0
  320 CONTINUE
```

```
      NOCHK=0
      GO TO 340
C   INPUT FOR CONTINUATION RUN
 335  READ(MREAD,800)ITT,NIMPCT
      READ (MREAD,810)(DIS(I),I=1,NDT)
      READ (MREAD,810)(DISP(I),I=1,NDT)
      READ (MREAD,810)(VEL(I),I=1,NDT)
      READ (MREAD,810)(FLVA(I),I=1,NDT)
      IF(IOP5 EQ 1 AND NB GT 0)READ (MREAD,810)(UCF2(I),I=1,NB)
      READ (MREAD,810)(((TAUSS(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      READ (MREAD,810)(((TAUEE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      READ (MREAD,810)(((TAUSE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      IF(NXST EQ 0)GO TO 336
      READ (MREAD,810)(((TAGSS(I,J,K),I=1,NXST),J=1,MNSLT4),K=1,3)
 336  CONTINUE
      IF(NYST EQ 0)GO TO 337
      READ (MREAD,810)(((TSGEE(I,J,K),I=1,NYST),J=1,MNSLT4),K=1,3)
 337  READ(MREAD,810)XF,YF,ZF,(VF(J),J=1,3),(OMEGF(II),II=1,3)
      NNT=NDT/6
      DO 338 I=1,NNT
      NN=(I-1)*6
      XG(I)=XGI(I)+DISP(NN+1)
      YG(I)=YGI(I)+DISP(NN+2)
 338  ZG(I)=ZG(I)+DISP(NN+3)
      NOCHK=0
      FKE=0 0
      DO 339 I=1,3
 339  FKE=FKE+0.5D0*(FMASS*VF(I)*VF(I)+FMOI*OMEGF(I)*OMEGF(I))
      IF(FKE LT.FKEC)NOCHK=1
C   BEGIN LOOP OVER TIME STEPS
 340  ITT=ITT+1
      NSINC=0
      DO 350 J=1,NDT
      DELD(J)=DIS(J)-DISP(J)
      DISM1(J)=DISP(J)
      DISP(J)=DIS(J)
      FLN(J)=FLVA(J)
      FLVM(J)=0 0
 350  FLVA(J)=0 0
      NNT=NDT/6
      DO 351 J=1,NNT
      NN=(J-1)*6
      XG(J)=XG(J)+DELD(NN+1)
      YG(J)=YG(J)+DELD(NN+2)
 351  ZG(J)=ZG(J)+DELD(NN+3)
      IF(NOCHK  EQ  1 .OR. ITT .EQ. 1) GO TO 343
      XF=XF+VF(1)*DELTAT
      YF=YF+VF(2)*DELTAT
      ZF=ZF+VF(3)*DELTAT
 343  CONTINUE
      IF(IOP5 EQ 0.OR NB.EQ.0)GO TO 353
      DO 352 I=1,NB
 352  UCF1(I)=UCF2(I)
 353  CONTINUE
C   CALCULATE EQUIVALENT LOADS
C   FOR PLATE ELEMENT
      CALL STRESP(DISP,DELD,FLVA,ITT,NODE,MAXEL,TAUSS,TAUEE,TAUSE,
     2EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,PLASTW,XGI,YGI,NP,STDE,GBI,
     3NMSUB,DSR,PSR,SUBW,SND ,NZSUBL)
C   FOR X-DIRECTION STIFFENERS
      IF(NXST EQ 0)GO TO 360
      CALL STRESA(DISP,DELD,FLVA,ITT,NODE,MAXEL,MNXST,TAGSS,PLASTW,XGI,
     2YGI,NP,STDE,GBI,NMSUB,DSR,PSR,SUBW,SNO ,NXST,XSPROP,LNXS,MATXS,
     3NZSUBL)
 360  CONTINUE
C   FOR Y-DIRECTION STIFFENERS
```

```
      IF(NYST.EQ 0)GO TO 370
      CALL STRESC(DISP,DELD,FLVA,ITT,NODE,MAXEL,MNYST,TSGEE,PLASTW,XGI,
     2YGI,NP,STDE,GBI,NMSUB,DSR,PSR,SUBW,SNO ,NYST,YSPROP,LNYS,MATYS,
     3NZSUBL)
  370 CONTINUE
C   PRINT OUTPUT
      ITIME=ITT-1
      TIME=ITIME*DELTAT
      CALL IPRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,XGI,
     2YGI,NP,GBI,STDE ,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,NMSUB,SUBW,
     3TAUSS,TAUEE,TAUSE,MAXEL,NZSUBL,NXST,LNXS,MATXS,XSPROP,TAGSS,MNXST,
     4NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,EK,
     5AMASS,VEL,NNSA,NVSA,NCON)
C   FORM RHS
  372 CONTINUE
      NSINC=NSINC+1
      IF(NSINC GT 20)WRITE(MWRITE,900)
      IF(NSINC GT 20)RETURN
  900 FORMAT('0','MORE THAN 20 ITERATIONS REQUIRED TO CONVERGE AN IMPACT
     2 SOLUTION--RUN HAS BEEN TERMINATED')
      DO 380 I=1,NDT
  380 FLVM(I)=-(2.0D0*FLVA(I)-FLN(I))   +2.0D0*AMASS(I)*(VEL(I)+DISP(I)
     2/DELTAT)/DELTAT
      IF(IOP5 EQ 0 OR NB EQ.0)GO TO 389
      DO 388 I=1,NB
      II=NBC(I)
  388 UCF2(I)=FLVM(II)
C   CONSTRAIN RHS
  389 CALL RCON(FLVM,NBC)
C   SOLVE FOR DISPLACEMENTS
      CALL SOLV(STF,FLVM,DIS,ICOL,KROW,NDEX)
      DO 390 I=1,NDT
  390 DELD(I)-DIS(I)-DISP(I)
C   CHECK FOR SKIP OF IMPACT INSPECTION/CORRECTION ROUTINE.
      IMPCT1=0
      IF(NOCHK EQ 1)GO TO 394
C   CHECK FOR IMPACT
      IF(NSYM EQ 0)CALL IMPACT(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,SI,
     1  NEFF,ALPHA,IMPCT1,NIAN)
      IF(NSYM EQ 1) CALL IMPSS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,
     1SI,NEFF,ALPHA,IMPCT1,NIAN)
      IF(NSYM EQ 2) CALL IMPDS(XG,YG,ZG,DELD,VEL,NP,VN,VNB,PMASS,NBC,
     1SI,NEFF,ALPHA,IMPCT1,NIAN)
C   IF IMPCT1=1, IMPACT HAS OCCURED.  CALCULATE NEW TRIAL DISPLACEMENT,
C   DIS
      IF(IMPCT1 EQ 1)GO TO 372
C   IF IMPCT1=0, THEN NO IMPACTS.  TRIAL DIS IS CORRECT.  PROCEED TO
C   NEXT TIME
      IF(NSINC.GT.1)NIMPCT=NIMPCT+1
      IF(NSINC GT 1)WRITE(MWRITE,830)NIMPCT
      IF(NSINC EQ.1)GO TO 394
C   SPECIAL CALCULATION OF VELOCITY IF IMPACT HAS OCCURED.
      DO 392 I=1,NDT
  392 VEL(I)=2 0D0*(DIS(I)-DISP(I))/DELTAT-VEL(I)
      GO TO 398
  394 CONTINUE
C   NORMAL UPDATE OF VELOCITY.
      DO 396 I=1,NDT
  396 VEL(I)=(3 0D0*DIS(I)-4.0D0*DISP(I)+DISM1(I))/(2.0D0*DELTAT)
  398 CONTINUE
      FKE=0.0
      DO 399 I=1,3
  399 FKE=FKE+0 5D0*(FMASS*VF(I)*VF(I)+FMOI*OMEGF(I)*OMEGF(I))
      IF(FKE.LT.FKEC)NOCHK=1
C   CHECK FOR CONTINUATION
      IF(ITIME.LT.ITIMEF.AND.TIME.LT.TIMEF)GO TO 340
```

```
      IF(IPUNCH.EQ.0)GO TO 420
      WRITE(MPUNCH,800)ITT,NIMPCT
      WRITE(MPUNCH,810)(DIS(I),I=1,NDT)
      WRITE(MPUNCH,810)(DISP(I),I=1,NDT)
      WRITE(MPUNCH,810)(VEL(I),I=1,NDT)
      WRITE(MPUNCH,810)(FLVA(I),I=1,NDT)
      IF(IOP5.EQ 1.AND.NB.GT.0)WRITE(MPUNCH,810)(UCF2(I),I=1,NB)
      WRITE(MPUNCH,810)(((TAUSS(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      WRITE(MPUNCH,810)(((TAUEE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      WRITE(MPUNCH,810)(((TAUSE(I,J,K),I=1,NET),J=1,MNSLT4),K=1,9)
      IF(NXST.EQ.0)GO TO 400
      WRITE(MPUNCH,810)(((TAGSS(I,J,K),I=1,NXST),J=1,MNSLT4),K=1,3)
  400 CONTINUE
      IF(NYST.EQ.0)GO TO 410
      WRITE(MPUNCH,810)(((TSGEE(I,J,K),I=1,NYST),J=1,MNSLT4),K=1,3)
  410 WRITE(MPUNCH,810)XF,YF,ZF,(VF(J),J=1,3),(OMEGF(II),II=1,3)
      WRITE(MWRITE,820)ITT
  420 RETURN
  800 FORMAT(2I10)
  810 FORMAT(5D15.7)
  820 FORMAT('0','CONTINUATION CARDS HAVE BEEN PUNCHED AT ITT=',I7)
  830 FORMAT(' ',10X,'THIS IS IMPACT NUMBER',I7)
      END
      SUBROUTINE IPRINT(DISP,XG,YG,ZG,EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,
     2EPSEO,XGI,YGI,NP,GBI,STDEP,NODE,RFM,NBE,MBWE,ILAST,UCF1,NBC,ICOL,
     3NMSUB,SUBW,TAUSS,TAUEE,TAUSE,MAXEL,NZS,NXST,LNXS,MATXS,XSPROP,
     4TAGSS,MNXST,NYST,LNYS,MATYS,YSPROP,TSGEE,MNYST,NELES,LNRS,ISRS,SC,
     5EK,AMASS,VEL,MNSA,NVSA,NCON)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ASRL TR 154-14 ......ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE CIVM PLATE PROGRAM.
C
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER
C
      DIMENSION DISP(1),XG(1),YG(1),ZG(1),EPSSI(9,1),EPSSO(9,1),EPEEI(9
     2,1),EPEEO(9,1),EPSEI(9,1),EPSEO(9,1),XGI(1),YGI(1),NP(4,1),GBI(24
```

```
     3,24),STDEP(9,24),NODE(1),EP(9),ELONG(2),RFM(NBE,MBWE),ILAST(1),
     4UCF1(1),NBC(1),ICOL(1),RF(6),NMSUB(1),AGAUS(6),AGAUW(6),ZGAUS(6),
     5ZCAUW(6),SUBW(5,1),TAUSS(MAXEL,NZS,9),TAUEE(MAXEL,NZS,9),TAUSE
     6(MAXEL,NZS,9),LNXS(1),MATXS(1),XSPROP(7,1),TAGSS(MNXST,NZS,3),
     7LNYS(1),MATYS(1),YSPROP(7,1),TSGEE(MNYST,NZS,3),LNRS(1),ISRS(1),
     8SC(5,1),SS(5),EK(24,24),EQ(24),STOR(24),AMASS(1),VEL(1),NVSA(1),
     9NCON(4,1)
      COMMON/PLATE/XDIS,YDIS,TH,HX,HY,NEAD,NECD,DSP,EPAN,ANUP,THALF
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON/ASP/SLASP(50),ELASP(50),EDIR(2,50),NASP,LNASP(50)
      COMMON/PRNT/IOP1,IOP2,IOP3,IOP4,IOP5,IOP6,IOP7,IOP8
      COMMON/GSOUT/NEGS,LNGS(200)
      COMMON/SSOUT/NXSS,NYSS,INXSS(200),INYSS(200)
      COMMON/FRAG/XF,YF,ZF,DELF(3),VF(3),DTHETF(3),OMEGF(3),RADF,FMASS,
     2FMOI,FKED,FKEC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C THIS SUBROUTINE CONTROLS THE CALCULATION AND OUTPUT OF ALL QUANTITIES
C (DISPLACEMENTS, STRAINS, ENERGIES, ETC.) AS REQUESTED BY THE USER AT
C REGULAR INTERVALS DURING THE TIMEWISE SOLUTION.
C
      IF(ITIME.LE.1)GO TO 10
      III=ITIME/INCRT*INCRT-ITIME
      IF(III.NE.0)GO TO 500
 10   CONTINUE
      WRITE(MWRITE,600)ITIME,TIME
C   NODAL DISPLACEMENTS AND LOCATION.
      IF(IOP1.EQ.0)GO TO 30
      WRITE(MWRITE,610)
      NNT=NDT/6
      DO 20 NN=1,NNT
      JJ=(NN-1)*6
      J1=JJ+1
      J2=JJ+6
 20   WRITE(MWRITE,620)NN,(DISP(I),I=J1,J2) ,XG(NN),YG(NN),ZG(NN)
 30   CONTINUE
      WRITE(MWRITE,940)
      WRITE(MWRITE,950)
      WRITE(MWRITE,960)XF,YF,ZF,(VF(I),I=1,3),(OMEGF(J),J=1,3)
C   REACTION FORCES AT CONSTRAINED NODES.
      IF(IOP5.EQ.0 OR NB.EQ.0)GO TO 39
      WRITE(MWRITE,550)
      WRITE(MWRITE,560)
C   DETERMINE NODE NO. OF FIRST CONSTRAINED D.O.F..
      NC=NBC(1)
      NN1=(NC-1)/6+1
      DO 31 I=1,6
 31   RF(I)=0.0D0
C   LOOP OVER CONSTRAINED D.O.F..
      DO 38 INB=1,NB
C   DETERMINE NODE NO. OF PRESENT CONSTRAINED D.O.F..
      NC=NBC(INB)
      NN2=(NC-1)/6+1
C   IF NN2.NE.NN1, CALC. OF RF FOR NODE NN1 IS COMPLETE--PRINT OUT
C   VALUES
      IF(NN2.EQ.NN1)GO TO 33
      WRITE(MWRITE,570)NN1,(RF(J),J=1,6)
      NN1=NN2
```

```
      DO 32 I=1,6
   32 RF(I)=0.0D0
   33 CC=-UCF1(INB)
      NN=0
      IB=ICOL(NC)
      IE=ILAST(INB)
      DO 34 I=IB,IE
      NN=NN+1
   34 CC=CC+RFM(INB,NN)*DISP(I)
      NDF=NC-(NN1-1)*6
      RF(NDF)=CC
   38 CONTINUE
      WRITE(MWRITE,570)NN1,(RF(J),J=1,6)
   39 CONTINUE
C   CENTROIDAL STRAIN INFORMATION.
      IF(IOP2.EQ 0)GO TO 50
      WRITE(MWRITE,630)
      WRITE(MWRITE,640)
      WRITE(MWRITE,644)
      DO 40 LNUM=1,NET
      G11=EPSSI(5,LNUM)
      G22=EPEEI(5,LNUM)
      G12=EPSEI(5,LNUM)
      CALL PSTRN(G11,G22,G12,SI,DIRI)
      G11=EPSSO(5,LNUM)
      G22=EPEEO(5,LNUM)
      G12=EPSEO(5,LNUM)
      CALL PSTRN(G11,G22,G12,SO,DIRO)
   40 WRITE(MWRITE,650)LNUM,EPSSI(5,LNUM),EPSSO(5,LNUM),EPEEI(5,LNUM),
     2EPEEO(5,LNUM),EPSEI(5,LNUM),EPSEO(5,LNUM),SI,SO,DIRI,DIRO
   50 CONTINUE
C   NODAL AVERAGE STRAINS.
      IF(IOP3 EQ.0)GO TO 55
      WRITE(MWRITE,900)
      WRITE(MWRITE,640)
      WRITE(MWRITE,920)
      DO 53 II=1,NNSA
      G11U=0 0D0
      G11L=0 0D0
      G22U=0.0D0
      G22L=0.0D0
      G12U=0.0D0
      G12L=0.0D0
      NNUM=NVSA(II)
      DENOM=0.0D0
      DO 52 JJ=1,4
      IF(NCON(JJ,II) EQ 0)GO TO 52
      DENOM=DENOM+1.0D0
      LNUM=NCON(JJ,II)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      SL=0 0D0
      EL=0.0D0
      IF(JJ EQ 2 OR.JJ.EQ 3)SL=1.0D0
      IF(JJ EQ 3.OR.JJ EQ 4)EL=1.0D0
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)
```

```
      DO 51 J=1,9
      EP(J)=0.0D0
      DO 51 K=1,24
      NNN=(LNUM-1)*24+K
      NN=NODE(NNN)
 51   EP(J)=EP(J)+STDEP(J,K)*DISP(NN)
      EPSSM=EP(1)+EP(7)*EP(7)/2.0D0+EP(9)*EP(9)/2.0D0
      EPEEM=EP(2)+EP(8)*EP(8)/2.0D0+EP(9)*EP(9)/2.0D0
      EPSEM=EP(3)+EP(7)*EP(8)
      G11L=G11L+EPSSM+THALF*EP(4)
      G11U=G11U+EPSSM-THALF*EP(4)
      G22L=G22L+EPEEM+THALF*EP(5)
      G22U=G22U+EPEEM-THALF*EP(5)
      G12L=G12L+EPSEM+THALF*EP(6)
      G12U=G12U+EPSEM-THALF*EP(6)
 52   CONTINUE
      G11L=G11L/DENOM
      G11U=G11U/DENOM
      G22L=G22L/DENOM
      G22U=G22U/DENOM
      G12L=G12L/DENOM
      G12U=G12U/DENOM
      CALL PSTRN(G11L,G22L,G12L,SI,DIRI)
      CALL PSTRN(G11U,G22U,G12U,SO,DIRO)
 53   WRITE(MWRITE,650)NNUM,G11L,G11U,G22L,G22U,G12L,G12U,SI,SO,DIRI,
     + DIRO
 55   CONTINUE
C   GAUSS STATION STRAIN PRINTOUT.
      IF(IOP3 EQ.0)GO TO 70
      WRITE(MWRITE,660)
      DO 60 NN=1,NEGS
      LNUM=LNGS(NN)
      WRITE(MWRITE,670) LNUM
      WRITE(MWRITE,640)
      WRITE(MWRITE,645)
      DO 60 NG=1,9
      G11=EPSSI(NG,LNUM)
      G22=EPEEI(NG,LNUM)
      G12=EPSEI(NG,LNUM)
      CALL PSTRN(G11,G22,G12,SI,DIRI)
      G11=EPSSO(NG,LNUM)
      G22=EPEEO(NG,LNUM)
      G12=EPSEO(NG,LNUM)
      CALL PSTRN(G11,G22,G12,SO,DIRO)
 60   WRITE(MWRITE,650)NG,EPSSI(NG,LNUM),EPSSO(NG,LNUM),EPEEI(NG,LNUM),
     2EPEEO(NG,LNUM),EPSEI(NG,LNUM),EPSEO(NG,LNUM),SI,SO,DIRI,DIRO
 70   CONTINUE
C   ADDITIONAL STRAIN POINTS.
      IF(IOP4.EQ.0)GO TO 110
      WRITE(MWRITE,690)
      WRITE(MWRITE,695)
      DO 100 KK=1,NASP
      LNUM=LNASP(KK)
      SL=SLASP(KK)
      EL=ELASP(KK)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
```

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      YL=YGI(NNN4)-YGI(NNN1)                                            IPRT0193
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)                                  IPRT0194
      DO 80 J=1,9                                                       IPRT0195
      EP(J)=0.D0                                                        IPRT0196
      DO 80 K=1,24                                                      IPRT0197
      NNN=(LNUM-1)*24+K                                                 IPRT0198
      NN-NODE(NNN)                                                      IPRT0199
   80 EP(J)=EP(J)+STDEP(J,K)*DISP(NN)                                   IPRT0200
      EPSSM=EP(1)+EP(7)*EP(7)/2.0+EP(9)*EP(9)/2.0                       IPRT0201
      EPEEM=EP(2)+EP(8)*EP(8)/2.0+EP(9)*EP(9)/2.0                       IPRT0202
      EPSEM=EP(3)+EP(7)*EP(8)                                           IPRT0203
      DO 100 II=1,2                                                     IPRT0204
      FCT=1 0D0                                                         IPRT0205
      IF(II EQ 2)FCT=-1 0D0                                             IPRT0206
      G11=EPSSM+FCT*THALF*EP(4)                                         IPRT0207
      G22=EPEEM+FCT*THALF*EP(5)                                         IPRT0208
      G12=EPSEM+FCT*THALF*EP(6)                                         IPRT0209
      CALL PSTRN(G11,G22,G12,SI,DIRI)                                   IPRT0210
      DO 90 I=1,2                                                       IPRT0211
      THET=EDIR(I,KK)*0 1745329251994D-01                               IPRT0212
      CT=DCOS(THET)                                                     IPRT0213
      ST-DSIN(THET)                                                     IPRT0214
      ELONG(I)=G11*CT*CT+G22*ST*ST+G12*CT*ST                            IPRT0215
   90 ELONG(I)=DSQRT(1 0+2.0*ELONG(I))-1.0                              IPRT0216
      IF(II EQ 1)WRITE(MWRITE,700)KK,G11,G22,G12,ELONG(1),ELONG(2),SI,  IPRT0217
     + DIRI                                                             IPRT0218
      IF(II EQ.2)WRITE(MWRITE,710)KK,G11,G22,G12,ELONG(1),ELONG(2),SI,  IPRT0219
     + DIRI                                                             IPRT0220
  100 CONTINUE                                                          IPRT0221
  110 CONTINUE                                                          IPRT0222
C    CALCULATION AND PRINTOUT OF GAUSS STATION STRAINS ON STIFFENERS.   IPRT0223
      IF(IOP6.EQ 0)GO TO 190                                            IPRT0224
      NA=3                                                              IPRT0225
      CALL GAUSS(NA,AGAUS,AGAUW)                                        IPRT0226
C    FOR X-DIRECTION STIFFENERS.                                        IPRT0227
      IF(NXSS EQ 0)GO TO 150                                            IPRT0228
      WRITE(MWRITE,780)                                                 IPRT0229
      WRITE(MWRITE,785)                                                 IPRT0230
      WRITE(MWRITE,790)                                                 IPRT0231
      DO 140 NX=1,NXSS                                                  IPRT0232
      NST=INXSS(NX)                                                     IPRT0233
      LNUM=LNXS(NST)                                                    IPRT0234
      NNN1=NP(1,LNUM)                                                   IPRT0235
      NNN2=NP(2,LNUM)                                                   IPRT0236
      NNN4=NP(4,LNUM)                                                   IPRT0237
      XL=XGI(NNN2)-XGI(NNN1)                                            IPRT0238
      YL=YGI(NNN4)-YGI(NNN1)                                            IPRT0239
      EL=XSPROP(6,NST)                                                  IPRT0240
      DO 130 NG=1,NA                                                    IPRT0241
      SL=(1.0D00+AGAUS(NG))/2.0D0                                       IPRT0242
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)                                  IPRT0243
      EP(1)=0.0                                                         IPRT0244
      EP(4)=0 0                                                         IPRT0245
      EP(7)=0.0                                                         IPRT0246
      DO 120 K=1,24                                                     IPRT0247
      NNN=(LNUM-1)*24+K                                                 IPRT0248
      NN=NODE(NNN)                                                      IPRT0249
      EP(1)=EP(1)+STDEP(1,K)*DISP(NN)                                   IPRT0250
      EP(4)=EP(4)+STDEP(4,K)*DISP(NN)                                   IPRT0251
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```
 120  EP(7)=EP(7)+STDEP(7,K)*DISP(NN)
      NN=(NG-1)*2+1
      STOR(NN)=EP(1)+EP(7)*EP(7)/2.0+(0.5*XSPROP(1,NST)+XSPROP(7,NST))
     2*EP(4)
      STOR(NN+1)=EP(1)+EP(7)*EP(7)/2.0+(XSPROP(7,NST)-0.5*XSPROP(1,NST))
     2*EP(4)
 130  CONTINUE
 140  WRITE(MWRITE,800)NST,(STOR(J),J=1,6)
 150  CONTINUE
C     FOR Y-DIRECTION STIFFENERS.
      IF(NYSS EQ.0)GO TO 190
      WRITE(MWRITE,810)
      WRITE(MWRITE,785)
      WRITE(MWRITE,790)
      DO 180 NY=1,NYSS
      NST=INYSS(NY)
      LNUM=LNYS(NST)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      SL=YSPROP(6,NST)
      DO 170 NG=1,NA
      EL=(1.0+AGAUS(NG))/2.0
      CALL SFDM(SL,EL,XL,YL,GBI,STDEP)
      EP(2)=0 0
      EP(5)=0 0
      EP(8)=0 0
      DO 160 K=1,24
      NNN=(LNUM-1)*24+K
      NN=NODE(NNN)
      EP(2)=EP(2)+STDEP(2,K)*DISP(NN)
      EP(5)=EP(5)+STDEP(5,K)*DISP(NN)
 160  EP(8)=EP(8)+STDEP(8,K)*DISP(NN)
      NN=(NG-1)*2+1
      STOR(NN)=EP(2)+EP(8)*EP(8)/2.0+(0.5*YSPROP(1,NST)+YSPROP(7,NST))
     2*EP(5)
      STOP(NN+1)=EP(2)+EP(8)*EP(8)/2.0+(YSPROP(7,NST)-0.5*YSPROP(1,NST))
     2*EP(5)
 170  CONTINUE
 180  WRITE(MWRITE,800)NST,(STOR(J),J=1,6)
 190  CONTINUE
C     CALCULATION AND PRINTOUT OF SYSTEM ENERGIES.
      IF(IOP7 EQ 0)GO TO 440
C     ELASTIC STRAIN ENERGY STORED IN STRUCTURE.
C     FOR PLATE ELEMENTS.
      NA=3
      NZ=4
      NSUBL=NMSUB(1)
      CALL GAUSS(NA,AGAUS,AGAUW)
      CALL GAUSS(NZ,ZGAUS,ZGAUW)
      ELAST=0 0
      S1=1.0/EPAN
      S2=-ANUP*S1
      S3=2 0*(1.0+ANUP)/EPAN
      DO 240 LNUM=1,NET
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
```

```
      NNN4=NP(4,LNUM)                                                   IPRT0311
      HXL=(XGI(NNN2)-XGI(NNN1))/2.0D0                                   IPRT0312
      HYL=(YGI(NNN4)-YGI(NNN1))/2.0D0                                   IPRT0313
      DO 240 KS=1,NA                                                    IPRT0314
      SW=HXL*AGAUW(KS)                                                  IPRT0315
      DO 240 KE=1,NA                                                    IPRT0316
      EW=HYL*AGAUW(KE)                                                  IPRT0317
      JA=KE+(KS-1)*NA                                                   IPRT0318
      STOR1=0.0                                                         IPRT0319
      DO 230 LZ=1,NZ                                                    IPRT0320
      ST1=0.0                                                           IPRT0321
      ST2=0.0                                                           IPRT0322
      ST3=0.0                                                           IPRT0323
      DO 220 LS=1,NSUBL                                                 IPRT0324
      JZ=LZ+(LS-1)*NZ                                                   IPRT0325
      ST1=ST1+SUBW(1,LS)*TAUSS(LNUM,JZ,JA)                              IPRT0326
      ST2=ST2+SUBW(1,LS)*TAUEE(LNUM,JZ,JA)                              IPRT0327
  220 ST3=ST3+SUBW(1,LS)*TAUSE(LNUM,JZ,JA)                              IPRT0328
      PROD=S1*(ST1*ST1+ST2*ST2)+2.0*S2*ST1*ST2+S3*ST3*ST3               IPRT0329
  230 STOR1=STOR1+PROD*0.5*TH*ZGAUW(LZ)                                 IPRT0330
  240 ELAST=ELAST+STOR1*SW*EW/2.0                                       IPRT0331
C   ELASTIC ENERGY IN X-DIRECTION STIFFENERS.                           IPRT0332
      IF(NXST.EQ.0)GO TO 280                                            IPRT0333
      DO 270 NST=1,NXST                                                 IPRT0334
      LNUM=LNXS(NST)                                                    IPRT0335
      NNN1=NP(1,LNUM)                                                   IPRT0336
      NNN2=NP(2,LNUM)                                                   IPRT0337
      HXL=(XGI(NNN2)-XGI(NNN1))/2.0D0                                   IPRT0338
      MAT=MATXS(NST)                                                    IPRT0339
      NSUBL=NMSUB(MAT)                                                  IPRT0340
      S1=1.0/XSPROP(3,NST)                                              IPRT0341
      EW=XSPROP(2,NST)                                                  IPRT0342
      DO 270 KE=1,NA                                                    IPRT0343
      SW=HXL*AGAUW(KE)                                                  IPRT0344
      STOR1=0.0                                                         IPRT0345
      DO 260 LZ=1,NZ                                                    IPRT0346
      ST1=0.0                                                           IPRT0347
      DO 250 LS=1,NSUBL                                                 IPRT0348
      JZ=LZ+(LS-1)*NZ                                                   IPRT0349
  250 ST1=ST1+SUBW(MAT,LS)*TAGSS(NST,JZ,KE)                             IPRT0350
      PROD=S1*ST1*ST1                                                   IPRT0351
  260 STOR1=STOR1+PROD*0.5*XSPROP(1,NST)*ZGAUW(LZ)                      IPRT0352
  270 ELAST=ELAST+STOR1*EW*SW/2.0                                       IPRT0353
  280 CONTINUE                                                          IPRT0354
C   ELASTIC ENERGY STORED IN Y-DIRECTION STIFFENERS.                    IPRT0355
      IF(NYST.EQ.0)GO TO 320                                            IPRT0356
      DO 310 NST=1,NYST                                                 IPRT0357
      LNUM=LNYS(NST)                                                    IPRT0358
      NNN4=NP(4,LNUM)                                                   IPRT0360
      HYL=(YGI(NNN4)-YGI(NNN1))/2.0D0                                   IPRT0361
      MAT=MATYS(NST)                                                    IPRT0362
      NSUBL=NMSUB(MAT)                                                  IPRT0363
      S1=1.0/YSPROP(3,NST)                                              IPRT0364
      SW=YSPROP(2,NST)                                                  IPRT0365
      DO 310 KS=1,NA                                                    IPRT0366
      EW=HYL*AGAUW(KS)                                                  IPRT0367
      STOR1=0.0                                                         IPRT0368
      DO 300 LZ=1,NZ                                                    IPRT0369
      ST1=0.0                                                           IPRT0370
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      DO 290 LS=1,NSUBL
      JZ=LZ+(LS-1)*NZ
  290 ST1=ST1+SUBW(MAT,LS)*TSGEE(NST,JZ,KS)
      PROD=S1*ST1*ST1
  300 STOR1=STOR1+PROD*0.5*YSPROP(1,NST)*ZGAUW(LZ)
  310 ELAST=ELAST+STOR1*EW*SW/2.0
  320 CONTINUE
C     CALCULATION OF ENERGY STORED IN ELASTIC RESTORING SPRINGS.
      SPREN=0.0
      IF(NELES.EQ.0)GO TO 380
      DO 370 NS=1,NELES
      LNUM=LNRS(NS)
      ISIDE=ISRS(NS)
      NNN1=NP(1,LNUM)
      NNN2=NP(2,LNUM)
      NNN4=NP(4,LNUM)
      XL=XGI(NNN2)-XGI(NNN1)
      YL=YGI(NNN4)-YGI(NNN1)
      DO 330 I=1,5
  330 SS(I)=SC(I,NS)
      CALL SPRING(EK,SS,ISIDE,XL,YL)
      L=(LNUM-1)*24
      DO 340 I=1,24
      LPI=L+I
      LL=NODE(LPI)
  340 EQ(I)=DISP(LL)
      DO 350 I=1,24
      STOR(I)=0.0
      DO 350 K=1,24
  350 STOR(I)=STOR(I)+EK(I,K)*EQ(K)
      STOR1=0 0
      DO 360 I=1,24
  360 STOR1=STOR1+EQ(I)*STOR(I)
  370 SPREN=SPREN+STOR1/2 0
  380 CONTINUE
C     CALCULATE KINETIC ENERGY OF THE STRUCTURE.
      CINET=0.0
      DO 400 I=1,NDT
  400 CINET=CINET+AMASS(I)*VEL(I)*VEL(I)/2.0
C     CALCULATE KINETIC ENERGY OF THE FRAGMENT.
      CINFRT=0.D0
      CINFTN=0.D0
      DO 420 I=1,3
      CINFRT=CINFRT+0 5D0*FMOI*OMEGF(I)*OMEGF(I)
  420 CINFTN=CINFTN+0 5D0*FMASS*VF(I)*VF(I)
C     WORK INPUT TO STRUCTURE EQUAL TO DIFFERENCE BETWEEN INITIAL
C     AND CURRENT FRAGMENT ENERGIES.
      EWORK=FKEO-CINFRT-CINFTN
C     CALCULATE PLASTIC WORK BY SUBTRACTION.
      PLASTW=EWORK-ELAST-SPREN-CINET
C     PRINTOUT ENERGIES.
      WRITE(MWRITE,720)
      WRITE(MWRITE,725) CINFTN
      WRITE(MWRITE,726) CINFRT
      WRITE(MWRITE,730) EWORK
      WRITE(MWRITE,740) CINET
      WRITE(MWRITE,750) ELAST
      WRITE(MWRITE,760) PLASTW
      WRITE(MWRITE,770) SPREN
```

```
440  CONTINUE                                                            IPRT0430
500  CONTINUE                                                            IPRT0431
550  FORMAT('0','REACTION FORCES AT CONSTRAINED NODES')                  IPRT0432
560  FORMAT('0',2X,'NODE',4X,'RX(LBS)',8X,'RY(LBS)',8X,'RZ(LBS)',7X,     IPRT0433
    2'MX(LBS-IN)',5X,'MY(LBS-IN)',5X,'MXY(LBS-IN-IN)')                   IPRT0434
570  FORMAT(' ',I6,6D15.5)                                               IPRT0435
600  FORMAT('0','***** INCR  NO.=',I5,2X,'TIME=',D12.4,1X,'SEC.')        IPRT0436
610  FORMAT('0','NODE',6X,'U',12X,'V',12X,'W',11X,'PSIX',9X,'PSIY',9X,   IPRT0437
    2'TWIST',7X,'X-POS ',7X,'Y-POS.',7X,'Z-POS.')                        IPRT0438
620  FORMAT(' ',I4,9D13 5)                                               IPRT0439
630  FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECIPRT0440
    2TION AT CENTROID OF EACH ELEMENT')                                  IPRT0441
640  FORMAT('0',14X,'EPS-X STRAIN',12X,'EPS-Y STRAIN',12X,'SHEAR STRAINIPRT0442
    2',12X,'PRINC. STRAIN(T)',10X,'DIRECTION(DEG.)')                     IPRT0443
644  FORMAT(' ','ELEM ',5(7X,'INNER',7X,'OUTER'))                        IPRT0444
645  FORMAT(' ','STA. ',5(7X,'INNER',7X,'OUTER'))                        IPRT0445
650  FORMAT(' ',I4,1X,10D12 4)                                           IPRT0446
660  FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECIPRT0447
    2TION AT GAUSS STATIONS FOR SELECTED ELEMENTS')                      IPRT0448
670  FORMAT('0','RESULTS FOR ELEMENT NUMBER',I5)                         IPRT0449
690  FORMAT('0','STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECIPRT0450
    2TION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS')              IPRT0451
695  FORMAT('0','POINT NO.',2X,'SURFACE',2X.'EPS-X STRAIN'.2X 'EPS-Y STIPRT0452
    2RAIN',2X,'SHEAR STRAIN',1X,'ELONG.(DIR.1)',1X,'ELONG.(DIR.2)',1X,   IPRT0453
    3'PRINC. STRN(T)',1X,'DIRECTION(DEG.)')                              IPRT0454
700  FORMAT(' ',I7,5X,'INNER',1X,7D14.5)                                 IPRT0455
710  FORMAT(' ',I7,5X,'OUTER',1X,7D14 5)                                 IPRT0456
720  FORMAT('0','SYSTEM ENERGIES(IN-LB)')                                IPRT0457
725  FORMAT(' ',5X,'FRAG. TRANSLATIONAL KINETIC ENERGY =',D15.7)         IPRT0458
726  FORMAT(' ',5X,'FRAG  ROTATIONAL KINETIC ENERGY    =',D15.7)         IPRT0459
730  FORMAT(' ',5X,'WORK INPUT TO STRUCTURE            =',D15.7)         IPRT0460
740  FORMAT(' ',5X,'STRUCTURE KINETIC ENERGY           =',D15.7)         IPRT0461
750  FORMAT(' ',5X,'STRUCTURE ELASTIC ENERGY           =',D15.7)         IPRT0462
760  FORMAT(' ',5X,'STRUCTURE PLASTIC ENERGY           =',D15.7)         IPRT0463
770  FORMAT(' ',5X,'ENERGY STORED IN ELASTIC RESTRAINTS=',D15.7)         IPRT0464
780  FORMAT('0','STRAIN, EPS-XX, AT GAUSSIAN STATIONS ON SPECIFIED X-DIIPRT0465
    2RECTION STIFFENERS')                                                IPRT0466
785  FORMAT('0','STIFFENER',14X,'STATION 1',15X,'STATION 2',15X,         IPRT0467
    2'STATION 3')                                                        IPRT0468
790  FORMAT(' ',' NUMBER',5X,3(7X,'INNER',7X ,'OUTER'))                  IPRT0469
800  FORMAT(' ',I5,7X,6D12 4)                                            IPRT0470
810  FORMAT('0','STRAIN, EPS-YY, AT GAUSSIAN STATIONS ON SPECIFIED Y-DIIPRT0471
    2RECTION STIFFENERS')                                                IPRT0472
900  FORMAT('0','STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIREIPRT0473
    2CTION AT USER-SPECIFIED NODES')                                     IPRT0474
920  FORMAT(' ','NODE ',5(7X,'INNER',7X,'OUTER'))                        IPRT0475
940  FORMAT('0','FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS')      IPRT0476
950  FORMAT(' ',9X,'X-LOC',9X,'Y-LOC',9X,'Z-LOC',9X,'VEL-X',9X,'VEL-Y',IPRT0477
    29X,'VEL-Z',7X,'OMEGA-X',7X,'OMEGA-Y',7X,'OMEGA-Z')                  IPRT0478
960  FORMAT(' ',9D14 6)                                                  IPRT0479
     RETURN                                                              IPRT0480
     END                                                                 IPRT0481
```

```
C   CIVM-PLATE 1 COMPUTER CODE.                                         IMAN0000
C                                                                       IMAN0001
C THIS DUMMY MAIN PROGRAM FOR THE CIVM PLATE PROGRAM REQUIRES           IMAN0002
C THESE SUBROUTINES:                                                    IMAN0003
C        ARRT,ASSEMK,BCONMK,COLLSN,EAROW,FACT,GAUSS,IFRAG,              IMAN0004
C        IMAINP,IMPACT,IMPDS,IMPSS,INTRAC,IPRINT,LMASSP,                IMAN0005
C        MESHPA,MESHPM,OMULT,PROTOP,PSTRN,RCON,RPLATE,SFDM,             IMAN0006
C        SOLV,SPRING,STRLSA,STRESC,STRESP,TSTEP.                        IMAN0007
C                                                                       IMAN0008
C ILLUSTRATIVE EXAMPLE FOR CIVM PLATE PROGRAM - FULL MODEL.             IMAN0009
C                                                                       IMAN0010
C SMALL STRAIN FORMULATION VERSION WRITTEN BY R. L. SPILKER             IMAN0011
C                                                                       IMAN0012
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM            IMAN0013
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980              IMAN0014
C                                                                       IMAN0015
C   DUMMY MAIN PROGRAM WHICH PROVIDES DIMENSIONS FOR THOSE ARRAYS       IMAN0016
C   (VECTORS) WHOSE DIMENSIONS ARE PROBLEM DEPENDENT.                   IMAN0017
C   THE FOLLOWING ARRAYS MUST BE DIMENSIONED BY THE USER AS SHOWN;      IMAN0018
C     DIMENSION DELD(NDT),DIS(NDT),DISP(NDT),DISM1(NDT),DISM2(NDT),     IMAN0019
C     FLN(NDT),FLVA(NDT),FLVM(NDT),FLVP(NDT),VEL(NDT),ICOL(NDT),        IMAN0020
C     INUM(NDT),KROW(NDT),NDEX(NDT),STF(MNC),AMASS(NDT ),NP(4,MAXEL),   IMAN0021
C     NODE(24*MAXEL),TAUSS(MAXEL,4*MNSL,9),TAUSE(MAXEL,4*MNSL,9),       IMAN0022
C     TAUEE(MAXEL,4*MNSL,9),EPSSI(9,MAXEL),EPSSO(9,MAXEL),EPEEI(9,MAXEL)IMAN0023
C     ,EPEEO(9,MAXEL),EPSEI(9,MAXEL),EPSEO(9,MAXEL),NBC(NB),BC(NB),     IMAN0024
C     RFM(NBE,MBWE),ILAST(NBE),UCF1(NBE),UCF2(NBE),XG(NNT),YG(NNT),     IMAN0025
C     ZG(NNT),XGI(NNT),YGI(NNT),                                        IMAN0026
C     TAGSS(MNXST,4*MNSL,3),LNXS(MNXST),XSPROP(7,MNXST),MATXS(MNXST),   IMAN0027
C     TSGEE(MNYST,4*MNSL,3),LNYS(MNYST),YSPROP(7,MNYST),MATYS(MNYST),   IMAN0028
C     LNRS(NRSS),ISRS(NRSS),SC(5,NRSS),NVSA(MNNSA),NCON(4,MNNSA),       IMAN0029
C     PMASS(NNT),VN(3,NIAN),VNB(3,NIAN),SI(NIAN),NEFF(NIAN),ALPHA(NIAN) IMAN0030
C                                                                       IMAN0031
C   WHERE,                                                              IMAN0032
C   NNT     = TOTAL NUMBER OF NODES IN THE ASSEMBLED FINITE-ELEMENT MODELIMAN0033
C   NDT     = TOTAL NUMBER OF DEGREES OF FREEDOM IN THE ASSEMBLED FINITE-IMAN0034
C             ELEMENT MODEL = 6*NNT.                                    IMAN0035
C   MNC     = ESTIMATED NUMBER OF WORDS OF STORAGE FOR THE ASSEMBLED    IMAN0036
C             STIFFNESS MATRIX (LOWER TRIANGLE STORED BY ROWS FROM FIRST IMAN0037
C             NONZERO TERM).                                            IMAN0038
C   MAXEL   = NUMBER OF ELEMENTS IN FINITE-ELEMENT MODEL.               IMAN0039
C   MNSL    = MAXIMUM NUMBER OF MECHANICAL SUBLAYERS EMPLOYED.          IMAN0040
C             NOTE-- MNSL MUST BE LESS THAN OR EQUAL TO 5.              IMAN0041
C   NB      = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM PRIOR  IMAN0042
C             TO ELIMINATION OF DUPLICATES.                             IMAN0043
C   NBE     = ESTIMATED NUMBER OF CONSTRAINED DEGREES OF FREEDOM AFTER  IMAN0044
C             ALL DUPLICATES HAVE BEEN ELIMINATED (IF IOP5 = 1).        IMAN0045
C           = 1 IF IOP5 = 0                                             IMAN0046
C   MBWE    = ESTIMATED MAXIMUM BANDWIDTH OF ASSEMBLED STIFFNESS MATRIX IMAN0047
C             AT A CONSTRAINED DEGREE OF FREEDOM (IF IOP5 = 1).         IMAN0048
C           = 1 IF IOP5 = 0                                             IMAN0049
C   MNXST   = MAXIMUM NUMBER OF X-DIRECTION STIFFENERS.                 IMAN0050
C           = 1 IF NO X-DIRECTION STIFFENERS.                           IMAN0051
C   MNYST   = MAXIMUM NUMBER OF Y-DIRECTION STIFFENERS.                 IMAN0052
```

```
C            = 1 IF NO Y-DIRECTION STIFFENERS.
C     NRSS   = TOTAL NUMBER OF ELEMENT SIDES ON WHICH LINE RESTORING
C              SPRINGS ARE LOCATED.
C            = 1 IF NO RESTORING SPRINGS ARE PRESENT.
C     MNSLT4 = 4+MNSL
C     MNNSA  = MAXIMUM NUMBER OF NODES REQUESTED FOR STRAIN OUTPUT
C            = (IF IOP8=1).
C     NIAN   = ESTIMATED MAXIMUM NUMBER OF NODES WHICH WILL BE AFFECTED
C              BY A SINGLE IMPACT.
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DELD(450),DIS(450),DISP(450),DISM1(450),DISM2(450),
     2FLN(450),FLVA(450),FLVM(450),FLVP(450),VEL(450),ICOL(450),
     3INUM(450),KROW(450),NDEX(450),STF(16335),AMASS(450),
     4NP(4,56),NODE(1344),TAUSS(56,12,9),TAUSE(56,12,9),TAUEE(56,12,9),
     5EPSSI(9,56),EPSSO(9,56),EPEEI(9,56),EPEEO(9,56),EPSEI(9,56),
     6EPSEO(9,56),NBC(60),BC(60),RFM(1,1),ILAST(1),UCF1(1),UCF2(1),
     7XG(75),YG(75),ZG(75),XGI(75),YGI(75),TAGSS(1,12,3),LNXS(1),
     8XSPROP(7,1),MATXS(1),TSGEE(28,12,3),LNYS(28),YSPROP(7,28),MATYS(28
     9),LNRS(28),ISRS(28),SC(5,28),NVSA(8),NCON(4,8),PMASS(75),VN(3,75),
     A VNB(3,75),SI(75),NEFF(75),ALPHA(75)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C DEFINE CARD READER, LINE PRINTER AND CARD PUNCH UNIT NUMBERS.
      MREAD = 5
      MWRITE = 6
      MPUNCH = 7
      READ(MREAD,500)MAXEL,MNSL,MNXST,MNYST,NBE,MBWE,MNC,NIAN
  500 FORMAT(8I10)
      WRITE(MWRITE,600)
      WRITE(MWRITE,610) MAXEL
      WRITE(MWRITE,620) MNSL
      WRITE(MWRITE,630) MNXST
      WRITE(MWRITE,640) MNYST
      WRITE(MWRITE,650) NBE
      WRITE(MWRITE,660) MBWE
      WRITE(MWRITE,670) MNC
      WRITE(MWRITE,680) NIAN
  600 FORMAT('1 CIVM-PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) : USER
     +INPUT FOR ARRAY DIMENSIONS')
  610 FORMAT(' ','MAXEL  =',I10)
  620 FORMAT(' ','MNSL   =',I10)
  630 FORMAT(' ','MNXST  =',I10)
  640 FORMAT(' ','MNYST  =',I10)
  650 FORMAT(' ','NBE    =',I10)
  660 FORMAT(' ','MBWE   =',I10)
  670 FORMAT(' ','MNC    =',I10)
  680 FORMAT(' ','NIAN   =',I10)
      MNSLT4=MNSL+4
      CALL      IMAINP(DELD,DIS,DISP,DISM1,DISM2,FLN,FLVA,FLVM,FLVP,
     2VEL,ICOL,INUM,KROW,NDEX,      STF,AMASS,NP,NODE,TAUSS,TAUSE,TAUEE,
     3EPSSI,EPSSO,EPEEI,EPEEO,EPSEI,EPSEO,NBC,BC,RFM,ILAST,UCF1,UCF2,
     4XG,YG,ZG,XGI,YGI,TAGSS,LNXS,XSPROP,MATXS,TSGEE,LNYS,YSPROP,MATYS,
     5LNRS,ISRS,SC,MAXEL,MNSL,MNSLT4,NBE,MBWE,MNXST,MNYST,NVSA,NCON,
```

```
     6PMASS,VN,VNB,SI,NEFF,ALPHA,NIAN)                              IMAN0106
      CALL EXIT                                                     IMAN0107
      END                                                           IMAN0108
```

SECTION 7

PLATE 1 CODE EXAMPLE: TRANSIENTLY-LOADED NARROW RECTANGULAR PLATE

To illustrate the use of the PLATE 1 computer program, a narrow rectangular plate with both ends clamped is analyzed.* The plate is subjected to a prescribed distributed vertical-direction transient external loading. Three alternate finite-element modelings are described, and the associated input data for each are shown. However, for conciseness, solution output data are given for only one of these models; these results can be employed by the user to check the adaptation of PLATE 1 to his particular computing facilities.

## 7.1 Problem Description

A narrow rectangular uniform-thickness aluminum plate with both ends ideally clamped and both sides free is subjected to a uniform w-direction triangular time-pulse pressure loading $F_z^A \equiv p_z \equiv p_o = 21{,}934.4$ psi for a 25-microsecond duration, as depicted in Fig. 16. This pressure loading covers the entire width of the plate and a 1.80-in spanwise region centered at the midspan station. This plate is of 0.102-in thickness, 1.497-in width, and 8.00-in span.

The initial mass per unit volume $\rho_o$ of this plate material is assumed to be $0.25384 \times 10^{-3}$ $(\text{lb-sec}^2)/\text{in}^4$. The uniaxial stress-strain $(\sigma,\varepsilon)$ properties of the plate material are represented by a 3-sublayer version of the mechanical-sublayer material model, defined by $\sigma_1,\varepsilon_1$ = 44,200 psi, 0.00442 in/in; $\sigma_2,\varepsilon_2$ = 49,200 psi, 0.0760; and $\sigma_3,\varepsilon_3$ = 76,400 psi, 0.615; this behavior is described as elastic, strain hardening (EL-SH). In addition, it is assumed that this material yields in a rate-dependent fashion with the strain-rate constants assumed to be $d = 6500$ $\text{sec}^{-1}$ and $p = 4$; this material is thus described concisely as of EL-SH-SR type where SR denotes that the material is strain-rate sensitive. Also, it is assumed that this initially-isotropic material has an elastic (Young's) modulus of $10 \times 10^6$ psi and a Poisson ratio of 1/3.

*Additional examples and transient response predictions obtained by using the PLATE 1 program are described in Subsection 7.2 of Ref. 14 as the "small strain" calculations.

The generalized nodal loads for a plate element of dimensions $(x,y) = (a,b)$ subjected to externally-applied loads $F_x^L = F_y^L = F_z^L = F_x^B = F_y^B = F_z^B = F_x^A = F_y^A = 0$ and $F_z^A \equiv p_o$ (psi) -- (see Subsection A.1.3.4 and Eqs. A.54) -- can be shown for the present LLC plate element to be:

Degree of Freedom (DOF)+

| | $u$ | $v$ | $w$ | $\frac{\partial w}{\partial x}$ | $\frac{\partial w}{\partial y}$ | $\frac{\partial^2 w}{\partial x \partial y}$ |
|---|---|---|---|---|---|---|
| Node | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 0 | $\frac{ab}{4}$ | $\frac{a^2b}{24}$ | $\frac{ab^2}{24}$ | $\frac{a^2b^2}{144}$ |
| 2 | 0 | 0 | $\frac{ab}{4}$ | $-\frac{a^2b}{24}$ | $\frac{ab^2}{24}$ | $-\frac{a^2b^2}{144}$ |
| 3 | 0 | 0 | $\frac{ab}{4}$ | $-\frac{a^2b}{24}$ | $-\frac{ab^2}{24}$ | $\frac{a^2b^2}{144}$ |
| 4 | 0 | 0 | $\frac{ab}{4}$ | $\frac{a^2b}{24}$ | $-\frac{ab^2}{24}$ | $-\frac{a^2b^2}{144}$ |

These values apply for a unit value of uniform pressure loading $p_o$; hence, if $p_o = p_o(t)$, the appropriate corresponding element nodal loads are obtained by multiplying the above values by $p_o(t)$.

For each of the three modelings employed, it is necessary to provide a specific version each of subroutines EXTL (prescribed transient external loading) and INCOND (initial conditions). A listing of the subroutines for each of these three example modelings follows.

+See Fig. 2 for definitions and signs of these generalized displacements; the positive generalized loads are defined consistent with these directions.

```
      SUBROUTINE EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C    COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
C THIS VERSION IS FOR THE EXAMPLE OF A RECTANGULAR PLATE, CLAMPED
C ON THE TOP AND BOTTOM EDGES, FREE ON THE OTHER TWO EDGES  WITH AN
C IMPULSIVE LOAD APPLIED SYMMETRICALLY TO ITS CENTRAL REGION.
      DIMENSION ELV(1),NP(4,1),NODE(1),XGI(1),YGI(1),GBI(24,24)
C
C THIS USER-PREPARED SUBROUTINE IS USED TO GENERATE THE NODAL
C GENERALIZED FORCES CORRESPONDING TO PRESCRIBED EXTERNALLY-APPLIED
C FORCES AT TIME T SUB M (=M*DELTA T) AS DESCRIBED IN SUBSECTION 4.6.2
C AND CORRESPONDING TO THE DEVELOPMENT IN SUBSECTION A.1.3.4.
C
C DEFINE SIZE OF VECTOR CONTAINING UNIT LOAD OVER ALL DOF IN CENTRAL
C REGION OF PLATE. WIDTH OF LOADED REGION IS 1.497 IN.,LENGTH IS 1.8 IN.
      DIMENSION ALOAD(150),AL1(72),AL2(78)
      EQUIVALENCE (AL1(1),ALOAD(1)),(AL2(1),ALOAD(73))
C
      COMMON /BAS/ NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C DEFINE TIME AT WHICH LOAD DROPS TO ZERO AND MAX VALUE OF IMPULSE.
      DATA TEND/25.D-6/,AMAXP/219344DO/
C DEFINE FIRST AND LAST DOF IN LOADED REGION.
      DATA IDOF/121/,LDOF/270/
C DEFINE VALUES OF UNIT LOAD AT EACH DOF IN LOADED REGION.
      DATA AL1/2*0.D0,.4210312D-1, .2626182D-2, .3157734D-2, .1969637D-3
     1        ,2*0.D0,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0
     2        ,2*0.D0,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0
     3        ,2*0.D0,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0
     4        ,2*0.D0,.4210312D-1,-.2626182D-2, .3157734D-2,-.1969637D-3
     5        ,2*0.D0,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0
     6        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     7        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     8        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     9        ,2*0.D0,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     A        ,2*0.D0,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0
     B       ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0/
      DATA AL2/2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     1        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     2        ,2*0.D0,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     3        ,2*0.D0,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0
     4        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     5        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     6        ,2*0.D0,.1684125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     7        ,2*0.D0,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     8        ,2*0.D0,.4210312D-1, .2626182D-2,-.3157734D-2,-.1969637D-3
     9        ,2*0.D0,.8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0
     A        ,2*0.D0,.8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0
     B        ,2*0.D0,.8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0
     C       ,2*0.D0,.4210312D-1,-.2626182D-2,-.3157734D-2, .1969637D-3/
C ZERO OUT LOAD VECTOR.
      DO 1 I=1,NDT
    1 ELV(I)=0.0D0
```

```
C IF THIS TIME STEP IS BEYOND LAST TIME OF IMPULSE RETURN WITH
C EXTERNAL LOAD VECTOR SET TO ZERO.
      TYM=(ITT-1)*DELTAT
      IF(TYM  .GT. TEND) RETURN
C INTERPOLATE TO FIND MAGNITUDE OF IMPULSE AT THIS TIME.
      AMULT=AMAXP*(TEND-TYM )/TEND
C FILL IN VALUES OF LOAD IN CENTRAL REGION OF PLATE.
      DO 2 I=IDOF,LDOF
      J=I-IDOF+1
    2 ELV(I)=AMULT*ALOAD(J)
C OUTPUT LOADS IF THIS IS A REGULAR PRINTING CYCLE.
      III=ITIME/INCRT*INCRT-ITIME
      IF(III.NE.0) RETURN
      NNT=NDT/6
      WRITE(MWRITE,100) TYM ,(J,J=1,6)
100   FORMAT(1H0/1H0,10X,'TIME=',D16.6,' SEC.'/1H ,33(1H.),'EXTERNALLY A
     +PPLIED USER SPECIFIED LOADING',33(1H.)/1H ,6X,'NODE',6(7X,'DOF#',
     & I1,3X)/8X,3(10X,'(LBS)'),1X,3(7X,'(IN-LBS)'))
      I1=1
      DO 3 I=1,NNT
      I2=6*I
      WRITE(MWRITE,101) I,(ELV(J),J=I1,I2)
      I1=I2+1
  3   CONTINUE
  101 FORMAT(6X,I3,2X,6D15.6)
      RETURN
      END
      SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
      DIMENSION DISP(1),VEL(1),NP(4,1),NODE(1),XGI(1),YGI(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
C
C THIS USER-PREPARED SUBROUTINE IS USED TO SPECIFY THE INITIAL (TIME
C ZERO) CONDITIONS ON THE PLATE NODAL DISPLACEMENT AND VELOCITIES,
C (Q ZERO)* AND (Q DOT ZERO)*, RESPECTIVELY, REQUIRED FOR THE FINITE-
C DIFFERENCE SOLUTION OF THE GOVERNING EQUATIONS OF MOTION (SEE
C SUBSECTION A.3).
C
C   DUMMY ROUTINE - SETS VELOCITY AND DISPLACEMENT TO ZERO INITIALLY.
C
      DO 10 I=1,NDT
      DISP(I)=0.0D0
 10   VEL(I)=0.0D0
      RETURN
      END
```

```
      SUBROUTINE EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
C THIS VERSION IS FOR THE EXAMPLE OF A RECTANGULAR PLATE, CLAMPED
C ON THE TOP AND BOTTOM EDGES, FREE ON THE OTHER TWO EDGES  WITH AN
C IMPULSIVE LOAD APPLIED SYMMETRICALLY TO ITS CENTRAL REGION.
C
C ONLY THE RIGHT HALF OF THE PLATE IS MODELLED USING THE SYMMETRY
C CONDITION ON THE LEFT EDGE. THIS IS THE SINGLE SYMMETRY CASE.
      DIMENSION ELV(1),NP(4,1),NODE(1),XGI(1),YGI(1),GBI(24,24)
C
C THIS USER-PREPARED SUBROUTINE IS USED TO GENERATE THE NODAL
C GENERALIZED FORCES CORRESPONDING TO PRESCRIBED EXTERNALLY-APPLIED
C FORCES AT TIME T SUB M (=M*DELTA T) AS DESCRIBED IN SUBSECTION 4.6.2
C AND CORRESPONDING TO THE DEVELOPMENT IN SUBSECTION A.1.3.4.
C
C DEFINE SIZE OF VECTOR CONTAINING UNIT LOAD OVER ALL DOF IN CENTRAL
C REGION OF PLATE. WIDTH OF LOADED REGION IS .7485 IN, LENGTH IS 1.8 IN.
      DIMENSION ALOAD(90)
      COMMON /BAS/ NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C USE DATA STATEMENTS FOR VALUES WHICH WILL NOT CHANGE THROUGHOUT RUN.
C
C DEFINE TIME AT WHICH LOAD DROPS TO ZERO AND MAX VALUE OF IMPULSE.
      DATA TEND/25.D-6/,AMAXP/21934.400/
C DEFINE FIRST AND LAST DOF IN LOADED REGION.
      DATA IDOF/73/, LDOF/162/
C DEFINE VALUES OF UNIT LOAD AT EACH DOF IN LOADED REGION.
      DATA ALOAD/2*0 D0,.4210312D-1, 2626182D-2,.3157734D-2,.1969637D-3
     1        ,2*0 D0,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0
     2        ,2*0 D0, 4210312D-1,-.2626182D-2, .3157734D-2,-.1969637D-3
     3        ,2*0.D0,.8420625D-1,  5252365D-2, .0000000D+0, .0000000D+0
     4        ,2*0 D0,.1684125D+0,  0000000D+0, .0000000D+0, .0000000D+0
     5        ,2*0 D0,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     6        ,2*0 D0, 8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0
     7        ,2*0.D0,.1684125D+0, .0000000D+0,  0000000D+0, .0000000D+0
     8        ,2*0.D0, 8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     9        ,2*0 D0, 8420625D-1, .5252335D-2,  0000000D+0, .0000000D+0
     A        ,2*0.D0, 1681125D+0, .0000000D+0, .0000000D+0, .0000000D+0
     B        ,2*0 D0,.8420625D-1,-.5252365D-2,  0000000D+0, .0000000D+0
     C        ,2*0.D0, 4210312D-1,  2626182D-2,-.3157734D-2,-.1969637D-3
     D        ,2*0 D0, 8420625D-1,  0000000D+0,- 6315468D-2, .0000000D+0
     E       ,2*0 D0,.4210312D-1,-.2626182D-2,-.3157734D-2, .1969637D-3/
C ZERO OUT LOAD VECTOR.
      DO 1 I=1,NDT
    1 ELV(I)=0 0D0
      TYM=(ITT-1)*DELTAT
C IF THIS TIME STEP IS BEYOND LAST TIME OF IMPULSE RETURN WITH
C EXTERNAL LOAD VECTOR SET TO ZERO.
      IF(TYM  GT. TEND) RETURN
C INTERPOLATE TO FIND MAGNITUDE OF IMPULSE AT THIS TIME.
      AMULT=AMAXP*(TEND-TYM )/TEND
```

```
C FILL IN VALUES OF LOAD IN CENTRAL REGION OF PLATE.
      DO 2 I=IDOF,LDOF
      J=I-IDOF+1
    2 ELV(I)=AMULT*ALOAD(J)
C OUTPUT LOADS IF THIS IS A REGULAR PRINTING CYCLE.
      III=ITIME/INCRT*INCRT-ITIME
      IF(III.NE.0) RETURN
      NNT=NDT/6
      WRITE(MWRITE,100) TYM ,(J,J=1,6)
100   FORMAT(1H0/1H0,10X,'TIME=',D16.6,' SEC.'/1H ,33(1H.),'EXTERNALLY A
     +PPLIED USER SPECIFIED LOADING',33(1H.)/1H ,6X,'NODE',6(7X,'DOF#',
     & I1,3X)/8X,3(10X,'(LBS)'),1X,3(7X,'(IN-LBS)'))
      I1=1
      DO 3 I=1,NNT
      I2=6*I
      WRITE(MWRITE,101) I,(ELV(J),J=I1,I2)
      I1=I2+1
  3   CONTINUE
  101 FORMAT(6X,I3,2X,6D15.6)
      RETURN
      END
      SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
      DIMENSION DISP(1),VEL(1),NP(4,1),NODE(1),XGI(1),YGI(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
C
C THIS USER-PREPARED SUBROUTINE IS USED TO SPECIFY THE INITIAL (TIME
C ZERO) CONDITIONS ON THE PLATE NODAL DISPLACEMENT AND VELOCITIES,
C (Q ZERO)* AND (Q DOT ZERO)*, RESPECTIVELY, REQUIRED FOR THE FINITE-
C DIFFERENCE SOLUTION OF THE GOVERNING EQUATIONS OF MOTION (SEE
C SUBSECTION A.3).
C
C   DUMMY ROUTINE - SETS VELOCITY AND DISPLACEMENT TO ZERO INITIALLY.
C
      DO 10 I=1,NDT
      DISP(I)=0.0D0
 10   VEL(I)=0.0D0
      RETURN
      END
```

```
      SUBROUTINE EXTL(ELV,ITT,NP,NODE,XGI,YGI,GBI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C   COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM.
C
C THIS VERSION IS FOR THE EXAMPLE OF A RECTANGULAR PLATE, CLAMPED
C ON THE TOP AND BOTTOM EDGES, FREE ON THE OTHER TWO EDGES  WITH AN
C IMPULSIVE LOAD APPLIED SYMMETRICALLY TO ITS CENTRAL REGION.
C
C ONLY THE UPPER RIGHT QUADRANT IS MODELLED USING THE SYMMETRY CONDITION
C ONE BOTH THE BOTTOM AND LEFT EDGES. THIS IS THE DOUBLE SYMMETRY CASE.
      DIMENSION ELV(1),NP(4,1),NODE(1),XGI(1),YGI(1),GBI(24,24)
C
C THIS USER-PREPARED SUBROUTINE IS USED TO GENERATE THE NODAL
C GENERALIZED FORCES CORRESPONDING TO PRESCRIBED EXTERNALLY-APPLIED
C FORCES AT TIME T SUB M (=M*DELTA T) AS DESCRIBED IN SUBSECTION 4.6.2
C AND CORRESPONDING TO THE DEVELOPMENT IN SUBSECTION A.1.3.4.
C
C DEFINE SIZE OF VECTOR CONTAINING UNIT LOAD OVER ALL DOF IN CENTRAL
C REGION OF PLATE  WIDTH OF LOADED REGION IS .7465 IN.,LENGTH IS .9 IN.
      DIMENSION ALOAD(54)
C
      COMMON /BAS/ NDT,NET,MN,NB,NIRREG,MNC
      COMMON/TIM/DELTAT,TIMEF,TIME,ITIMEF,INCRT,IOUT,ITIME
      COMMON /INOUT/ MREAD,MWRITE,MPUNCH
C
C USE DATA STATEMENTS FOR VALUES WHICH WILL NOT CHANGE THROUGHOUT RUN.
C
C DEFINE TIME AT WHICH LOAD DROPS TO ZERO AND MAX VALUE OF IMPULSE.
      DATA TEND/25.D-6/,AMAXP/21934.4DO/
C DEFINE FIRST AND LAST DOF IN LOADED REGION.
      DATA IDOF/1/,LDOF/54/
C DEFINE VALUES OF UNIT LOAD AT EACH DOF IN LOADED REGION.
      DATA ALOAD/2*0 DO,.4210312D-1,.2626182D-2,.3157734D-2,.1969637D-3
     1        ,2*0.DO,.8420625D-1, .0000000D+0, .6315468D-2, .0000000D+0
     2        ,2*0.DO, 4210312D-1,-.2626182D-2, .3157734D-2,-.1969637D-3
     3        ,2*0.DO,.8420625D-1, .5252365D-2, .0000000D+0, .0000000D+0
     4        ,2*0.DO,.1684125D+0, .0000000D+0,  0000000D+0, .0000000D+0
     5        ,2*0.DO,.8420625D-1,-.5252365D-2, .0000000D+0, .0000000D+0
     6        ,2*0 DO, 4210312D-1, .2626182D-2,-.3157734D-2,-.1969637D-3
     7        ,2*0 DO, 8420625D-1, .0000000D+0,-.6315468D-2, .0000000D+0
     8       ,2*0.DO,.4210312D-1,-.2626182D-2,-.3157734D-2, .1969637D-3/
C ZERO OUT LOAD VECTOR.
      DO 1 I=1,NDT
    1 ELV(I)=0 ODO
C IF THIS TIME STEP IS BEYOND LAST TIME OF IMPULSE RETURN WITH
C EXTERNAL LOAD VECTOR SET TO ZERO.
      TYM=(ITT-1)*DELTAT
      IF(TYM   GT. TEND) RETURN
C INTERPOLATE TO FIND MAGNITUDE OF IMPULSE AT THIS TIME.
      AMULT=AMAXP*(TEND-TYM )/TEND
C FILL IN VALUES OF LOAD IN CENTRAL REGION OF PLATE.
      DO 2 I=IDOF,LDOF
      J=I-IDOF+1
    2 ELV(I)=AMULT*ALOAD(J)
C OUTPUT LOADS IF THIS IS A REGULAR PRINTING CYCLE.
```

```
      III=ITIME/INCRT*INCRT-ITIME
      IF(III.NE.0) RETURN
      NNT=NDT/6
      WRITE(MWRITE,100) TYM ,(J,J=1,6)
100   FORMAT(1H0/1H0,10X,'TIME=',D16.6,' SEC.'/1H ,33(1H.),'EXTERNALLY A
     +PPLIED USER SPECIFIED LOADING',33(1H.)/1H ,6X,'NODE',6(7X,'DOF#',
     & I1,3X)/8X,3(10X,'(LBS)'),1X,3(7X,'(IN-LBS)'))
      I1=1
      DO 3 I=1,NNT
      I2=6*I
      WRITE(MWRITE,101) I,(ELV(J),J=I1,I2)
      I1=I2+1
    3 CONTINUE
  101 FORMAT(6X,I3,2X,6D15.6)
      RETURN
      END
      SUBROUTINE INCOND(DISP,VEL,NP,NODE,XGI,YGI)
      IMPLICIT REAL*8(A-H,O-Z)
C
C    ASRL TR 154-14.......ORIGINAL REPORT VERSION OF PROGRAM
C COPYRIGHT (C) MASSACHUSETTS INSTITUTE OF TECHNOLOGY 1980
C
C THIS SUBROUTINE IS USED EXCLUSIVELY BY THE PLATE PROGRAM
C
      DIMENSION DISP(1),VEL(1),NP(4,1),NODE(1),XGI(1),YGI(1)
      COMMON/BAS/NDT,NET,MN,NB,NIRREG,MNC
C
C THIS USER-PREPARED SUBROUTINE IS USED TO SPECIFY THE INITIAL (TIME
C ZERO) CONDITIONS ON THE PLATE NODAL DISPLACEMENT AND VELOCITIES,
C (Q ZERO)* AND (Q DOT ZERO)*, RESPECTIVELY, REQUIRED FOR THE FINITE-
C DIFFERENCE SOLUTION OF THE GOVERNING EQUATIONS OF MOTION (SEE
C SUBSECTION A.3).
C
C   DUMMY ROUTINE - SETS VELOCITY AND DISPLACEMENT TO ZERO INITIALLY.
      DO 10 I=1,NDT
      DISP(I)=0.0D0
   10 VEL(I)=0.0D0
      RETURN
      END
```

The FORTRAN listing of the main program for the full plate example is given on pages 175 and 176. For the half plate and the quarter plate models, the associated main programs are the same except for the following DIMENSION statement:

```
      DIMENSION DELD(390),DIS(390),DISP(390),DISM1(390),DISM2(390),     MAIN0065
     2FLN(390),FLVA(390),FLVM(390),FLVP(390),ELV(390),VEL(390),ICOL(390)MAIN0066
     3,INUM(390),KROW(390),NDEX(390),STOR(390),STF(14037),AMASS(14037), MAIN0067
     4NP(4,48),NODE(1152),TAUSS(48,12,9),TAUSE(48,12,9),TAUEE(48,12,9), MAIN0068
     5EPSSI(9,48),EPSSO(9,48),EPEEI(9,48),EPEEO(9,48),EPSEI(9,48),      MAIN0069
     6EPSEO(9,48),NBC(60),BC(60),RFM(60,60),ILAST(60),UCF1(60),UCF2(60),MAIN0070
     7XG(65),YG(65),ZG(65),XGI(65),YGI(65),TAGSS(1,12,3),LNXS(1),       MAIN0071
     8XSPROP(7,1),MATXS(1),TSGEE(1,12,3),LNYS(1),YSPROP(7,1),MATYS(1),  MAIN0072
     9LNRS(1),ISRS(1),SC(5,1),NVSA(6 ),NCON(4,6 )                       MAIN0073
```

For the half-plate example, replace the original above DIMENSION statement with this DIMENSION statement:

```
      DIMENSION DELD(234),DIS(234),DISP(234),DISM1(234),DISM2(234),     MAIN1067
     2FLN(234),FLVA(234),FLVM(234),FLVP(234),ELV(234),VEL(234),ICOL(234)MAIN1068
     3,INUM(234),KROW(234),NDEX(234),STOR(234),STF(5643),AMASS(5643),   MAIN1069
     4NP(4,24),NODE(576),TAUSS(24,12,9),TAUSE(24,12,9),TAUEE(24,12,9),  MAIN1070
     5EPSSI(9,24),EPSSO(9,24),EPEEI(9,24),EPEEO(9,24),EPSEI(9,24),      MAIN1071
     6EPSEO(9,24),NBC(75),BC(75),RFM(69,48),ILAST(69),UCF1(69),UCF2(69),MAIN1072
     7XG(39),YG(39),ZG(39),XGI(39),YGI(39),TAGSS(1,12,3),LNXS(1),       MAIN1073
     8XSPROP(7,1),MATXS(1),TSGEE(1,12,3),LNYS(1),YSPROP(7,1),MATYS(1),  MAIN1074
     9LNRS(1),ISRS(1),SC(5,1),NVSA(6 ),NCON(4, 6)                       MAIN1075
```

For the quarter-plate example, replace the original DIMENSION statement with this DIMENSION statement:

```
      DIMENSION DELD(126),DIS(126),DISP(126),DISM1(126),DISM2(126),     MAIN2065
     2FLN(126),FLVA(126),FLVM(126),FLVP(126),ELV(126),VEL(126),ICOL(126)MAIN2066
     3,INUM(126),KROW(126),NDEX(126),STOR(126),STF(2889),AMASS(2889),   MAIN2067
     4NP(4,12),NODE(288),TAUSS(12,12,9),TAUSE(12,12,9),TAUEE(12,12,9),  MAIN2068
     5EPSSI(9,12),EPSSO(9,12),EPEEI(9,12),EPEEO(9,12),EPSEI(9,12),      MAIN2069
     6EPSEO(9,24),NBC(75),BC(75),RFM(44,48),ILAST(44),UCF1(44),UCF2(44),MAIN2070
     7XG(21),YG(21),ZG(21),XGI(21),YGI(21),TAGSS(1,12,3),LNXS(1),       MAIN2071
     8XSPROP(7,1),MATXS(1),TSGEE(1,12,3),LNYS(1),YSPROP(7,1),MATYS(1),  MAIN2072
     9LNRS(1),ISRS(1),SC(5,1),NVSA(6 ),NCON(4, 6)                       MAIN2073
```

## 7.2 Modeling and Input Data

For illustrative purposes, three different structural models of the narrow-plate specimen depicted in Fig. 16 are described in the following. First, the entire plate is modeled by 48 LLC plate elements as shown in Fig. 17. Next, since symmetry exists about the y axis, the half-plate is modeled with the same basic finite-element breakdown, with symmetry about the y axis invoked; this results in a 24-element model (Fig. 17). Finally, advantage is taken of the fact that there are, in fact, two axes of symmetry; hence, the third model is the 12-element quarter-plate model shown also in Fig. 17. For each of these three models, the selected numbering of elements and nodes is shown in Fig. 17.

Pilot calculations show that the largest natural frequency in the linear-system mathematical model of the whole plate is $\omega_{max} = 3,052,755$ rad/sec. Hence, if the timewise central-difference model were used to predict the linear transient response of this model, a $\Delta t$ not exceeding $\Delta t = 2/\omega_{max} = 0.655$ μsec would be required. However, since the PLATE 1 program employs the Houbolt operator, a reasonable choice is $\Delta t = 2.5$ μsec, as discussed in Subsection 4.8.2; reasonable behavior for $\Delta t \lesssim 6[0.8(2/\omega_{max})] \doteq 3.14$ μsec would be expected.

Solution output data will be requested at the end of every 10 time steps or cycles; however, for conciseness these data will be included here only for the end of time cycles (or time steps) 0, 1, 2, 10, 20, 140, and 150 (last). The desired solution output data are indicated as the following print options (Card 51 of Subsection 4.2):

- IOP1 — Displacements and current (global) X,Y,Z coordinates for each node.
- IOP2 — Strain (Green) components as well as principal (tensile) strain and direction at the upper and lower surface evaluated at the centroid of each element.
- IOP4 — Strain (Green) components, principal (tensile) strain and direction, and elongations (in user-specified directions) at additional spanwise locations (evaluated at both upper and lower surfaces at each spanwise location) as specified by the user.

| | |
|---|---|
| IOP5 | Reaction forces at all nodes where displacement constraint conditions have been imposed. |
| IOP7 | System energies (i.e., work input to structure, structure kinetic energy, structure elastic energy, structure plastic energy, and energy stored in elastic restoring springs). |
| IOP8 | Strain (Green) components, principal (tensile) strain and direction at the upper and lower plate surfaces at user-specified nodes (by nodal averaging). |

Print options IOP3 and IOP6 are not desired; hence, set these quantities equal to 0 on Card 51 (of Subsection 4.2). Summarized concisely as follows are the affected element or node numbers for each of the three finite-element models (see Fig. 17):

| Output Option | Full-Plate Model | Half-Plate Model | Quarter-Plate Model |
|---|---|---|---|
| IOP1 | All Nodes | All Nodes | All Nodes |
| IOP2 | All Elements | All Elements | All Elements |
| IOP4 | Elements 2,6,10,14, 26,34,38,42 and 46 | Elements 1,3,5,7, 11,17,19,21 and 23 | Elements 1,5,7, 9 and 11 |
| IOP5 | Degrees of Freedom 1 through 30 inclusive and 361 through 390 inclusive | Degrees of Freedom 1 through 19 inclusive, 22,24, 37,40,42,55,58,60, 73,76,78,91,94,96, 109,112,114,127, 130,132,145,148, 150,163,166,168, 181,184,186,199, 202,204, and 217 through 234 inclusive | Degrees of Freedom+ 1,2,4,5,6,8,11,12, 14,17,18,19,22,24, 37,40,42,55,58,60, 73,76,78,91,94,96, and 109 through 126 inclusive |

+See Fig. 17c.

| | Full-Plate Model | Half-Plate Model | Quarter-Plate Model |
|---|---|---|---|
| IOP7 | Entire Structure | Entire Structure | Entire Structure |
| IOP8 | Nodes 33,35,38,40, 48,50 | Nodes 19,21,22,24, 28,30 | Nodes 1,3,4,6,10, 12 |

The associated input data on each card are described for the full-plate, the half-plate, and the quarter-plate model in Subsections 7.2.1, 7.2.2, and 7.2.3, respectively. However, solution output data are presented in Subsection 7.3 only for the full-plate model.

### 7.2.1 Input Data for the Full-Plate Model

The values to be punched on the data cards are as follows (see Fig. 17a):

Card 1 8I10

MAXEL = 48
MNSL = 3
MNXST = 1+
MNYST = 1+
NBE = 1+
MBWE = 1+
MNC = 14037

Card 2 20I4

IMESH = 0
IMASS = 2
IMCONT = 0
IPUNCH = 0

Card 3 20I4

NEAD = 4
NECD = 12

Card 4a 5D16.7

EPAN = 0.1D8
ANUP = .3333333D0

+As noted on page 82, these quantities must be set equal to 1 in accordance with FORTRAN rules because they are used as dimensioning variables; they must not be set equal to zero.

DENSP = 0.25384D-3

TH = 0.102D0

XDIST = 1.4970D0

Card 4b 5D16.7

YDIST = 8.0D0

ZPOS = 0.0D0

Card 5 5D16.7

XG(1) = 0.0D0

XG(2) = .37425D0

XG(3) = .7485D0

XG(4) = 1.12275D0

XG(5) = 1.4970D0

Card 6a 5D16.7

YG(1) = 0.0D0

YG(2) = .775D0

YG(3) = 1.55D0

YG(4) = 2.325D0

YG(5) = 3.1D0

Card 6b 5D16.7

YG(6) = 3.55D0

YG(7) = 4.0D0

YG(8) = 4.45D0

YG(9) = 4.9D0

YG(10) = 5.675D0

Card 6c 5D16.7

YG(11) = 6.45D0

YG(12) = 7.225D0

YG(13) = 8.0D0

Card 7 20I4

NCSB(1) = 2

NCSB(2) = 0

NCSB(3) = 2

NCSB(4) = 0

| | | |
|---|---|---|
| Card 8 | | 20I4 |
| NAST | = 0 | |
| NCST | = 0 | |
| Card 25* | | 20I4 |
| NAXS | = 0 | |
| NAYS | = 0 | |
| Card 44* | | 20I4 |
| NMAT | = 1 | |
| Card 45 | | 20I4 |
| NSUB | = 3 | |
| Card 46 | | 5D16.7 |
| SIG(1) | = 44.2D3 | |
| SIG(2) | = 49.2D3 | |
| SIG(3) | = 76.4D3 | |
| Card 47 | | 5D16.7 |
| EPS(1) | = .00442D0 | |
| EPS(2) | = .0760D0 | |
| EPS(3) | = 0.615D0 | |
| Card 48 | | 5D16.7 |
| DSR(1) | = 6500.D0 | |
| PSR(1) | = 4.0D0 | |
| Card 49 | | 20I4 |
| ITIMEF | = 150 | |
| INCRT | = 1 | |
| IOUT | = 1 | |
| Card 50 | | 5D16.7 |
| DELTAT | = 2.5D-06 | |
| TIMEF | = 3.75D-04 | |
| Card 51 | | 20I4 |
| IOP1 | = 1 | |
| IOP2 | = 1 | |

*For omitted cards, see page 251.

IOP3 = 0

IOP4 = 1

IOP5 = 1

IOP6 = 0

IOP7 = 1

IOP8 = 1

| | |
|---|---|
| Card 54* | 20I4 |
| NASP = 11 | |
| Card 55a | I4,2D16.7 |
| LNASP(1) = 2 | |
| SLASP(1) = 1.0D0 | |
| ELASP(1) = 0.0D0 | |
| Card 55b | I4,2D16.7 |
| LNASP(2) = 2 | |
| SLASP(2) = 1.0D0 | |
| ELASP(2) = .2580645D0 | |
| Card 55c | I4,2D16.7 |
| LNASP(3) = 6 | |
| SLASP(3) = 1.0D0 | |
| ELASP(3) = .2903226D0 | |
| Card 55d | I4,2D16.7 |
| LNASP(4) = 10 | |
| SLASP(4) = 1.0D0 | |
| ELASP(4) = .3225806D0 | |
| Card 55e | I4,2D16.7 |
| LNASP(5) = 14 | |
| SLASP(5) = 1.0D0 | |
| ELASP(5) = .3548387D0 | |
| Card 55f | I4,2D16.7 |
| LNASP(6) = 26 | |
| SLASP(6) = 1.0D0 | |
| ELASP(6) = 0.0D0 | |

*For omitted cards, see page 251.

| | |
|---|---|
| Card 55g | I4,2D16.7 |
| LNASP(7) = 34 | |
| SLASP(7) = 1.0D0 | |
| ELASP(7) = .6451613D0 | |
| Card 55h | I4,2D16.7 |
| LNASP(8) = 38 | |
| SLASP(8) = 1.0D0 | |
| ELASP(8) = .6774194D0 | |
| Card 55i | I4,2D16.7 |
| LNASP(9) = 42 | |
| SLASP(9) = 1.0D0 | |
| ELASP(9) = .7096774D0 | |
| Card 55j | I4,2D16.7 |
| LNASP(10) = 46 | |
| SLASP(10) = 1.0D0 | |
| ELASP(10) = .7419355D0 | |
| Card 55k | I4,2D16.7 |
| LNASP(11) = 46 | |
| SLASP(11) = 1.0D0 | |
| ELASP(11) = 1.0D0 | |
| Card 56 | 20I4 |
| NASPE = 0 | |
| Card 61* | 20I4 |
| NNSA = 6 | |
| Card 62 | 20I4 |
| NVSA(1) = 33 | |
| NVSA(2) = 35 | |
| NVSA(3) = 38 | |
| NVSA(4) = 40 | |
| NVSA(5) = 48 | |
| NVSA(6) = 50 | |
| Card 63 | 20I4 |
| NELES = 0 | |

*For omitted cards, see page 251.

Cards 9 through 16 are omitted because NAST on Card 8 is zero (that is, there are no complete X-direction stiffeners).

Cards 17 through 24 are omitted because NCST on Card 8 is zero (that is, there are no complete Y-direction stiffeners).

Cards 26 through 34 are omitted because NAXS on Card 24 is zero (that is, there are no additional X-direction stiffeners).

Cards 35 through 43 are omitted because NAYS on Card 24 is zero (that is, there are no additional Y-direction stiffeners).

Cards 52 and 53 are omitted because IOP3 = 3 on Card 51.

Card 57 is omitted because NASPE = 0 on Card 55. Cards 58 through 60 are omitted because IOP5 = 0 on Card 51.

The following is the complete input deck for this example:

```
      48          3          1          1          1          1     14037
  0   2   0   0
  4  12
         0.1D8     .333333D0     .25384D-3       .102D0      1.497D0
        8 0D0        0.0D0
        0.0D0      .3742D0        .7485D0     1.12275D0      1.497D0
        0 0D0      .7750D0         1.55D0       2.325D0        3.1D0
        3.55D0       4.0D0         4.45D0         4.9D0      5.675D0
        6.45D0     7.225D0          8.0D0
  2   0   2   0
  0   0
  0   0
  1
  3
       44.2D3       49.2D3        76.4D3
     .00442D0      .0760D0        0.615D0
      6500.D0        4.0D0
150   1   1
   2.5      D-06      3.75D-04
  1   1   0   1   1   0   1   1
 11
  2          1.0D0        0 0D0
  2          1.0D0     .258064 5D0
  6          1 0D0     .290322 6D0
 10          1 0D0     .322580 6D0
 14          1.0D0     .354838 7D0
 26          1 0D0        0.0D0
 34          1.0D0     .645161 3D0
 38          1.0D0     .677419 4D0
 42          1.0D0     .709677 4D0
 46          1.0D0     .741935 5D0
 46          1.0D0        1.0D0
  0
  6
 33  35  38  40  48  50
  0
```

### 7.2.2 Input Data for the Half-Plate Model*

The values to be punched on the data cards are as follows (see Fig. 17b):

Card 1 8I10

MAXEL = 24

MNSL = 3

MNXST = 1

MNYST = 1

NBE = 1

MBWE = 1

MNC = 5643

Card 2 20I4

IMESH = 0

IMASS = 2

IMCONT = 0

IPUNCH = 0

Card 3 20I4

NEAD = 2

NECD = 12

Card 4a 5D16.7

EPAN = 0.1D8

ANUP = .3333333D0

DENSP = 0.25384D-3

TH = 0.102D0

XDIST = 1.4970D0

Card 4b 5D16.7

YDIST = 8.0D0

ZPOS = 0.0D0

Card 5 5D16.7

XG(1) = 0.0D0

XG(2) = .37425D0

XG(3) = .7485D0

*Certain input cards are omitted; see page 251.

Card 6a 5D16.7

YG(1) = 0.0D0

YG(2) = .775D0

YG(3) = 1.55D0

YG(4) = 2.325D0

YG(5) = 3.1D0

Card 6b 5D16.7

YG(6) = 3.55D0

YG(7) = 4.0D0

YG(8) = 4.45D0

YG(9) = 4.9D0

YG(10) = 5.675D0

Card 6c 5D16.7

YG(11) = 6.45D0

YG(12) = 7.225D0

YG(13) = 8.0D0

Card 7 20I4

NCSB(1) = 2

NCSB(2) = 0

NCSB(3) = 2

NCSB(4) = 1

Card 8 20I4

NAST = 0

NCST = 0

Card 25 20I4

NAYS = 0

NAYS = 0

Card 44 20I4

NMAT = 1

Card 45 20I4

NSUB = 3

| | | |
|---|---|---|
| Card 46 | | 5D16.7 |
| SIG(1) | = 44.2D3 | |
| SIG(2) | = 49.2D3 | |
| SIG(3) | = 76.4D3 | |
| Card 47 | | 5D16.7 |
| EPS(1) | = .00442D0 | |
| EPS(2) | = .0760D0 | |
| EPS(3) | = 0.615D0 | |
| Card 48 | | 5D16.7 |
| DSR(1) | = 6500.D0 | |
| PSR(1) | = 4.0D0 | |
| Card 49 | | 20I4 |
| ITIMEF | = 150 | |
| INCRT | = 1 | |
| IOUT | = 1 | |
| Card 50 | | 5D16.7 |
| DELTAT | = 2.5D-06 | |
| TIMEF | = 3.75D-04 | |
| Card 51 | | 20I4 |
| IOP1 | = 1 | |
| IOP2 | = 1 | |
| IOP3 | = 0 | |
| IOP4 | = 1 | |
| IOP5 | = 1 | |
| IOP6 | = 0 | |
| IOP7 | = 1 | |
| IOP8 | = 1 | |
| Card 54 | | 20I4 |
| NASP | = 11 | |
| Card 55a | | I4,2D16.7 |
| LNASP(1) | = 1 | |
| SLASP(1) | = 0.0D0 | |
| ELASP(1) | = 0.0D0 | |

Card 55b I4,2D16.7

LNASP(2) = 1

SLASP(2) = 0.0D0

ELASP(2) = .2580645D0

Card 55c I4,2D16.7

LNASP(3) = 3

SLASP(3) = 0.0D0

ELASP(3) = .2903226D0

Card 55d I4,2D16.7

LNASP(4) = 5

SLASP(4) = 0.0D0

ELASP(4) = .3225806D0

Card 55e I4,2D16.7

LNASP(5) = 7

SLASP(5) = 0.0D0

ELASP(5) = .3548387D0

Card 55f I4,2D16.7

LNASP(6) = 11

SLASP(6) = 0.0D0

ELASP(6) = 1.0D0

Card 55g I4,2D16.7

LNASP(7) = 17

SLASP(7) = 0.0D0

ELASP(7) = .6451613D0

Card 55h I4,2D16.7

LNASP(8) = 19

SLASP(8) = 0.0D0

ELASP(8) = .6774194D0

Card 55i I4,2D16.7

LNASP(9) = 21

SLASP(9) = 0.0D0

ELASP(9) = .7096774D0

Card 55j I4,2D16.7

LNASP(10) = 23

SLASP(10) = 0.0D0

ELASP(10) = .7419355D0

Card 55k I4,2D16.7

LNASP(11) = 23

SLASP(11) = 0.0D0

ELASP(11) = 1.0D0

Card 56 20I4

NASPE = 0

Card 61 20I4

NNSA = 6

Card 62 20I4

NVSA(1) = 19

NVSA(2) = 21

NVSA(3) = 22

NVSA(4) = 24

NVSA(5) = 28

NVSA(6) = 30

Card 63 20I4

NELES = 0

The following is the complete input deck for this example:

```
       24          3          1          1          1          1       5643
 0   2   0   0
 2  12
          0.1D8       .3333333D0        .25384D-3          .102D0         .748500D0
          8.0D0          0.0D0
          0.0D0        .37425D0          .7485D0
          0.0D0        .7750D0           1.55D0          2.325D0             3.1D0
          3.55D0         4.0D0           4.45D0            4.9D0           5.675D0
          6.45D0       7.225D0            8.0D0
 2   0   2   1
 0   0
 0   0
 1
 3
          44 2D3        49.2D3           76.4D3
         0044200       .0760D0          0.615D0
         6500.D0         4.0D0
150   1   1
   2.5      D-06      3.75D-04
 1   1   0   1   1   0   1   1
11
 1             .0D0          0.0D0
 1             .0D0      .2580645D0
 3             .0D0      .2903226D0
 5             .0D0      .3225806D0
 7             0D0       .3548387D0
11             .0D0          1 0D0
17             .0D0       6451613D0
19             .0D0      .6774194D0
21             .0D0      .7096774D0
23             .0D0      .7419355D0
23             .0D0          1.0D0
 0
 6
19  21  22  24  28  30
 0
```

### 7.2.3 Input Data for the Quarter-Plate Model*

The values to be punched on the data cards are as follows (see Fig. 17c):

Card 1 — 8I10

MAXEL = 12
MNSL = 3
MNXST = 1
MNYST = 1
NBE = 1
MBWE = 1
MNC = 2889

Card 2 — 20I4

IMESH = 0
IMASS = 2
IMCONT = 0
IPUNCH = 0

Card 3 — 20I4

NEAD = 2
NECD = 6

Card 4a — 5D16.7

EPAN = 0.1D8
ANUP = .3333333D0
DENSP = 0.25384D-3
TH = 0.102D0
XDIST = .74850D0

Card 4b — 5D16.7

YDIST = 4.0D0
ZPOS = 0.0D0

Card 5 — 5D16.7

XG(1) = 0.0D0
XG(2) = .37425D0
XG(3) = .7485D0

---

* Certain input cards are omitted; see page 251.

Card 6a 5D16.7

YG(1) = 0.0D0

YG(2) = .4500D0

YG(3) = 0.90D0

YG(4) = 1.675D0

YG(5) = 2.45D0

Card 6b 5D16.7

YG(6) = 3.225D0

YG(7) = 4.0D0

Card 7 20I4

NCSB(1) = 1

NCSB(2) = 0

NCSB(3) = 2

NCSB(4) = 1

Card 8 20I4

NAST = 0

NCST = 0

Card 25 20I4

NAXS = 0

NAYS = 0

Card 44 20I4

NMAT = 1

Card 45 20I4

NSUB = 3

Card 46 5D16.7

SIG(1) = 44.2D3

SIG(2) = 49.2D3

SIG(3) = 76.4D3

Card 47 5D16.7

EPS(1) = .00442D0

EPS(2) = .0760D0

EPS(3) = 0.615D0

| | | |
|---|---|---|
| Card 48 | | 5D16.7 |
| DSR(1) | = 6500.D0 | |
| PSR(1) | = 4.0D0 | |
| Card 49 | | 20I4 |
| ITIMEF | = 150 | |
| INCRT | = 1 | |
| IOUT | = 1 | |
| Card 50 | | 5D16.7 |
| DELTAT | = 2.5D-06 | |
| TIMEF | = 3.75D-04 | |
| Card 51 | | 20I4 |
| IOP1 | = 1 | |
| IOP2 | = 1 | |
| IOP3 | = 0 | |
| IOP4 | = 1 | |
| IOP5 | = 1 | |
| IOP6 | = 0 | |
| IOP7 | = 1 | |
| IOP8 | = 1 | |
| Card 54 | | 20I4 |
| NASP | = 6 | |
| Card 55a | | I4,2D16.7 |
| LNASP(1) | = 1 | |
| SLASP(1) | = 0.0D0 | |
| ELASP(1) | = 0.0D0 | |
| Card 55b | | I4,2D16.7 |
| LNASP(2) | = 5 | |
| SLASP(2) | = 0.0D0 | |
| ELASP(2) | = .6451613D0 | |
| Card 55c | | I4,2D16.7 |
| LNASP(3) | = 7 | |
| SLASP(3) | = 0.0D0 | |
| ELASP(3) | = .6774194D0 | |

Card 55d I4,2D16.7

LNASP(4) = 9

SLASP(4) = 0.0D0

ELASP(4) = .7096774D0

Card 55e I4,2D16.7

LNASP(5) = 11

SLASP(5) = 0.0D0

ELASP(5) = .7419355D0

Card 55f I4,2D16.7

LNASP(6) = 11

SLASP(6) = 0.0D0

ELASP(6) = 1.0D0

Card 56 20I4

NASPE = 0

Card 61 20I4

NNSA = 6

Card 62 20I4

NVSA(1) = 1

NVSA(2) = 3

NVSA(3) = 4

NVSA(4) = 6

NVSA(5) = 10

NVSA(6) = 12

Card 63 20I4

NELES = 0

The following is the complete input deck for this example:

```
      12          3          1          1          1          1       2889
  0   2   0   0
  2   6
        0.1D8      .3333333D0      .25384D-3        .102D0        .7485000
        4.0D0          0.0D0
        0.0D0       .37425D0        .7485D0
        0.0D0        .4500D0        0.90D0        1.675D0          2.45D0
      3.225D0          4.0D0
  1   0   2   1
  0   0
  0   0
  1
  3
       44.2D3         49.2D3         76.4D3
     .00442D0        .0760D0        0.615D0
      6500.D0          4.0D0
150   1   1
   2.5     D-06       3.75D-04
  1   1   0   1   1   0   1   1
  6
  1            0 0D0          0 0D0
  5            0.0D0       .6451613D0
  7            0.0D0       .6774194D0
  9            0.0D0       .7096774D0
 11            0.0D0       .7419355D0
 11            0.0D0          1.0D0
  0
  6
  1   3   4   6  10  12
  0
```

## 7.3 Solution Output Data for the Full-Plate Model

The following is the output for 150 cycles (375 μsec) of response of the transiently-loaded narrow plate specimen depicted in Fig. 16, for the full-plate finite-element model shown in Fig. 17.

The first segment of the output describes the initial geometry, material properties and boundary conditions of the plate as well as element numbers and global node numbers of the finite elements used to model the structure. Noted also are the constrained degrees of freedom and the properties of each sublayer of the mechanical-sublayer material model: (a) stress, (b) strain, (c) yield stress, and (d) weighting factor.

The next segment describes the output to be given at every regular printout cycle. This output consists of (a) the generalized displacements of and the global location of each nodal station, (b) the reaction forces at each constrained node, (c) the strain components as well as the principal (tensile) strain and direction at the centroid of each element, (d) strain components as well as principal strains and associated direction at 6 nodes as described, (e) strain components, elongations in specified directions, and principal strain and associated direction at each of 11 identified "additional locations", and (f) a tabulation of the various energies of the system. Also given are the generalized nodal forces from the externally-applied loading whenever those forces are non-zero.

In the interest of conciseness, only a portion of the called-for output is shown. Included are: (1) all input-verification information and (2) scheduled output at the end of time cycles 0, 1, 2, 10, 20, 140, and 150 (last).[+] This output listing is intended for use in verifying the (successful) adaptation of the PLATE 1 program to a user's computing facility.

[+] An auxiliary computer run was made with printout every cycle to give the output shown at the end of time steps 1 and 2.

```
PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) : USER INPUT FOR ARRAY DIMENSIONS
MAXEL  =        48
MNSL   =         3
MNXST  =         1
MNYST  =         1
NBE    =        60
MBWE   =        60
MNC    =     14037

AUTO-GENERATED FINITE-ELEMENT MESH INFORMATION FOR STIFFENED OR UNSTIFFENED FLAT-PLATE PROBLEM


    X-LENGTH(IN)                  =  0.1497000D+01
    Y-LENGTH(IN)                  =  0.8000000D+01
    Z-POSITION(IN)                =  0.0
    THICKNESS(IN)                 =  0.1020000D+00
    YOUNGS MODULUS(PSI)           =  0.1000000D+08
    POISSONS RATIO                =  0.3333333D+00
    DENSITY(LB-SEC**2/IN**4)      =  0.2538400D-03
    NO  OF ELEM  IN X-DIRECTION   =              4
    NO  OF ELEM  IN Y-DIRECTION   =             12
    NO  OF D O F /ELEM            =             24
    TOTAL NO  OF ELEMENTS         =             48
    TOTAL NO  OF D O F            =            390
    NO  OF X-DIR  STIFFENERS      =              0
    NO  OF Y-DIR  STIFFENERS      =              0
THE GLOBAL NODE NUMBERS ASSOCIATED WITH EACH ELEMENT ARE AS FOLLOWS :
```

| ELEMENT | GLOBAL NODE NUMBERS | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 7 | 6 |
| 2 | 2 | 3 | 8 | 7 |
| 3 | 3 | 4 | 9 | 8 |
| 4 | 4 | 5 | 10 | 9 |
| 5 | 6 | 7 | 12 | 11 |
| 6 | 7 | 8 | 13 | 12 |
| 7 | 8 | 9 | 14 | 13 |
| 8 | 9 | 10 | 15 | 14 |
| 9 | 11 | 12 | 17 | 16 |
| 10 | 12 | 13 | 18 | 17 |
| 11 | 13 | 14 | 19 | 18 |
| 12 | 14 | 15 | 20 | 19 |
| 13 | 16 | 17 | 22 | 21 |
| 14 | 17 | 18 | 23 | 22 |
| 15 | 18 | 19 | 24 | 23 |
| 16 | 19 | 20 | 25 | 24 |
| 17 | 21 | 22 | 27 | 26 |
| 18 | 22 | 23 | 28 | 27 |
| 19 | 23 | 24 | 29 | 28 |
| 20 | 24 | 25 | 30 | 29 |
| 21 | 26 | 27 | 32 | 31 |
| 22 | 27 | 28 | 33 | 32 |
| 23 | 28 | 29 | 34 | 33 |
| 24 | 29 | 30 | 35 | 34 |
| 25 | 31 | 32 | 37 | 36 |
| 26 | 32 | 33 | 38 | 37 |
| 27 | 33 | 34 | 39 | 38 |
| 28 | 34 | 35 | 40 | 39 |
| 29 | 36 | 37 | 42 | 41 |
| 30 | 37 | 38 | 43 | 42 |
| 31 | 38 | 39 | 44 | 43 |
| 32 | 39 | 40 | 45 | 44 |
| 33 | 41 | 42 | 47 | 46 |
| 34 | 42 | 43 | 48 | 47 |

```
35          43   44   49   48
36          44   45   50   49
37          46   47   52   51
38          47   48   53   52
39          48   49   54   53
40          49   50   55   54
41          51   52   57   56
42          52   53   58   57
43          53   54   59   58
44          54   55   60   59
45          56   57   62   61
46          57   58   63   62
47          58   59   64   63
48          59   60   65   64

BOUNDARY CONDITIONS (SEE WRITEUP FOR CONVENTION FOR SIDE NUMBER AND BOUNDARY CONDITION)

SIDE NUMBER      BOUNDARY CONDITION
     1           IDEALLY CLAMPED
     2           FREE
     3           IDEALLY CLAMPED
     4           FREE

THE FOLLOWING    60  DEGREES OF FREEDOM ARE CONSTRAINED
    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16   17   18   19   20
   21   22   23   24   25   26   27   28   29   30  361  362  363  364  365  366  367  368  369  370
  371  372  373  374  375  376  377  378  379  380  381  382  383  384  385  386  387  388  389  390

PRESENT RUN USES CONSISTENT MASS MATRIX

  MAX SIZE IS  14037

PROPERTIES FOR USER-SPECIFIED MATERIALS

PROPERTIES FOR MATERIAL NUMBER  1
SUBLAYER       STRESS POINT      STRAIN POINT      YIELD STRESS   WEIGHTING FACTOR
    1        0 4420000D+05    0.4420000D-02     0.4420000D+05    0.9937861D+00
    2        0 4920000D+05    0.7600000D-01     0.8488500D+06    0.1725692D-02
    3        0 7640000D+05    0.6150000D+00     0.6909200D+07    0.4488190D-02
STRAIN RATE PARAMETERS  D=   0 6500000D+04  P=    0.4000000D+01

TIMEWISE SOLUTION PARAMETERS
TIME-STEP SIZE=    0 2500000D-05  SEC
RUN WILL TERMINATE AT TIME =   0 3750000D-03  SEC. OR AT CYCLE NUMBER       150
REGULAR PRINTOUT WILL BE GIVEN EVERY   10  CYCLES

***** OUTPUT CONTROL INFORMATION *****

THE FOLLOWING RESULTS WILL BE GIVEN AT EVERY REGULAR PRINTOUT CYCLE:
     NODAL DISPLACEMENTS AND LOCATION
     REACTION FORCES AT CONSTRAINED NODES
     STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION AT THE FOLLOWING    6  NODES (OBTAINED BY NODAL AVERAGING):
            33   35   38   40   48   50
     STRAIN COMPONENTS, ELONGATION IN SPECIFIED DIRECTIONS, PRINCIPAL STRAIN AND DIRECTION AT THE FOLLOWING   11  ADDTL. POINTS:
     ADDTL. POINT  ON ELEM.    PSI-LOCATION     ETA-LOCATION   ELONG  DIR.-1(DEG)  ELONG. DIR -2(DEG)
             1          2     0 1000000D+01   0 0                0 0                 0.9000000D+02
             2          2     0.1000000D+01   0.2580645D+00      0 0                 0 9000000D+02
             3          6     0 1000000D+01   0 2903226D+00      0 0                 0 9000000D+02
             4         10     0 1000000D+01   0 3225806D+00      0 0                 0 9000000D+02
             5         14     0.1000000D+01   0 3548387D+00      0 0                 0.9000000D+02
             6         26     0 1000000D+01   0 0                0 0                 0.9000000D+02
             7         34     0 1000000D+01   0 6451613D+00      0.0                 0 9000000D+02
             8         38     0 1000000D+01   0 6774194D+00      0 0                 0.9000000D+02
             9         42     0 1000000D+01   0.7096774D+00      0 0                 0 9000000D+02
            10         46     0.1000000D+01   0 7419355D+00      0 0                 0.9000000D+02
            11         46     0.1000000D+01   0 1000000D+01      0.0                 0.9000000D+02
```

SYSTEM ENERGIES

ROUNDING ERROR PARAMETER IN FACTORING ROW 150 = 0.11560148 NO OF NEGATIVE DIAG= 0

HIGHEST NATURAL FREQUENCY(RAD/SEC)= 0.3052755D+07

HIGHEST EIGENVECTOR NORMALIZED BY LARGEST VALUE

| NODE | U | V | W | PSIX | PSIY | TWIST |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 3 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 4 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 5 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 6 | 0 33785D-39 | -0 15059D-39 | -0 14396D-04 | 0.42661D-03 | -0.16797D-03 | 0.49778D-02 |
| 7 | -0 32374D-39 | 0 49596D-40 | -0 19032D-05 | 0.18868D-03 | -0 22206D-04 | 0 22019D-02 |
| 8 | 0 32202D-39 | 0.91215D-52 | -0 20335D-10 | 0.15294D-03 | -0.23733D-09 | 0.17849D-02 |
| 9 | -0 32374D-39 | -0 49596D-40 | 0 19031D-05 | 0.18868D-03 | 0 22205D-04 | 0.22019D-02 |
| 10 | 0 33785D-39 | 0 15059D-39 | 0 14396D-04 | 0 42661D-03 | 0 16797D-03 | 0 49778D-02 |
| 11 | -0 10874D-38 | 0.38119D-39 | -0 36548D-04 | 0 10842D-02 | -0 62817D-03 | 0.18624D-01 |
| 12 | 0 10623D-38 | -0 12626D-39 | -0 48299D-05 | 0 48067D-03 | -0 83031D-04 | 0 82463D-02 |
| 13 | -0 10593D-38 | -0 37017D-51 | -0 51892D-10 | 0 38994D-03 | -0 88945D-09 | 0.66869D-02 |
| 14 | 0 10623D-38 | 0 12626D-39 | 0 48298D-05 | 0 48066D-03 | 0.83029D-04 | 0 82462D-02 |
| 15 | -0 10874D-38 | -0 38119D-39 | 0 36548D-04 | 0.10842D-02 | 0 62816D-03 | 0 18624D-01 |
| 16 | 0 30294D-38 | -0 81416D-39 | -0 11186D-03 | 0 33205D-02 | -0 19494D-02 | 0 57840D-01 |
| 17 | -0 29977D-38 | 0 26985D-39 | -0 14781D-04 | 0.14712D-02 | -0 25762D-03 | 0 25650D-01 |
| 18 | 0.29940D-38 | 0 11917D-50 | -0 15931D-09 | 0 11965D-02 | -0 27697D-08 | 0 20811D-01 |
| 19 | -0 29977D-38 | -0 26985D-39 | 0 14780D-04 | 0 14742D-02 | 0 25762D-03 | 0 25650D-01 |
| 20 | 0 30294D-38 | 0 81418D-39 | 0 11185D-03 | 0 33205D-02 | 0 19494D-02 | 0 57839D-01 |
| 21 | -0 81171D-38 | 0 15157D-38 | -0 57385D-03 | 0.17021D-01 | -0 73302D-02 | 0.21748D+00 |
| 22 | 0 81067D-38 | -0 50439D-39 | -0 75851D-04 | 0 75436D-02 | -0 96878D-03 | 0 96441D-01 |
| 23 | -0 81060D-38 | -0 35006D-50 | -0 81410D-09 | 0 61188D-02 | -0 10413D-07 | 0 78243D-01 |
| 24 | 0 81067D-38 | 0 50439D-39 | 0 75849D-04 | 0 75435D-02 | 0 96875D-03 | 0 96440D-01 |
| 25 | -0 81171D-39 | -0 15157D-38 | 0 57384D-03 | 0 17021D-01 | 0 73301D-02 | 0.21718D+00 |
| 26 | 0 18540D-37 | -0 16502D-38 | -0 45469D-03 | 0 13474D-01 | -0 25868D-01 | 0 76705D+00 |
| 27 | -0 18642D-37 | 0 55309D-39 | -0 60132D-04 | 0 59584D-02 | -0 34198D-02 | 0 33971D+00 |
| 28 | 0 18656D-37 | 0 85192D-50 | -0 64189D-09 | 0 48292D-02 | -0 36641D-07 | 0 27548D+00 |
| 29 | -0 18642D-37 | -0 55309D-39 | 0 60131D-04 | 0 59584D-02 | 0 34197D-02 | 0 33971D+00 |
| 30 | 0 18540D-37 | 0 16502D-38 | 0 45468D-03 | 0.13474D-01 | 0 25868D-01 | 0 76704D+00 |
| 31 | -0 22625D-37 | -0 87298D-50 | 0 16310D-12 | -0 48016D-11 | -0 33729D-01 | 0 10000D+01 |
| 32 | 0.22801D-37 | 0 10055D-49 | 0 21428D-13 | -0 21355D-11 | -0 44595D-02 | 0 44272D+00 |
| 33 | -0 22820D-37 | -0.10604D-49 | 0 32050D-15 | -0 17284D-11 | -0 47737D-07 | 0.35897D+00 |
| 34 | 0 22801D-37 | 0 10072D-49 | -0 20312D-13 | -0 21070D-11 | 0 44594D-02 | 0 44272D+00 |
| 35 | -0 22625D-37 | -0 87759D-50 | -0 15775D-12 | -0 46809D-11 | 0 33729D-01 | 0 99999D+00 |
| 36 | 0 18540D-37 | 0 16502D-38 | 0 45469D-03 | -0 13474D-01 | -0 25868D-01 | 0 76705D+00 |
| 37 | -0 18642D-37 | -0 55309D-39 | 0 60132D-04 | -0 59584D-02 | -0 34198D-02 | 0.33971D+00 |
| 38 | 0 18653D-37 | 0 84830D-50 | 0 64189D-09 | -0 48292D-02 | -0 36641D-07 | 0 27548D+00 |
| 39 | -0 18642D-37 | 0 55309D-39 | -0 60131D-01 | -0 59584D-02 | 0 34197D-02 | 0.33971D+00 |
| 40 | 0 18540D-37 | -0 16502D-38 | -0 45468D-03 | -0 13474D-01 | 0 25868D-01 | 0 76704D+00 |
| 41 | -0 81171D-38 | -0 15157D-38 | 0 57385D-03 | -0 17021D-01 | -0 73302D-02 | 0 21748D+00 |
| 42 | 0 81067D-38 | 0 50439D-39 | 0 75851D-04 | -0 75436D-02 | -0 96878D-03 | 0 96441D-01 |
| 43 | -0 81060D-38 | -0 34672D-50 | 0 81410D-09 | -0 61108D-02 | -0 10413D-07 | 0 78243D-01 |
| 44 | 0 81067D-38 | -0 50439D-39 | -0 75849D-04 | -0 75435D-02 | 0 96875D-03 | 0 96440D-01 |
| 45 | -0 81171D-38 | 0 15157D-38 | -0 57384D-03 | -0 17021D-01 | 0 73301D-02 | 0 21748D+00 |
| 46 | 0 30291D-38 | 0 81418D-39 | 0 11186D-03 | -0 33205D-02 | -0 19494D-02 | 0 57840D-01 |
| 47 | -0 29977D-38 | -0 26985D-39 | 0 14781D-04 | -0 14712D-02 | -0 25762D-03 | 0 25650D-01 |
| 48 | 0 29940D-38 | 0 11731D-50 | 0 15931D-09 | -0 11965D-02 | -0 27697D-08 | 0 20811D-01 |
| 49 | -0 29977D-38 | 0 26985D-39 | -0 14780D-04 | -0 14742D-02 | 0 25762D-03 | 0 25650D-01 |
| 50 | 0 30294D-38 | -0 81418D-39 | -0 11185D-03 | -0 33205D-02 | 0 19491D-02 | 0 57839D-01 |
| 51 | -0 10874D-38 | -0 38119D-39 | 0 36548D-04 | -0 10842D-02 | -0.62817D-03 | 0 18624D-01 |
| 52 | 0 10623D-38 | 0 12626D-39 | 0 48299D-05 | -0.48067D-03 | -0.83031D-04 | 0.82463D-02 |
| 53 | -0 10593D-38 | -0 36061D-51 | 0 51892D-10 | -0 38994D-03 | -0 88944D-09 | 0 66869D-02 |
| 54 | 0 10623D-38 | -0 12626D-39 | -0 48298D-05 | -0 48066D-03 | 0 83029D-04 | 0 82462D-02 |
| 55 | -0 10874D-38 | 0 38119D-39 | -0 36548D-04 | -0 10842D-02 | 0 62816D-03 | 0 18624D-01 |
| 56 | 0 33785D-39 | 0 15059D-39 | 0 14396D-04 | -0 42661D-03 | -0 16797D-03 | 0 49778D-02 |
| 57 | -0.32374D-39 | -0 49596D-40 | 0 19032D-05 | -0 16868D-03 | -0.22206D-04 | 0.22019D-02 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 58 | 0.32202D-39 | 0 86867D-52 | 0.20335D-10 | -0.15294D-03 | -0.23733D-09 | 0.17849D-02 |
| 59 | -0 32374D-39 | 0 49596D-40 | -0.19031D-05 | -0.18868D-03 | 0.22205D-04 | 0.22019D-02 |
| 60 | 0 33785D-39 | -0.15059D-39 | -0 14396D-04 | -0 42661D-03 | 0 16797D-03 | 0.49778D-02 |
| 61 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 62 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 63 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 64 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 65 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 |

AFTER 106 ITERATIONS, CONVERGENCE RATIO= 0 9539402D-07
CENTRAL-DIFFERENCE BOUND ON TIME-STEP(SEC) FOR LINEAR SYSTEM= 0.6551460D-06
ACTUAL TIME-STEP BOUND (SEC) WOULD BE= 0 5241168D-06

TIME= 0.0 SEC.

....... . .. .. .... ...........EXTERNALLY APPLIED USER SPECIFIED LOADING..................................

| NODE | DOF#1 (LBS) | DOF#2 (LBS) | DOF#3 (LBS) | DOF#4 (IN-LBS) | DOF#5 (IN-LBS) | DOF#6 (IN-LBS) |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 2 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 3 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 6 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 |
| 7 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 8 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 9 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 10 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 11 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 12 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 |
| 13 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 14 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 15 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 16 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 17 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 |
| 18 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 19 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 20 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 |
| 21 | 0 0 | 0.0 | 0 923507D+03 | 0.576037D+02 | 0 692630D+02 | 0.432028D+01 |
| 22 | 0.0 | 0.0 | 0 184701D+04 | 0 0 | 0 138526D+03 | 0 0 |
| 23 | 0 0 | 0 0 | 0.184701D+04 | 0 0 | 0 138526D+03 | 0 0 |
| 24 | 0 0 | 0 0 | 0 184701D+04 | 0 0 | 0 138526D+03 | 0 0 |
| 25 | 0.0 | 0.0 | 0 923507D+03 | -0 576037D+02 | 0 692630D+02 | -0 432028D+01 |
| 26 | 0 0 | 0 0 | 0 184701D+04 | 0 115207D+03 | 0 0 | 0 0 |
| 27 | 0.0 | 0 0 | 0 369403D+04 | 0 0 | 0 0 | 0 0 |
| 28 | 0 0 | 0.0 | 0.369403D+04 | 0 0 | 0 0 | 0 0 |
| 29 | 0 0 | 0.0 | 0 369403D+04 | 0.0 | 0 0 | 0 0 |
| 30 | 0 0 | 0.0 | 0 184701D+04 | -0 115207D+03 | 0 0 | 0 0 |
| 31 | 0 0 | 0.0 | 0 184701D+04 | 0 115207D+03 | 0 0 | 0.0 |
| 32 | 0 0 | 0 0 | 0 369403D+04 | 0 0 | 0 0 | 0 0 |
| 33 | 0.0 | 0 0 | 0 369403D+04 | 0 0 | 0 0 | 0 0 |
| 34 | 0.0 | 0 0 | 0 369403D+04 | 0 0 | 0 0 | 0 0 |
| 35 | 0 0 | 0.0 | 0 184701D+04 | -0 115207D+03 | 0.0 | 0 0 |
| 36 | 0 0 | 0 0 | 0 184701D+04 | 0.115207D+03 | 0 0 | 0 0 |
| 37 | 0 0 | 0.0 | 0 369403D+04 | 0.0 | 0 0 | 0.0 |
| 38 | 0.0 | 0 0 | 0.369403D+04 | 0 0 | 0 0 | 0 0 |
| 39 | 0.0 | 0.0 | 0 369403D+04 | 0 0 | 0 0 | 0 0 |
| 40 | 0.0 | 0 0 | 0 184701D+04 | -0 115207D+03 | 0 0 | 0 0 |
| 41 | 0 0 | 0.0 | 0 923507D+03 | 0 576037D+02 | -0 692630D+02 | -0 432028D+01 |
| 42 | 0.0 | 0.0 | 0 184701D+04 | 0 0 | -0.138526D+03 | 0 0 |
| 43 | 0.0 | 0.0 | 0 184701D+04 | 0 0 | -0 138526D+03 | 0.0 |
| 44 | 0 0 | 0 0 | 0 184701D+04 | 0 0 | -0 138526D+03 | 0 0 |
| 45 | 0 0 | 0.0 | 0 923507D+03 | -0 576037D+02 | -0 692630D+02 | 0.432028D+01 |
| 46 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 47 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 48 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 49 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 50 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 51 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 52 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 53 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 54 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 55 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 |
| 56 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 |
| 57 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 58 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 59 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 |
| 60 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 61 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 62 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 63 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 64 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 65 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 |

***** INCR. NO.= 0 TIME= 0.0 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.11227D+01 | 0.0 | 0.0 |
| 5 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.14970D+01 | 0.0 | 0.0 |
| 6 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.77500D+00 | 0.0 |
| 7 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.77500D+00 | 0.0 |
| 8 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 74850D+00 | 0 77500D+00 | 0.0 |
| 9 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0 77500D+00 | 0.0 |
| 10 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.14970D+01 | 0 77500D+00 | 0.0 |
| 11 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 15500D+01 | 0.0 |
| 12 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 37425D+00 | 0 15500D+01 | 0.0 |
| 13 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.74850D+00 | 0 15500D+01 | 0 0 |
| 14 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 11227D+01 | 0 15500D+01 | 0.0 |
| 15 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 14970D+01 | 0 15500D+01 | 0.0 |
| 16 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 23250D+01 | 0 0 |
| 17 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 37425D+00 | 0 23250D+01 | 0.0 |
| 18 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 74850D+00 | 0 23250D+01 | 0.0 |
| 19 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 11227D+01 | 0 23250D+01 | 0.0 |
| 20 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 14970D+01 | 0 23250D+01 | 0.0 |
| 21 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 31000D+01 | 0.0 |
| 22 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 37425D+00 | 0 31000D+01 | 0.0 |
| 23 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 74850D+00 | 0 31000D+01 | 0 0 |
| 24 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 11227D+01 | 0.31000D+01 | 0 0 |
| 25 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 14970D+01 | 0 31000D+01 | 0 0 |
| 26 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 35500D+01 | 0.0 |
| 27 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.37425D+00 | 0 35500D+01 | 0.0 |
| 28 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 74850D+00 | 0 35500D+01 | 0.0 |
| 29 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 11227D+01 | 0 35500D+01 | 0.0 |
| 30 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.14970D+01 | 0 35500D+01 | 0.0 |
| 31 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+01 | 0 0 |
| 32 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 37425D+00 | 0 40000D+01 | 0.0 |
| 33 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.74850D+00 | 0 40000D+01 | 0.0 |
| 34 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 11227D+01 | 0 40000D+01 | 0.0 |
| 35 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.14970D+01 | 0 40000D+01 | 0 0 |
| 36 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 44500D+01 | 0 0 |
| 37 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 37425D+00 | 0 44500D+01 | 0.0 |
| 38 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.74850D+00 | 0 44500D+01 | 0.0 |
| 39 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 11227D+01 | 0 44500D+01 | 0.0 |
| 40 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.14970D+01 | 0 44500D+01 | 0.0 |
| 41 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 49000D+01 | 0.0 |
| 42 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 37425D+00 | 0 49000D+01 | 0.0 |
| 43 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 74850D+00 | 0.49000D+01 | 0.0 |
| 44 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0.49000D+01 | 0.0 |
| 45 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.14970D+01 | 0.49000D+01 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 46 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.56750D+01 | 0.0 |
| 47 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.56750D+01 | 0.0 |
| 48 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.56750D+01 | 0.0 |
| 49 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.56750D+01 | 0.0 |
| 50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.56750D+01 | 0.0 |
| 51 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.64500D+01 | 0.0 |
| 52 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.64500D+01 | 0.0 |
| 53 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.64500D+01 | 0.0 |
| 54 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.64500D+01 | 0.0 |
| 55 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.64500D+01 | 0.0 |
| 56 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.72250D+01 | 0.0 |
| 57 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.72250D+01 | 0.0 |
| 58 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.72250D+01 | 0.0 |
| 59 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.72250D+01 | 0.0 |
| 60 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.72250D+01 | 0.0 |
| 61 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.80000D+01 | 0.0 |
| 62 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.80000D+01 | 0.0 |
| 63 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.80000D+01 | 0.0 |
| 64 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.80000D+01 | 0.0 |
| 65 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 61 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 62 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 63 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 64 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 65 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 10 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 11 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 13 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 14 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 15 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 16 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 17 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 18 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 19 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 20 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 21 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 22 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 23 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 24 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 25 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 26 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 27 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 28 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 29 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 30 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 31 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 32 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 33 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 4500D+02 | 0.4500D+02 |
| 34 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.4500D+02 | 0 4500D+02 |
| 35 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 4500D+02 | 0.4500D+02 |
| 36 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 37 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 38 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 4500D+02 | 0 4500D+02 |
| 39 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.4500D+02 | 0 4500D+02 |
| 40 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 4500D+02 | 0 4500D+02 |
| 41 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.4500D+02 | 0 4500D+02 |
| 42 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 43 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 4500D+02 | 0.4500D+02 |
| 44 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 4500D+02 | 0.4500D+02 |
| 45 | 0 0 | 0 0 | C 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 46 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 47 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.4500D+02 | 0 4500D+02 |
| 48 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 4500D+02 | 0.4500D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 35 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 38 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.4500D+02 | 0 4500D+02 |
| 40 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 4500D+02 | 0 4500D+02 |
| 48 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 50 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |



STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 45000D+02 |
| 1 | OUTER | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.45000D+02 |
| 2 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 45000D+02 |
| 2 | OUTER | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 3 | INNER | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 45000D+02 |
| 3 | OUTER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 4 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45 )0D+02 |
| 4 | OUTER | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 45000D+02 |
| 5 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 5 | OUTER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 45000D+02 |
| 6 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 6 | OUTER | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 7 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 7 | OUTER | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 8 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 45000D+02 |
| 8 | OUTER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.45000D+02 |
| 9 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 45000D+02 |
| 9 | OUTER | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.45000D+02 |
| 10 | INNER | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.45000D+02 |
| 10 | OUTER | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 11 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 11 | OUTER | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |

SYSTEM ENERGIES(IN-LB)
WORK INPUT TO STRUCTURE = 0.0
STRUCTURE KINETIC ENERGY = 0.0
STRUCTURE ELASTIC ENERGY = 0.0
STRUCTURE PLASTIC ENERGY = 0.0
ENERGY STORED IN ELASTIC RESTRAINTS= 0 0

TIME= 0.250000D-05 SEC.

..................................EXTERNALLY APPLIED USER SPECIFIED LOADING..................................

| NODE | DOF#1 (LBS) | DOF#2 (LBS) | DOF#3 (LBS) | DOF#4 (IN-LBS) | DOF#5 (IN-LBS) | DOF#6 (IN-LBS) |
|---|---|---|---|---|---|---|
| 1 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 |
| 4 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 6 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 7 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 8 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 9 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 10 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 11 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 12 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 13 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 14 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 15 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 16 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 17 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 18 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 19 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 |
| 20 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 21 | 0 0 | 0 0 | 0 831156D+03 | 0 518434D+02 | 0 623367D+02 | 0 388825D+01 |
| 22 | 0.0 | 0.0 | 0 166231D+04 | 0 0 | 0.124673D+03 | 0 0 |
| 23 | 0 0 | 0 0 | 0 166231D+04 | 0 0 | 0 124673D+03 | 0 0 |
| 24 | 0.0 | 0 0 | 0 166231D+04 | 0 0 | 0 124673D+03 | 0 0 |
| 25 | 0 0 | 0.0 | 0 831156D+03 | -0 518434D+02 | 0 623367D+02 | -0 388825D+01 |
| 26 | 0.0 | 0.0 | 0 166231D+04 | 0 103687D+03 | 0.0 | 0 0 |
| 27 | 0 0 | 0.0 | 0 332462D+04 | 0 0 | 0 0 | 0 0 |
| 28 | 0 0 | 0.0 | 0 332462D+04 | 0 0 | 0.0 | 0 0 |
| 29 | 0 0 | 0.0 | 0 332462D+04 | 0 0 | 0 0 | 0 0 |
| 30 | 0 0 | 0.0 | 0 166231D+04 | -0 103687D+03 | 0 0 | 0 0 |
| 31 | 0.0 | 0 0 | 0 166231D+04 | 0 103687D+03 | 0 0 | 0 0 |
| 32 | 0 0 | 0 0 | 0 332462D+04 | 0 0 | 0 0 | 0 0 |
| 33 | 0 0 | 0.0 | 0 332462D+04 | 0 0 | 0 0 | 0 0 |
| 34 | 0 0 | 0.0 | 0 332462D+04 | 0 0 | 0 0 | 0 0 |
| 35 | 0 0 | 0 0 | 0 166231D+04 | -0 103687D+03 | 0 0 | 0 0 |
| 36 | 0 0 | 0.0 | 0 166231D+04 | 0 103687D+03 | 0 0 | 0.0 |
| 37 | 0.0 | 0.0 | 0.332462D+04 | 0.0 | 0 0 | 0 0 |
| 38 | 0 0 | 0 0 | 0.332462D+04 | 0 0 | 0 0 | 0.0 |
| 39 | 0.0 | 0 0 | 0 332462D+04 | 0.0 | 0 0 | 0 0 |
| 40 | 0 0 | 0.0 | 0 166231D+04 | -0 103687D+03 | 0.0 | 0 0 |
| 41 | 0 0 | 0.0 | 0 831156D+03 | 0 518434D+02 | -0.623367D+02 | -0 388825D+01 |
| 42 | 0 0 | 0 0 | 0 166231D+04 | 0 0 | -0 124673D+03 | 0 0 |
| 43 | 0 0 | 0.0 | 0 166231D+04 | 0 0 | -0 124673D+03 | 0 0 |
| 44 | 0.0 | 0.0 | 0 166231D+04 | 0 0 | -0 124673D+03 | 0 0 |
| 45 | 0 0 | 0.0 | 0 831156D+03 | -0 518434D+02 | -0 623367D+02 | 0 388825D+01 |
| 46 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 47 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 48 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 |
| 49 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 50 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 51 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 52 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 53 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 54 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 |
| 55 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 56 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 57 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 58 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 59 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 |
| 60 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 61 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 62 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 63 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 64 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 65 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

ROUNDING ERROR PARAMETER IN FACTORING ROW 150 = 0.28916102 NO OF NEGATIVE DIAG= 0

MAXIMUM BANDWIDTH OF 60 IS FOUND FOR (CONSTRAINED) DEGREE-OF-FREEDOM NUMBER 372

ROUNDING ERROR PARAMETER IN FACTORING ROW 150 = 0.34295915 NO OF NEGATIVE DIAG= 0

***** INCR. NO.= 1 TIME= 0.2500D-05 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |
| 4 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.11227D+01 | 0.0 | 0.0 |
| 5 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 14970D+01 | 0.0 | 0.0 |
| 6 | 0 0 | 0.0 | 0.51519D-05 | -0.14519D-04 | 0 55451D-04 | -0.16130D-03 | 0.0 | 0.77500D+00 | 0.51519D-05 |
| 7 | 0.0 | 0 0 | 0 43962D-05 | -0 14371D-05 | 0 47145D-04 | -0.16538D-04 | 0.37425D+00 | 0 77500D+00 | 0 43962D-05 |
| 8 | 0 0 | 0.0 | 0 43375D-05 | 0.44170D-16 | 0 46477D-04 | 0.51387D-15 | 0.74850D+00 | 0.77500D+00 | 0.43375D-05 |
| 9 | 0 0 | 0 0 | 0 43962D-05 | 0.14371D-05 | 0 47145D-04 | 0.16539D-04 | 0 11227D+01 | 0.77500D+00 | 0.43962D-05 |
| 10 | 0 0 | 0 0 | 0.51519D-05 | 0 14519D-04 | 0.55451D-04 | 0.16130D-03 | 0.14970D+01 | 0 77500D+00 | 0.51519D-05 |
| 11 | 0 0 | 0 0 | 0 19922D-04 | -0.46461D-04 | 0 27121D-03 | -0.69738D-03 | 0 0 | 0.15500D+01 | 0.19922D-04 |
| 12 | 0 0 | 0 0 | 0.17431D-04 | -0.38937D-05 | 0.23448D-03 | -0.64409D-04 | 0 37425D+00 | 0.15500D+01 | 0 17431D-04 |
| 13 | 0 0 | 0 0 | 0.17262D-04 | 0.13365D-15 | 0 23179D-03 | 0 21406D-14 | 0 74850D+00 | 0 15500D+01 | 0 17262D-04 |
| 14 | 0 0 | 0 0 | 0 17431D-04 | 0 38937D-05 | 0.23448D-03 | 0.64409D-04 | 0.11227D+01 | 0 15500D+01 | 0.17431D-04 |
| 15 | 0 0 | 0 0 | 0.19922D-04 | 0 46461D-04 | 0 27121D-03 | 0.69738D-03 | 0 14970D+01 | 0.15500D+01 | 0 19922D-04 |
| 16 | 0 0 | 0.0 | 0 99179D-04 | -0 19027D-03 | 0 12524D-02 | -0 27066D-02 | 0 0 | 0 23250D+01 | 0.99179D-04 |
| 17 | 0 0 | 0 0 | 0.88537D-04 | -0 12370D-04 | 0 11050D-02 | -0 20861D-03 | 0 37425D+00 | 0 23250D+01 | 0.88537D-04 |
| 18 | 0 0 | 0 0 | 0.87982D-04 | 0 42699D-16 | 0 10959D-02 | 0.53380D-14 | 0.74850D+00 | 0 23250D+01 | 0 87982D-04 |
| 19 | 0 0 | 0 0 | 0 88537D-04 | 0 12370D-04 | 0 11050D-02 | 0 20861D-03 | 0 11227D+01 | 0.23250D+01 | 0.88537D-04 |
| 20 | 0 0 | 0 0 | 0.99179D-04 | 0 19027D-03 | 0.12524D-02 | 0.27066D-02 | 0 14970D+01 | 0 23250D+01 | 0.99179D-04 |
| 21 | 0 0 | 0.0 | 0 13479D-02 | -0 14959D-03 | 0 10897D-01 | -0 94515D-02 | 0 0 | 0 31000D+01 | 0.13479D-02 |
| 22 | 0 0 | 0 0 | 0 13400D-02 | -0 12765D-04 | 0 10366D-01 | -0 50944D-03 | 0.37425D+00 | 0 31000D+01 | 0 13400D-02 |
| 23 | 0 0 | 0 0 | 0 13395D-02 | 0 64591D-15 | 0 10341D-01 | 0.77473D-14 | 0.74850D+00 | 0 31000D+01 | 0.13395D-02 |
| 24 | 0 0 | 0 0 | 0.13400D-02 | 0 12765D-04 | 0 10366D-01 | 0 50944D-03 | 0 11227D+01 | 0 31000D+01 | 0.13400D-02 |
| 25 | 0 0 | 0 0 | 0 13479D-02 | 0 14959D-03 | 0 10897D-01 | 0.94515D-02 | 0 14970D+01 | 0 31000D+01 | 0 13479D-02 |
| 26 | 0 0 | 0.0 | 0 25893D-02 | -0 45265D-04 | -0 51140D-03 | 0.12476D-02 | 0 0 | 0.35500D+01 | 0.25893D-02 |
| 27 | 0 0 | 0.0 | 0 25857D-02 | 0 11586D-04 | -0 42594D 03 | -0 54027D-04 | 0.37425D+00 | 0 35500D+01 | 0 25857D-02 |
| 28 | 0 0 | 0 0 | 0 25861D-02 | 0 29626D-15 | -0 42629D-03 | 0 27136D-13 | 0 74850D+00 | 0 35500D+01 | 0 25861D-02 |
| 29 | 0 0 | 0 0 | 0 25857D-02 | -0 11586D-04 | -0 42594D-03 | 0 54027D-04 | 0 11227D+01 | 0 35500D+01 | 0.25857D-02 |
| 30 | 0 0 | 0 0 | 0 25893D-02 | 0 45265D-04 | -0 51140D-03 | -0.12476D-02 | 0.14970D+01 | 0.35500D+01 | 0.25893D-02 |
| 31 | 0 0 | 0 0 | 0 25651D-02 | -0 23795D-04 | -0 21068D-14 | 0 60101D-13 | 0 0 | 0 40000D+01 | 0 25651D-02 |
| 32 | 0 0 | 0 0 | 0 25637D-02 | -0 18478D-05 | -0 54644D-16 | 0 36420D-13 | 0 37425D+00 | 0 40000D+01 | 0.25637D-02 |
| 33 | 0 0 | 0.0 | 0 25636D-02 | -0 95735D-16 | -0 25587D-15 | 0 31861D-13 | 0 74850D+00 | 0 40000D+01 | 0.25636D-02 |
| 34 | 0 0 | 0 0 | 0 25637D-02 | 0 18478D-05 | -0 36084D-15 | 0 20530D-13 | 0.11227D+01 | 0 40000D+01 | 0.25637D-02 |
| 35 | 0 0 | 0 0 | 0 25651D-02 | 0 23795D-04 | 0 68869D-15 | 0 25161D-13 | 0 14970D+01 | 0 40000D+01 | 0 25651D-02 |
| 36 | 0 0 | 0 0 | 0 25893D-02 | -0 45265D-04 | 0 51140D 03 | -0 12476D-02 | 0 0 | 0.44500D+01 | 0 25893D-02 |
| 37 | 0 0 | 0.0 | 0.25857D-02 | 0 11586D-04 | 0 42594D 03 | 0 54027D-04 | 0 37425D+00 | 0 44500D+01 | 0 25857D-02 |
| 38 | 0.0 | 0 0 | 0.25861D-02 | -0 81218D-15 | 0 42629D-03 | 0.19021D-13 | 0 74850D+00 | 0 44500D+01 | 0 25861D-02 |
| 39 | 0 0 | 0 0 | 0 25857D-02 | -0 11586D-04 | 0 42594D-03 | -0.54027D-04 | 0 11227D+01 | 0 44500D+01 | 0 25857D-02 |
| 40 | 0 0 | 0 0 | 0 25893D-02 | 0 45265D-04 | 0 51140D-03 | 0 12476D-02 | 0 14970D+01 | 0.44500D+01 | 0.25893D-02 |
| 41 | 0 0 | 0 0 | 0 13479D-02 | -0 14959D-03 | -0 10897D-01 | 0 94515D-02 | 0 0 | 0 49000D+01 | 0 13479D-02 |
| 42 | 0 0 | 0 0 | 0 13400D-02 | -0 12765D-04 | -0 10366D-01 | 0 50944D-03 | 0 37425D+00 | 0 49000D+01 | 0.13400D-02 |
| 43 | 0 0 | 0 0 | 0 13395D-02 | 0 28531D-15 | -0 10341D-01 | -0 16253D-13 | 0.74850D+00 | 0 49000D+01 | 0.13395D-02 |
| 44 | 0 0 | 0 0 | 0.13400D-02 | 0 12765D-04 | -0 10366D-01 | -0.50944D-03 | 0.11227D+01 | 0.49000D+01 | 0.13400D-02 |
| 45 | 0 0 | 0.0 | 0 13479D-02 | 0 14959D-03 | -0 10897D-01 | -0 94515D-02 | 0 14970D+01 | 0 49000D+01 | 0.13479D-02 |
| 46 | 0 0 | 0.0 | 0.99179D-04 | -0 19027D-03 | -0 12524D-02 | 0 27066D-02 | 0 0 | 0 56750D+01 | 0.99179D-04 |
| 47 | 0 0 | 0 0 | 0 88537D-04 | -0 12370D-04 | -0 11050D 02 | 0.20861D-03 | 0.37425D+00 | 0.56750D+01 | 0 88537D-04 |
| 48 | 0 0 | 0 0 | 0.87982D-04 | 0 32499D-15 | -0 10959D-02 | -0.23433D-14 | 0 74850D+00 | 0.56750D+01 | 0.87982D-04 |
| 49 | 0 0 | 0 0 | 0 88537D-04 | 0 12370D-04 | -0 11050D-02 | -0 20861D-03 | 0.11227D+01 | 0 56750D+01 | 0.88537D-04 |
| 50 | 0 0 | 0 0 | 0 99179D-04 | 0 19027D-03 | -0 12524D-02 | -0 27066D-02 | 0 14970D+01 | 0 56750D+01 | 0.99179D-04 |
| 51 | 0 0 | 0 0 | 0.19922D-04 | -0 46461D-04 | -0 27121D-03 | 0.69738D-03 | 0.0 | 0.64500D+01 | 0.19922D-04 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 52 | 0.0 | 0.0 | 0.17431D-04 | -0.38937D-05 | -0 23448D-03 | 0.64409D-04 | 0 37425D+00 | 0.64500D+01 | 0.17431D-04 |
| 53 | 0.0 | 0 0 | 0.17262D-04 | 0 16121D-16 | -0 23179D-03 | -0 54427D-15 | 0.74850D+00 | 0.64500D+01 | 0.17262D-04 |
| 54 | 0 0 | 0 0 | 0 17431D-04 | 0 38937D-05 | -0 23448D-03 | -0.64409D-04 | 0.11227D+01 | 0.64500D+01 | 0.17431D-04 |
| 55 | 0.0 | 0.0 | 0.19922D-04 | 0 46461D-04 | -0 27121D-03 | -0 69738D-03 | 0.14970D+01 | 0.64500D+01 | 0.19922D-04 |
| 56 | 0.0 | 0 0 | 0.51519D-05 | -0.14519D-04 | -0.55451D-04 | 0 16130D-03 | 0 0 | 0 72250D+01 | 0.51519D-05 |
| 57 | 0 0 | 0 0 | 0 43962D-05 | -0.14371D-05 | -0 47145D-04 | 0 16538D-04 | 0 37425D+00 | 0.72250D+01 | 0.43962D-05 |
| 58 | 0 0 | 0 0 | 0.43375D-05 | 0 12340D-16 | -0 46477D-04 | -0 81810D-16 | 0 74850D+00 | 0.72250D+01 | 0.43375D-05 |
| 59 | 0 0 | 0 0 | 0 43962D-05 | 0 14371D-05 | -0 47145D-04 | -0.16538D-04 | 0.11227D+01 | 0.72250D+01 | 0 43962D-05 |
| 60 | 0 0 | 0 0 | 0 51519D-05 | 0 14519D-04 | -0 55451D-04 | -0.16130D-03 | 0 14970D+01 | 0.72250D+01 | 0.51519D-05 |
| 61 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 80000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 37425D+00 | 0 80000D+01 | 0.0 |
| 63 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 74850D+00 | 0 80000D+01 | 0.0 |
| 64 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 11227D+01 | 0.80000D+01 | 0.0 |
| 65 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 19108D+01 | 0.12929D+00 | 0 33697D+00 | 0.22568D-01 |
| 2 | 0 0 | 0.0 | 0 36567D+01 | 0.80131D-02 | 0.64390D+00 | 0 13107D-02 |
| 3 | 0.0 | 0 0 | 0.36947D+01 | -0.36310D-14 | 0 65001D+00 | -0 20456D-14 |
| 4 | 0 0 | 0.0 | 0 36567D+01 | -0 80131D-02 | 0 64390D+00 | -0 13107D-02 |
| 5 | 0 0 | 0 0 | 0 19108D+01 | -0 12929D+00 | 0.33697D+00 | -0 22568D-01 |
| 61 | 0 0 | 0.0 | 0,19108D+01 | 0.12929D+00 | -0.33697D+00 | -0.22568D-01 |
| 62 | 0.0 | 0.0 | 0 36567D+01 | 0.80131D-02 | -0 64390D+00 | -0.13107D-02 |
| 63 | 0 0 | 0 0 | 0.36947D+01 | 0.33005D-13 | -0.65001D+00 | -0.60392D-14 |
| 64 | 0 0 | 0.0 | 0 36567D+01 | -0 80131D-02 | -0.64390D+00 | 0.13107D-02 |
| 65 | 0.0 | 0 0 | 0.19108D+01 | -0 12929D+00 | -0.33697D+00 | 0.22568D-01 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT



| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0.1020D-05 | 0 1020D-05 | 0.2930D-05 | -0 2930D-05 | -0 9521D-07 | 0.9521D-07 | 0.2931D-05 | 0 1020D-05 | -0.8931D+02 | 0.6905D+00 |
| 2 | -0.1204D-06 | 0 1204D-06 | 0.3030D-05 | -0 3030D-05 | -0.1264D-07 | 0.1264D-07 | 0.3030D-05 | 0.1204D-06 | -0 8989D+02 | 0.1150D+00 |
| 3 | -0 1204D-06 | 0.1204D-06 | 0.3030D-05 | -0 3030D-05 | 0 1264D-07 | -0.1264D-07 | 0.3030D-05 | 0 1204D-06 | 0.8989D+02 | -0.1150D+00 |
| 4 | -0 1020D-05 | 0 1020D-05 | 0.2930D-05 | -0 2930D-05 | 0 9521D-07 | -0 9521D-07 | 0.2931D-05 | 0 1020D-05 | 0 8931D+02 | -0.6905D+00 |
| 5 | -0 2653D-05 | 0 2653D-05 | 0.1176D-04 | -0 1176D-04 | -0.1062D-05 | 0 1062D-05 | 0.1178D-04 | 0 2673D-05 | -0.8789D+02 | 0.2107D+01 |
| 6 | -0 2688D-06 | 0 2688D-06 | 0 1211D-04 | -0 1211D-04 | -0 1380D-06 | 0 1380D-06 | 0.1212D-04 | 0 2691D-06 | -0 8968D+02 | 0 3193D+00 |
| 7 | -0 2688D-06 | 0 2688D-06 | 0 1211D-04 | -0 1211D-04 | 0 1380D-06 | -0 1380D-06 | 0 1212D-04 | 0 2691D-06 | 0 8968D+02 | -0 3193D+00 |
| 8 | -0 2653D-05 | 0 2653D-05 | 0 1176D-04 | -0 1176D-04 | 0 1062D-05 | -0.1062D-05 | 0 1178D-04 | 0 2673D-05 | 0 8789D+02 | -0 2107D+01 |
| 9 | -0 9598D-05 | 0 9598D-05 | 0 5520D-04 | -0 5517D-04 | -0 3554D-05 | 0 3555D-05 | 0 5525D-04 | 0 9647D-05 | -0 8843D+02 | 0 1571D+01 |
| 10 | -0 7955D-06 | 0 7955D-06 | 0 5665D-04 | -0 5661D-04 | -0 4219D-06 | 0 4220D-06 | 0 5665D-04 | 0 7963D-06 | -0 8979D+02 | 0 2106D+00 |
| 11 | -0 7955D-06 | 0 7955D-06 | 0 5665D-04 | -0 5661D-04 | 0 4219D-06 | -0 4220D-06 | 0 5665D-04 | 0 7963D-06 | 0 8979D+02 | -0.2106D+00 |
| 12 | -0 9598D-05 | 0 9598D-05 | 0 5520D-04 | -0 5517D-04 | 0 3554D-05 | -0 3555D-05 | 0 5525D-04 | 0 9647D-05 | 0 8843D+02 | -0 1571D+01 |
| 13 | -0 6363D-04 | 0 6363D-04 | 0 6023D-03 | -0 6021D-03 | -0 1258D-04 | 0 1259D-04 | 0.6024D-03 | 0 6369D-04 | -0 8946D+02 | 0 5417D+00 |
| 14 | -0 2259D-05 | 0 2259D-05 | 0 6081D-03 | -0 6079D-03 | -0 1085D-05 | 0.1085D-05 | 0 6081D-03 | 0 2259D-05 | -0 8995D+02 | 0 5094D-01 |
| 15 | -0 2259D-05 | 0.2259D-05 | 0 6081D-03 | -0 6079D-03 | 0 1085D-05 | -0 1085D-05 | 0 6081D-03 | 0 2259D-05 | 0 8995D+02 | -0 5094D-01 |
| 16 | -0 6363D-04 | 0.6363D-04 | 0 6023D-03 | -0 6021D-03 | 0 1258D-04 | -0 1259D-04 | 0 6021D-03 | 0 6369D-04 | 0 8946D+02 | -0 5417D+00 |
| 17 | 0 9172D-04 | -0 9172D-04 | -0 1202D-02 | 0 1205D-02 | -0 1538D-04 | 0.1544D-04 | 0 9176D-04 | 0 1205D-02 | -0 3404D+00 | 0 8966D+02 |
| 18 | 0 3571D-05 | -0 3571D-05 | -0 1218D-02 | 0 1221D-02 | -0.1921D-05 | 0 1923D-05 | 0 3572D-05 | 0 1221D-02 | -0 4505D-01 | 0 8996D+02 |
| 19 | 0.3571D-05 | -0 3571D-05 | -0 1218D-02 | 0 1221D-02 | 0 1921D-05 | -0 1923D-05 | 0 3572D-05 | 0 1221D-02 | 0 4505D-01 | -0 8996D+02 |
| 20 | 0 9172D-04 | -0 9172D-04 | -0 1202D-02 | 0 1205D-02 | 0 1538D-04 | -0 1544D-04 | 0 9176D-04 | 0.1205D-02 | 0.3404D+00 | -0 8966D+02 |
| 21 | -0 4608D-05 | 0 4608D-05 | 0 4622D-04 | -0 4621D-04 | 0 1128D-05 | -0 1128D-05 | 0 4622D-04 | 0 4614D-05 | 0 8936D+02 | -0 6358D+00 |
| 22 | -0 2494D-06 | 0 2494D-06 | 0 4858D-04 | -0 4858D-04 | 0 2263D-06 | -0 2263D-06 | 0 4858D-04 | 0 2496D-06 | 0 8987D+02 | -0 1328D+00 |
| 23 | -0 2494D-06 | 0 2494D-06 | 0 4858D-04 | -0 4858D-04 | -0 2263D-06 | 0 2263D-06 | 0 4858D-04 | 0 2496D-06 | -0.8987D+02 | 0 1328D+00 |
| 24 | -0 4608D-05 | 0 4608D-05 | 0 4622D-04 | -0 4621D-04 | -0 1128D-05 | 0 1128D-05 | 0 4622D-04 | 0 4614D-05 | -0 8936D+02 | 0 6358D+00 |
| 25 | -0.4608D-05 | 0 4608D-05 | 0 4622D-04 | -0 4621D-04 | -0.1128D-05 | 0 1128D-05 | 0 4622D-04 | 0 4614D-05 | -0 8936D+02 | 0 6358D+00 |
| 26 | -0 2191D-06 | 0.2494D-06 | 0 4858D-04 | -0 4858D-04 | -0 2263D-06 | 0.2263D-06 | 0 4858D-04 | 0 2496D-06 | -0 8987D+02 | 0 1328D+00 |
| 27 | -0 2494D-06 | 0 2494D-06 | 0 4858D-04 | -0 4858D-04 | 0 2263D-06 | -0 2263D-06 | 0 4858D-04 | 0 2496D-06 | 0 8987D+02 | -0.1328D+00 |
| 28 | -0 4608D-05 | 0 4608D-05 | 0.4622D-04 | -0 4621D-04 | 0.1128D-05 | -0 1128D-05 | 0 4622D-04 | 0.4614D-05 | 0.8936D+02 | -0.6358D+00 |
| 29 | 0 9172D-04 | -0 9172D-04 | -0 1202D-02 | 0 1205D-02 | 0 1538D-04 | -0 1544D-04 | 0 9176D-04 | 0 1205D-02 | 0.3404D+00 | -0 8966D+02 |
| 30 | 0 3571D-05 | -0 3571D-05 | -0 1218D-02 | 0.1221D-02 | 0.1921D-05 | -0 1923D-05 | 0 3572D-05 | 0.1221D-02 | 0.4505D-01 | -0.8996D+02 |
| 31 | 0 3571D-05 | -0 3571D-05 | -0 1218D-02 | 0 1221D-02 | -0.1921D-05 | 0 1923D-05 | 0 3572D-05 | 0 1221D-02 | -0 4505D-01 | 0.8996D+02 |
| 32 | 0.9172D-04 | -0.9172D-04 | -0 1202D-02 | 0 1205D-02 | -0.1538D-04 | 0.1544D-04 | 0.9176D-04 | 0.1205D-02 | -0.3404D+00 | 0.8966D+02 |
| 33 | -0.6363D-04 | 0.6363D-04 | 0 6023D-03 | -0 6021D-03 | 0.1258D-04 | -0 1259D-04 | 0.6024D-03 | 0 6369D-04 | 0.8946D+02 | -0.5417D+00 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 34 | -0 2259D-05 | 0.2259D-05 | 0 6081D-03 | -0 6079D-03 | 0.1085D-05 | -0.1085D-05 | 0.6081D-03 | 0.2259D-05 | 0 8995D+02 | -0.5094D-01 |
| 35 | -0 2259D-05 | 0 2259D-05 | 0.6081D-03 | -0.6079D-03 | -0.1085D-05 | 0.1085D-05 | 0.6081D-03 | 0.2259D-05 | -0.8995D+02 | 0.5094D-01 |
| 36 | -0 6363D-04 | 0 6363D-04 | 0.6023D-03 | -0 6021D-03 | -0 1258D-04 | 0.1259D-04 | 0.6024D-03 | 0 6369D-04 | -0.8946D+02 | 0.5417D+00 |
| 37 | -0 9598D-05 | 0 9598D-05 | 0 5520D-04 | -0.5517D-04 | 0 3554D-05 | -0 3555D-05 | 0.5525D-04 | 0 9647D-05 | 0 8843D+02 | -0.1571D+01 |
| 38 | -0 7955D-06 | 0.7955D-06 | 0.5665D-04 | -0 5661D-04 | 0 4219D-06 | -0.4220D-06 | 0.5665D-04 | 0.7963D-06 | 0 8979D+02 | -0.2106D+00 |
| 39 | -0 7955D-06 | 0 7955D-06 | 0 5665D-04 | -0 5661D-04 | -0 4219D-06 | 0 4220D-06 | 0 5665D-04 | 0 7963D-06 | -0.8979D+02 | 0 2106D+00 |
| 40 | -0.9598D-05 | 0 9598D-05 | 0.5520D-04 | -0 5517D-04 | -0 3554D-05 | 0 3555D-05 | 0 5525D-04 | 0 9647D-05 | -0 8843D+02 | 0.1571D+01 |
| 41 | -0 2653D-05 | 0 2653D-05 | 0 1176D-04 | -0 1176D-04 | 0.1062D-05 | -0 1062D-05 | 0 1178D-04 | 0.2673D-05 | 0 8789D+02 | -0 2107D+01 |
| 42 | -0 2688D-06 | 0 2688D-06 | 0.1211D-04 | -0 1211D-04 | 0.1380D-06 | -0 1380D-06 | 0 1212D-04 | 0 2691D-06 | 0 8968D+02 | -0 3193D+00 |
| 43 | -0 2688D-06 | 0 2688D-06 | 0 1211D-04 | -0 1211D-04 | -0 1380D-06 | 0.1380D-06 | 0 1212D-04 | 0 2691D-06 | -0 8968D+02 | 0.3193D+00 |
| 44 | -0 2653D-05 | 0 2653D-05 | 0 1176D-04 | -0 1176D-04 | -0.1062D-05 | 0 1062D-05 | 0.1178D-04 | 0.2673D-05 | -0.8789D+02 | 0.2107D+01 |
| 45 | -0 1020D-05 | 0 1020D-05 | 0 2930D-05 | -0 2930D-05 | 0 9521D-07 | -0 9521D-07 | 0 2931D-05 | 0.1020D-05 | 0 8931D+02 | -0 6905D+00 |
| 46 | -0 1204D-06 | 0 1204D-06 | 0 3030D-05 | -0 3030D-05 | 0 1264D-07 | -0 1264D-07 | 0 3030D-05 | 0.1204D-06 | 0 8989D+02 | -0.1150D+00 |
| 47 | -0 1204D-06 | 0 1204D-06 | 0.3030D-05 | -0 3030D-05 | -0 1264D-07 | 0 1264D-07 | 0 3030D-05 | 0.1204D-06 | -0 8989D+02 | 0.1150D+00 |
| 48 | -0 1020D-05 | 0.1020D-05 | 0 2930D-05 | -0 2930D-05 | -0 9521D-07 | 0 9521D-07 | 0 2931D-05 | 0.1020D-05 | -0 8931D+02 | 0.6905D+00 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0.2436D-06 | 0 2436D-06 | -0 6264D-04 | 0 6264D-04 | 0 3250D-14 | -0 3250D-14 | 0 0 | 0.6264D-04 | 0 0 | -0.9000D+02 |
| 35 | 0.1037D-04 | -0.1037D-04 | -0 7935D-04 | 0 7935D-04 | 0 2566D-14 | -0.2566D-14 | 0.1037D-04 | 0 7935D-04 | 0.8196D-09 | -0.9000D+02 |
| 38 | 0 2445D-05 | -0 2445D-05 | 0 2133D-03 | -0 2131D-03 | 0 1940D-14 | -0 1941D-14 | 0 2133D-03 | 0 0 | 0.9000D+02 | 0.0 |
| 40 | 0 1370D-04 | -0.1370D-04 | 0 2789D-03 | -0 2786D-03 | 0 1273D-03 | -0 1272D-03 | 0.2934D-03 | 0 7851D-06 | 0.7718D+02 | -0.1283D+02 |
| 48 | -0 2160D-05 | 0 2160D-05 | -0 3639D-03 | 0 3651D-03 | -0 2394D-15 | 0 2387D-15 | 0.0 | 0 3651D-03 | 0.0 | 0.9000D+02 |
| 50 | 0 8385D-04 | -0 8382D-04 | -0.4005D-03 | 0.4021D-03 | -0.2763D-03 | 0.2758D-03 | 0.1205D-03 | 0 4385D-03 | -0.1485D+02 | 0.7521D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR 1) | ELONG (DIR 2) | PRINC STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 0 | -0 39072D-05 | 0 0 | 0 0 | -0 39072D-05 | 0 0 | 0 0 |
| 1 | OUTER | 0 0 | 0 39072D-05 | 0 0 | 0 0 | 0.39072D-05 | 0 39072D-05 | 0.90000D+02 |
| 2 | INNER | 0 73124D-07 | -0 31195D-06 | -0 99024D-17 | 0 73124D-07 | -0 31195D-06 | 0.73124D-07 | -0.73671D-09 |
| 2 | OUTER | -0 73124D-07 | 0 31201D-06 | 0.99026D-17 | -0 73124D-07 | 0 31201D-06 | 0 31201D-06 | 0.90000D+02 |
| 3 | INNER | -0 15575D-06 | -0 80799D-05 | -0 52211D-16 | -0 15575D-06 | -0 80799D-05 | 0 0 | 0 0 |
| 3 | OUTER | 0 15575D-06 | 0.80625D-05 | 0 52209D-16 | 0 15575D-06 | 0.80824D-05 | 0 80825D-05 | 0 90000D+02 |
| 4 | INNER | -0 37069D-06 | -0 23329D-04 | -0 19224D-15 | -0 37068D-06 | -0 23329D-04 | 0 0 | 0 0 |
| 4 | OUTER | 0 37068D-06 | 0 23387D-04 | 0 19221D-15 | 0 37068D-06 | 0 23387D-04 | 0 23387D-04 | 0 90000D+02 |
| 5 | INNER | -0 10969D-05 | 0.13879D-03 | -0 17632D-15 | -0.10969D-05 | 0.13878D-03 | 0.13879D-03 | -0.90000D+02 |
| 5 | OUTER | 0 10969D-05 | -0.13720D-03 | 0 17546D-15 | 0 10969D-05 | -0.13721D-03 | 0 10969D-05 | 0.36347D-10 |
| 6 | INNER | -0 24361D-06 | -0.62636D-04 | 0 32498D-14 | -0 24361D-06 | -0.62638D-04 | 0 0 | 0 0 |
| 6 | OUTER | 0 24361D-06 | 0 62636D-04 | -0 32498D-14 | 0 21361D-06 | 0 62634D-04 | 0 62636D-04 | -0 90000D+02 |
| 7 | INNER | -0 10969D-05 | 0 13879D-03 | 0 56693D-15 | -0 10969D-05 | 0.13878D-03 | 0.13879D-03 | 0 90000D+02 |
| 7 | OUTER | 0 10969D-05 | -0.13720D-03 | -0 56805D-15 | 0 10969D-05 | -0 13721D-03 | 0 10969D-05 | -0 11767D-09 |
| 8 | INNER | -0 37068D-06 | -0 23329D-04 | 0 25099D-16 | -0 37068D-06 | -0 23329D-04 | 0.0 | 0 0 |
| 8 | OUTER | 0 37068D-06 | 0 23387D-04 | -0 25086D-16 | 0 37068D-06 | 0 23387D-04 | 0 23387D-04 | -0 90000D+02 |
| 9 | INNER | -0 15575D-06 | -0 80799D-05 | 0 16811D-16 | -0.15575D-06 | -0.80799D-05 | 0 0 | 0 0 |
| 9 | OUTER | 0 15575D-06 | 0 80825D-05 | -0 16811D-16 | 0.15575D-06 | 0.80824D-05 | 0 80825D-05 | -0.90000D+02 |
| 10 | INNER | 0 73124D-07 | -0 31195D-06 | 0 77592D-18 | 0 73124D-07 | -0 31195D-06 | 0 73124D-07 | 0.57726D-10 |
| 10 | OUTER | -0 73124D-07 | 0 31201D-06 | -0 77594D-18 | -0.73124D-07 | 0 31201D-06 | 0.31201D-06 | -0.90000D+02 |
| 11 | INNER | -0 12440D-22 | -0.39072D-05 | 0 86910D-21 | 0.0 | -0.39072D-05 | 0 0 | 0.0 |
| 11 | OUTER | 0 12440D-22 | 0.39072D-05 | -0 86910D-21 | 0.0 | 0.39072D-05 | 0.39072D-05 | -0.90000D+02 |

SYSTEM ENERGIES(IN-LB)

| | | |
|---|---|---|
| WORK INPUT TO STRUCTURE | = | 0.4011640D+03 |
| STRUCTURE KINETIC ENERGY | = | 0 1456689D+03 |
| STRUCTURE ELASTIC ENERGY | = | 0.1127671D+01 |
| STRUCTURE PLASTIC ENERGY | = | 0.2543675D+03 |
| ENERGY STORED IN ELASTIC RESTRAINTS | = | 0 0 |

TIME= 0.500000D-05 SEC.

...... ............. . ........ EXTERNALLY APPLIED USER SPECIFIED LOADING..................................

| NODE | DOF#1 | DOF#2 | DOF#3 | DOF#4 | DOF#5 | DOF#6 |
|---|---|---|---|---|---|---|
| | (LBS) | (LBS) | (LBS) | (IN-LBS) | (IN-LBS) | (IN-LBS) |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 |
| 3 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 |
| 4 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 5 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 |
| 7 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 8 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 9 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 10 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 11 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 12 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 13 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 14 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 15 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 16 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 |
| 17 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 |
| 18 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 19 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 20 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 21 | 0 0 | 0.0 | 0.738805D+03 | 0.460830D+02 | 0.554104D+02 | 0 345622D+01 |
| 22 | 0 0 | 0 0 | 0 147761D+04 | 0.0 | 0 110821D+03 | 0 0 |
| 23 | 0 0 | 0.0 | 0.147761D+04 | 0 0 | 0 110821D+03 | 0 0 |
| 24 | 0 0 | 0 0 | 0.147761D+04 | 0 0 | 0 110821D+03 | 0 0 |
| 25 | 0 0 | 0 0 | 0 738805D+03 | -0 460830D+02 | 0 554104D+02 | -0 345622D+01 |
| 26 | 0 0 | 0.0 | 0.147761D+04 | 0 921660D+02 | 0.0 | 0 0 |
| 27 | 0 0 | 0 0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 28 | 0 0 | 0.0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 29 | 0 0 | 0.0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 30 | 0 0 | 0.0 | 0.147761D+04 | -0 921660D+02 | 0 0 | 0.0 |
| 31 | 0.0 | 0 0 | 0 147761D+04 | 0 921660D+02 | 0 0 | 0 0 |
| 32 | 0 0 | 0.0 | 0 295522D+04 | 0 0 | 0.0 | 0 0 |
| 33 | 0 0 | 0.0 | 0.295522D+04 | 0 0 | 0 0 | 0 0 |
| 34 | 0 0 | 0 0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 35 | 0 0 | 0.0 | 0 147761D+04 | -0 921660D+02 | 0 0 | 0.0 |
| 36 | 0 0 | 0 0 | 0 147761D+04 | 0 921660D+02 | 0.0 | 0 0 |
| 37 | 0.0 | 0 0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 38 | 0 0 | 0.0 | 0 295522D+04 | 0 0 | 0 0 | 0 0 |
| 39 | 0 0 | 0 0 | 0.295522D+04 | 0 0 | 0 0 | 0 0 |
| 40 | 0.0 | 0.0 | 0 147761D+04 | -0 921660D+02 | 0 0 | 0 0 |
| 41 | 0 0 | 0.0 | 0 738805D+03 | 0 460830D+02 | -0 554104D+02 | -0 345622D+01 |
| 42 | 0 0 | 0 0 | 0 147761D+04 | 0 0 | -0 110821D+03 | 0 0 |
| 43 | 0.0 | 0.0 | 0 147761D+04 | 0 0 | -0 110821D+03 | 0 0 |
| 44 | 0.0 | 0.0 | 0.147761D+04 | 0 0 | -0 110821D+03 | 0 0 |
| 45 | 0 0 | 0 0 | 0 738805D+03 | -0.460830D+02 | -0 554104D+02 | 0 345622D+01 |
| 46 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 47 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 48 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 49 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 50 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 51 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 |
| 52 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 53 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 54 | 0 0 | 0 0 | 0 0 | 0 0 | 0,0 | 0 0 |
| 55 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 56 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 |
| 57 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 58 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 59 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 60 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 61 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 62 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 63 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 64 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| 65 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

***** INCR. NO.= 2 TIME= 0.5000D-05 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |
| 4 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.11227D+01 | 0.0 | 0.0 |
| 5 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.14970D+01 | 0.0 | 0.0 |
| 6 | -0 32055D-07 | 0 17451D-07 | 0 75092D-08 | -0 14477D-04 | -0 44967D-05 | -0 15187D-03 | -0.32055D-07 | 0.77500D+00 | 0.75092D-08 |
| 7 | 0 69323D-08 | -0 13693D-07 | -0 47200D-06 | -0 26689D-05 | -0 90374D-05 | -0 30124D-04 | 0.37425D+00 | 0 77500D+00 | -0.47200D-06 |
| 8 | -0 47720D-21 | 0 86680D-08 | -0 64130D-06 | -0 11025D-17 | -0 10875D-04 | -0 36194D-16 | 0.74850D+00 | 0 77500D+00 | -0.64130D-06 |
| 9 | -0 69323D-08 | -0 13693D-07 | -0 47200D-06 | 0 26689D-05 | -0 90374D 05 | 0 30124D-04 | 0.11227D+01 | 0.77500D+00 | -0.47200D-06 |
| 10 | 0 32055D-07 | 0 17451D-07 | 0 75092D-08 | 0 14477D-04 | -0 44967D-05 | 0.15187D-03 | 0.14970D+01 | 0.77500D+00 | 0.75092D-08 |
| 11 | 0 26745D-06 | 0 10613D-06 | 0 98713D-05 | -0.71806D-04 | 0 63672D-04 | -0 86432D-03 | 0 26745D-06 | 0 15500D+01 | 0 98713D-05 |
| 12 | 0 29660D-07 | 0 17536D-06 | 0 65589D-05 | -0 83695D-05 | 0 29290D-04 | -0 12722D-03 | 0.37425D+00 | 0.15500D+01 | 0.65589D-05 |
| 13 | 0 18118D-20 | 0 12084D-06 | 0 59073D-05 | 0 12039D-16 | 0 20394D-04 | 0 37937D-16 | 0.74850D+00 | 0.15500D+01 | 0.59073D-05 |
| 14 | -0 29660D-07 | 0 17536D-06 | 0 65589D-05 | 0 83695D-05 | 0 29290D-04 | 0 12722D-03 | 0.11227D+01 | 0 15500D+01 | 0 65589D-05 |
| 15 | -0 26745D-06 | 0 10613D-06 | 0 98713D-05 | 0 71806D-04 | 0 63672D-04 | 0 86432D-03 | 0 14970D+01 | 0.15500D+01 | 0.98713D-05 |
| 16 | -0 38160D-03 | 0 21993D-05 | 0 16867D-03 | -0 55483D-03 | 0 12984D-02 | -0 56278D-02 | -0 38160D-06 | 0 23250D+01 | 0.16867D-03 |
| 17 | 0 27500D-08 | 0 18533D-05 | 0 13247D-03 | -0.34472D-04 | 0 97851D-03 | -0 49099D-03 | 0 37425D+00 | 0 23250D+01 | 0.13247D-03 |
| 18 | -0 87913D-20 | 0 19584D-05 | 0 12946D-03 | 0 25625D-16 | 0 93867D-03 | 0 61686D-15 | 0 74850D+00 | 0.23250D+01 | 0.12946D-03 |
| 19 | -0 27500D-08 | 0 18533D-05 | 0.13247D-03 | 0 34472D-04 | 0 97851D-03 | 0 49099D-03 | 0 11227D+01 | 0 23250D+01 | 0 13247D-03 |
| 20 | 0 38160D-06 | 0 21993D-05 | 0 16867D-03 | 0 55483D-03 | 0 12984D-02 | 0 56278D-02 | 0 14970D+01 | 0 23250D+01 | 0.16867D-03 |
| 21 | 0 33972D-05 | 0 22839D-06 | 0 49834D-02 | -0 21907D-03 | 0 34504D-01 | -0 33045D-01 | 0 33972D-05 | 0.31000D+01 | 0 49834D-02 |
| 22 | 0 83089D-06 | 0 21707D-06 | 0 49715D-02 | -0 18310D-04 | 0 32072D-01 | -0 18330D-03 | 0 37425D+00 | 0 31000D+01 | 0.49715D-02 |
| 23 | 0 49578D-19 | 0 20695D-06 | 0 49703D-02 | 0 29664D-15 | 0 32119D-01 | 0 32110D-14 | 0 74850D+00 | 0 31000D+01 | 0.49703D-02 |
| 24 | -0 83689D-06 | 0 21707D-06 | 0 49715D-02 | 0 18310D-04 | 0 32072D-01 | 0 18330D-03 | 0.11227D+01 | 0 31000D+01 | 0.49715D-02 |
| 25 | -0 33972D-05 | 0 22839D-06 | 0 49831D-02 | 0 21907D-03 | 0 34504D-01 | 0.33045D-01 | 0 14970D+01 | 0 31000D+01 | 0.49834D-02 |
| 26 | 0 18129D-07 | -0 28943D-05 | 0.10305D-01 | -0 10664D-02 | -0 30850D-02 | 0.11445D-01 | 0 18129D-07 | 0 35500D+01 | 0.10305D-01 |
| 27 | 0 22826D-06 | -0 24871D-05 | 0.10206D-01 | 0 94412D-04 | -0 21704D-02 | -0.14922D-03 | 0 37425D+00 | 0 35500D+01 | 0.10206D-01 |
| 28 | -0 28624D-19 | -0 26217D-05 | 0 10219D-01 | 0 41368D-15 | -0 22001D-02 | 0.18014D-13 | 0 74850D+00 | 0 35500D+01 | 0.10219D-01 |
| 29 | -0 22826D-06 | -0 24871D-05 | 0 10206D-01 | -0.94412D-04 | -0 21704D-02 | 0.14922D-03 | 0.11227D+01 | 0 35500D+01 | 0 10206D-01 |
| 30 | -0 18129D-07 | -0 28943D-05 | 0 10305D-01 | 0 10664D-02 | -0 30850D-02 | -0.11445D-01 | 0 14970D+01 | 0 35500D+01 | 0.10305D-01 |
| 31 | 0 76874D-06 | 0.34053D-19 | 0 98308D-02 | 0 70954D-04 | -0 86302D-15 | 0.34514D-13 | 0 76874D-06 | 0.40000D+01 | 0.98308D-02 |
| 32 | 0.78726D-07 | 0 84439D-20 | 0 98400D-02 | -0 79478D-05 | 0 64185D-16 | 0.27315D-13 | 0 37425D+00 | 0 40000D+01 | 0.98400D-02 |
| 33 | 0 17333D-19 | 0 58631D-20 | 0 98395D-02 | 0 64835D-16 | -0 16133D-15 | 0 24703D-13 | 0 74850D+00 | 0 40000D+01 | 0.98395D-02 |
| 34 | -0 78723D-07 | 0 11832D-19 | 0 98400D-02 | 0 79478D-05 | -0 40072D-15 | 0.14564D-13 | 0 11227D+01 | 0 40000D+01 | 0.98400D-02 |
| 35 | -0 76874D-06 | 0.18132D-20 | 0 98308D-02 | -0 70954D-04 | 0 86736D-17 | 0.10064D-13 | 0 14970D+01 | 0 40000D+01 | 0.98308D-02 |
| 36 | 0 18129D-07 | 0 28943D-05 | 0 10305D-01 | -0 10664D-02 | 0 30850D 02 | -0 11445D-01 | 0 18129D-07 | 0 44500D+01 | 0 10305D-01 |
| 37 | 0 22826D-06 | 0 24871D-05 | 0 10206D-01 | 0 94412D-04 | 0 21704D-02 | 0 14922D-03 | 0 37425D+00 | 0 44500D+01 | 0.10206D-01 |
| 38 | -0 19181D-19 | 0 26217D-05 | 0 10219D-01 | -0 24069D-15 | 0 22001D-02 | 0.21647D-13 | 0 74850D+00 | 0 44500D+01 | 0 10219D-01 |
| 39 | -0 22826D-06 | 0 24871D-05 | 0 10206D-01 | -0 94412D-04 | 0 21704D-02 | -0.14922D-03 | 0 11227D+01 | 0 44500D+01 | 0 10206D-01 |
| 40 | -0 18129D-07 | 0 28913D-05 | 0 10305D-01 | 0 10664D-02 | 0 30850D-02 | 0.11445D-01 | 0 14970D+01 | 0 44500D+01 | 0.10305D-01 |
| 41 | 0 33972D-05 | -0 22839D-06 | 0 49834D-02 | -0 21907D-03 | -0 34504D-01 | 0 33045D-01 | 0 33972D-05 | 0 49000D+01 | 0.49834D-02 |
| 42 | 0 83089D-06 | -0 21707D-06 | 0 49715D-02 | -0 18310D-04 | -0 32072D-01 | 0.18330D-03 | 0 37425D+00 | 0 49000D+01 | 0.49715D-02 |
| 43 | 0 15284D-18 | -0 20695D-06 | 0 49703D-02 | -0 23581D-15 | -0 32119D-01 | -0.38840D-14 | 0.74850D+00 | 0 49000D+01 | 0.49703D-02 |
| 44 | -0 83689D-06 | -0 21707D-06 | 0 49715D-02 | 0.18310D-04 | -0 32072D-01 | -0 18330D-03 | 0.11227D+01 | 0 49000D+01 | 0 49715D-02 |
| 45 | -0 33972D-05 | -0 22839D-06 | 0 49834D-02 | 0 21907D-03 | -0 34504D-01 | -0.33045D-01 | 0.14970D+01 | 0 49000D+01 | 0.49834D-02 |
| 46 | -0 38160D-06 | -0 21993D-05 | 0 16867D-03 | -0 55483D-03 | -0 12984D-02 | 0 56278D-02 | -0 38160D-06 | 0 56750D+01 | 0.16867D-03 |
| 47 | 0 27500D-08 | -0 18533D-05 | 0 13247D-03 | -0 34472D-04 | -0 97851D-03 | 0 49099D-03 | 0 37425D+00 | 0 56750D+01 | 0 13247D-03 |
| 48 | -0 36600D-19 | -0 19584D-05 | 0 12946D-03 | 0 56389D-16 | -0 93867D-03 | 0 32896D-15 | 0 74850D+00 | 0 56750D+01 | 0 12946D-03 |
| 49 | -0 27500D-08 | -0 18533D-05 | 0 13247D-03 | 0 34472D-04 | -0 97851D-03 | -0 49099D-03 | 0 11227D+01 | 0 56750D+01 | 0.13247D-03 |
| 50 | 0 38160D-06 | -0 21993D-05 | 0 16867D-03 | 0 55483D-03 | -0 12984D-02 | -0 56278D-02 | 0 14970D+01 | 0 56750D+01 | 0.16867D-03 |
| 51 | 0 26745D-06 | -0 10613D-06 | 0 98713D-05 | -0 71806D-04 | -0 63672D-04 | 0 86432D-03 | 0 26745D-06 | 0 64500D+01 | 0.98713D-05 |
| 52 | 0 29660D-07 | -0 17536D-06 | 0 65589D-05 | -0 83695D-05 | -0 29290D-04 | 0 12722D-03 | 0.37425D+00 | 0 64500D+01 | 0.65589D-05 |
| 53 | 0 77846D-20 | -0 12084D-06 | 0 59073D-05 | -0 10963D-16 | -0 20394D-04 | 0.33630D-15 | 0 74850D+00 | 0 64500D+01 | 0.59073D-05 |
| 54 | -0 29660D-07 | -0.17536D-06 | 0 65589D-05 | 0 83695D-05 | -0 29290D-04 | -0 12722D-03 | 0 11227D+01 | 0 64500D+01 | 0 65589D-05 |
| 55 | -0 26745D-06 | -0 10613D-06 | 0 98713D-05 | 0 71806D-04 | -0.63672D-04 | -0 86432D-03 | 0.14970D+01 | 0 64500D+01 | 0.98713D-05 |
| 56 | -0 32055D-07 | -0 17451D-07 | 0 75092D-08 | -0 14477D-04 | 0 44967D-05 | 0 15187D-03 | -0 32055D-07 | 0.72250D+01 | 0.75092D-08 |
| 57 | 0.69323D-08 | 0 13693D-07 | -0 47200D-06 | -0 26689D-05 | 0 90374D-05 | 0 30124D-04 | 0 37425D+00 | 0 72250D+01 | -0.47200D-06 |
| 58 | -0.25499D-20 | -0 86560D-08 | -0 64130D-06 | -0 81090D-17 | 0.10875D-04 | 0.57772D-16 | 0.74850D+00 | 0 72250D+01 | -0.64130D-06 |
| 59 | -0 69323D-08 | 0 13693D-07 | -0 47200D-06 | 0 26689D-05 | 0 90374D-05 | -0.30124D-04 | 0 11227D+01 | 0.72250D+01 | -0.47200D-06 |
| 60 | 0 32055D-07 | -0.17451D-07 | 0 75092D-08 | 0 14477D-04 | 0 44967D-05 | -0 15187D-03 | 0 14970D+01 | 0 72250D+01 | 0.75092D-08 |
| 61 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+01 | 0.0 |
| 62 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.37425D+00 | 0.80000D+01 | 0.0 |
| 63 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.74850D+00 | 0.80000D+01 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 64 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.80000D+01 | 0.0 |
| 65 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | -0 14890D-01 | 0.21819D-02 | 0.34260D+01 | 0 22219D+00 | 0 59755D+00 | 0.39043D-01 |
| 2 | 0 19712D-01 | -0 12578D-01 | 0 65787D+01 | 0 12319D-01 | 0.11444D+01 | 0.21494D-02 |
| 3 | -0 86649D-15 | 0.82524D-02 | 0 66355D+01 | 0 21157D-12 | 0.11548D+01 | 0 37984D-13 |
| 4 | -0.19712D-01 | -0 12578D-01 | 0.65787D+01 | -0.12319D-01 | 0.11444D+01 | -0.21494D-02 |
| 5 | 0.14890D-01 | 0.21819D-02 | 0 34260D+01 | -0.22219D+00 | 0.59755D+00 | -0.39043D-01 |
| 61 | -0.14890D-01 | -0 21819D-02 | 0 34260D+01 | 0.22219D+00 | -0.59755D+00 | -0 39043D-01 |
| 62 | 0 19712D-01 | 0 12578D-01 | 0 65787D+01 | 0 12319D-01 | -0 11444D+01 | -0 21494D-02 |
| 63 | -0.36478D-14 | -0 82524D-02 | 0.66355D+01 | 0 57668D-13 | -0.11548D+01 | -0.11000D-13 |
| 64 | -0.19712D-01 | 0.12578D-01 | 0 65787D+01 | -0 12319D-01 | -0.11444D+01 | 0.21494D-02 |
| 65 | 0.14890D-01 | -0 21819D-02 | 0 34260D+01 | -0.22219D+00 | -0.59755D+00 | 0.39043D-01 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0.7506D-06 | 0 8548D-06 | -0 8177D-06 | 0 8225D-06 | -0.2871D-06 | 0 1715D-06 | 0.0 | 0 9259D-06 | 0.0 | 0.3968D+02 |
| 2 | -0 2251D-06 | 0 2066D-06 | -0.7512D-06 | 0 7447D-06 | 0 2786D-07 | 0.4083D-07 | 0 0 | 0 7454D-06 | 0.0 | 0.8783D+02 |
| 3 | -0 2251D-06 | 0.2066D-06 | -0 7512D-06 | 0 7447D-06 | -0 2786D-07 | -0 4083D-07 | 0 0 | 0 7454D-06 | 0.0 | -0.8783D+02 |
| 4 | -0.7506D-06 | 0 8548D-06 | -0 8177D-06 | 0 8225D-06 | 0 2871D-06 | -0.1715D-06 | 0 0 | 0 9259D-06 | 0 0 | -0 3968D+02 |
| 5 | -0 3262D-05 | 0 2731D-05 | 0.1789D-05 | -0 1430D-05 | -0.2375D-05 | 0 2893D-05 | 0 2054D-05 | 0 3184D-05 | -0.7741D+02 | 0.1740D+02 |
| 6 | -0 5785D-06 | 0 4808D-06 | 0 2185D-05 | -0 1797D-05 | -0 3460D-07 | -0 2198D-07 | 0 2186D-05 | 0.4808D-06 | -0.8964D+02 | -0.2765D+00 |
| 7 | -0 5785D-06 | 0 4808D-06 | 0 2185D-05 | -0 1797D-05 | 0 3460D-07 | 0 2198D-07 | 0 2186D-05 | 0 4808D-06 | 0 8964D+02 | 0.2765D+00 |
| 8 | -0 3262D-05 | 0 2731D-05 | 0 1789D-05 | -0 1430D-05 | 0 2375D-05 | -0 2893D-05 | 0 2054D-05 | 0 3184D-05 | 0 7741D+02 | -0.1740D+02 |
| 9 | -0 1811D-04 | 0.1850D-04 | 0 6075D-04 | -0 5588D-04 | -0 1082D-04 | 0 9203D-05 | 0 6112D-04 | 0 1878D-04 | -0 8610D+02 | 0.3527D+01 |
| 10 | -0 1927D-05 | 0 1840D-05 | 0 6259D-04 | -0 5806D-04 | 0 5120D-06 | -0 4117D-06 | 0 6260D-04 | 0 1841D-05 | 0 8977D+02 | -0.1969D+00 |
| 11 | -0 1927D-05 | 0.1840D-05 | 0 6259D-04 | -0 5806D-04 | -0 5120D-06 | 0 4117D-06 | 0 6260D-04 | 0.1841D-05 | -0.8977D+02 | 0.1969D+00 |
| 12 | -0 1811D-04 | 0.1850D-04 | 0 6075D-04 | -0 5588D-04 | 0.1082D-04 | -0 9203D-05 | 0 6112D-04 | 0 1878D-04 | 0 8610D+02 | -0.3527D+01 |
| 13 | -0 3198D-03 | 0 3140D-03 | 0.2029D-02 | -0 2032D-02 | 0.3506D-04 | -0 2965D-04 | 0 2029D-02 | 0 3141D-03 | 0 8957D+02 | -0.3621D+00 |
| 14 | 0 6537D-05 | -0 8780D-05 | 0 2048D-02 | -0 2052D-02 | -0 3822D-05 | 0 5089D-05 | 0.2048D-02 | 0 0 | -0.8995D+02 | 0.0 |
| 15 | 0 6537D-05 | -0 8780D-05 | 0 2048D-02 | -0 2052D-02 | 0 3822D-05 | -0 5089D-05 | 0 2048D-02 | 0 0 | 0 8995D+02 | 0.0 |
| 16 | -0 3198D-03 | 0 3140D-03 | 0 2029D-02 | -0 2032D-02 | -0 3506D-04 | 0 2965D-04 | 0 2029D-02 | 0 3141D-03 | -0.8957D+02 | 0 3621D+00 |
| 17 | 0 4304D-03 | -0 4367D-03 | -0 3791D-02 | 0 3879D-02 | -0 4672D-04 | 0 3492D-04 | 0 4306D-03 | 0 3879D-02 | -0.3170D+00 | 0 8977D+02 |
| 18 | -0 6347D-05 | 0 3501D-05 | -0 3841D-02 | 0 3929D-02 | 0 4160D-05 | -0 5297D-05 | 0 0 | 0 3929D-02 | 0.0 | -0.8996D+02 |
| 19 | -0 6347D-05 | 0 3501D-05 | -0 3841D-02 | 0 3929D-02 | -0 4160D-05 | 0 5297D-05 | 0.0 | 0 3929D-02 | 0 0 | 0 8996D+02 |
| 20 | 0 4301D-03 | -0 4367D-03 | -0 3791D-02 | 0 3879D-02 | 0 4672D-04 | -0 3492D-04 | 0 4306D-03 | 0 3879D-02 | 0 3170D+00 | -0.8977D+02 |
| 21 | -0 1580D-04 | 0 1452D-04 | 0 2425D-03 | -0 2301D-03 | 0 3902D-04 | -0 3657D-04 | 0 2440D-03 | 0 1588D-04 | 0 8571D+02 | -0 4252D+01 |
| 22 | -0 5158D-05 | 0.4337D-05 | 0 2544D-03 | -0 2425D-03 | -0 6869D-05 | 0 6166D-05 | 0 2544D-03 | 0 4376D-05 | -0 8924D+02 | 0 7155D+00 |
| 23 | -0 5158D-05 | 0 4337D-05 | 0 2544D-03 | -0 2425D-03 | 0 6869D-05 | -0 6166D-05 | 0 2544D-03 | 0 4376D-05 | 0 8924D+02 | -0 7155D+00 |
| 24 | -0 1580D-04 | 0.1452D-04 | 0 2425D-03 | -0 2301D-03 | -0 3902D-04 | 0 3657D-04 | 0.2440D-03 | 0 1588D-04 | -0 8571D+02 | 0.4252D+01 |
| 25 | -0 1580D-04 | 0 1452D-04 | 0 2425D-03 | -0 2301D-03 | -0.3902D-04 | 0 3657D-04 | 0 2440D-03 | 0.1588D-04 | -0 8571D+02 | 0 4252D+01 |
| 26 | -0 5158D-05 | 0 4337D-05 | 0 2544D-03 | -0 2425D-03 | 0 6869D-05 | -0 6166D-05 | 0 2544D-03 | 0 4376D-05 | 0 8924D+02 | -0.7155D+00 |
| 27 | -0 5158D-05 | 0 4337D-05 | 0 2544D-03 | -0 2425D-03 | -0 6869D-05 | 0 6166D-05 | 0 2544D-03 | 0.4376D-05 | -0.8924D+02 | 0 7155D+00 |
| 28 | -0 1580D-04 | 0 1452D-04 | 0 2425D-03 | -0 2301D-03 | 0 3902D-04 | -0 3657D-04 | 0 2440D-03 | 0 1588D-04 | 0 8571D+02 | -0.4252D+01 |
| 29 | 0 4304D-03 | -0 4367D-03 | -0 3791D-02 | 0 3879D-02 | 0 4672D-04 | -0 3192D-04 | 0 4306D-03 | 0 3879D-02 | 0 3170D+00 | -0.8977D+02 |
| 30 | -0 6347D-05 | 0 3501D-05 | -0 3841D-02 | 0 3929D-02 | -0 4160D-05 | 0 5297D-05 | 0 0 | 0 3929D-02 | 0 0 | 0 8996D+02 |
| 31 | -0 6347D-05 | 0 3501D-05 | -0 3841D-02 | 0 3929D-02 | 0 4160D-05 | -0 5297D-05 | 0 0 | 0 3929D-02 | 0 0 | -0 8996D+02 |
| 32 | 0 4304D-03 | -0 4367D-03 | -0 3791D-02 | 0 3879D-02 | -0 4672D-04 | 0 3492D-04 | 0.4306D-03 | 0 3879D-02 | -0 3170D+00 | 0 8977D+02 |
| 33 | -0 3198D-03 | 0 3140D-03 | 0 2029D-02 | -0 2032D-02 | -0 3506D-04 | 0 2965D-04 | 0 2029D-02 | 0 3141D-03 | -0 8957D+02 | 0.3621D+00 |
| 34 | 0 6537D-05 | -0 8780D-05 | 0 2048D-02 | -0 2052D-02 | 0 3822D-05 | 0 5089D-05 | 0 2048D-02 | 0 0 | 0 8995D+02 | 0 0 |
| 35 | 0 6537D-05 | -0.8780D-05 | 0 2048D-02 | -0 2052D-02 | -0 3822D-05 | 0 5089D-05 | 0 2048D-02 | 0 0 | -0 8995D+02 | 0.0 |
| 36 | -0 3198D-03 | 0 3140D-03 | 0 2029D-02 | -0 2032D-02 | 0 3506D-04 | -0 2965D-04 | 0 2029D-02 | 0 3141D-03 | 0.8957D+02 | -0 3621D+00 |
| 37 | -0 1811D-04 | 0.1850D-04 | 0 6075D-04 | -0 5588D-04 | 0 1082D-04 | -0 9203D-05 | 0 6112D-04 | 0 1878D-04 | 0 8610D+02 | -0.3527D+01 |
| 38 | -0 1927D-05 | 0 1810D-05 | 0 6259D-04 | -0 5806D-04 | -0 5120D-06 | 0 4117D-06 | 0 6260D-04 | 0.1841D-05 | -0 8977D+02 | 0.1969D+00 |
| 39 | -0.1927D-05 | 0 1840D-05 | 0 6259D-04 | -0 5806D-04 | 0 5120D-06 | -0 4117D-06 | 0.6260D-04 | 0.1841D-05 | 0 8977D+02 | -0.1969D+00 |
| 40 | -0 1811D-04 | 0.1850D-04 | 0 6075D-04 | -0 5588D-04 | -0 1082D-04 | 0 9203D-05 | 0 6112D-04 | 0 1878D-04 | -0 8610D+02 | 0.3527D+01 |
| 41 | -0 3262D-05 | 0 2731D-05 | 0 1789D-05 | -0 1430D-05 | 0 2375D-05 | -0 2893D-05 | 0 2054D-05 | 0.3184D-05 | 0.7741D+02 | -0.1740D+02 |
| 42 | -0 5785D-06 | 0 4808D-06 | 0 2185D-05 | -0 1797D-05 | 0 3460D-07 | 0.2198D-07 | 0 2186D-05 | 0.4808D-06 | 0 8964D+02 | 0.2765D+00 |
| 43 | -0 5785D-06 | 0 4906D-06 | 0 2185D-05 | -0 1797D-05 | -0 3460D-07 | -0 2198D-07 | 0 2186D-05 | 0 4808D-06 | -0 8964D+02 | -0.2765D+00 |
| 44 | -0 3262D-05 | 0 2731D-05 | 0.1769D-05 | -0 1430D-05 | -0 2375D-05 | 0 2893D-05 | 0 2054D-05 | 0.3184D-05 | -0.7741D+02 | 0.1740D+02 |
| 45 | -0.7506D-06 | 0.8548D-06 | -0 8177D-06 | 0.8225D-06 | 0.2871D-06 | -0.1715D-06 | 0.0 | 0.9259D-06 | 0.0 | -0.3968D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 46 | -0.2251D-06 | 0 2066D-06 | -0.7512D-06 | 0.7447D-06 | -0 2786D-07 | -0.4083D-07 | 0.0 | 0.7454D-06 | 0.0 | -0.8783D+02 |
| 47 | -0.2251D-06 | 0 2066D-06 | -0 7512D-06 | 0 7447D-06 | 0.2786D-07 | 0.4083D-07 | 0.0 | 0.7454D-06 | 0.0 | 0.8783D+02 |
| 48 | -0 7506D-06 | 0 8548D-06 | -0.8177D-06 | 0 8225D-06 | -0.2871D-06 | 0.1715D-06 | 0.0 | 0.9259D-06 | 0.0 | 0.3968D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0 1097D-05 | 0 6767D-06 | 0 8008D-04 | -0.6843D-04 | 0 2520D-14 | -0.2520D-14 | 0.8008D-04 | 0.6767D-06 | 0.9000D+02 | -0.1045D-08 |
| 35 | -0 1826D-04 | 0 1458D-04 | 0.2419D-04 | -0 1132D-04 | 0.1026D-14 | -0.1026D-14 | 0.2419D-04 | 0.1458D-04 | 0.9000D+02 | -0.1135D-08 |
| 38 | -0 1338D-05 | 0.1180D-06 | -0 6095D-03 | 0 6139D-03 | 0.2208D-14 | -0.2208D-14 | 0.0 | 0.6139D-03 | 0.0 | -0.9000D+02 |
| 40 | 0.3407D-03 | -0 3334D-03 | -0.4646D-03 | 0.4736D-03 | 0 1169D-02 | -0.1166D-02 | 0.6478D-03 | 0.7780D-03 | 0.2772D+02 | -0.6243D+02 |
| 48 | -0 2828D-05 | 0 2813D-05 | -0.9102D-03 | 0 9111D-03 | 0.3338D-16 | -0.3372D-16 | 0.0 | 0.9111D-03 | 0.0 | -0.9000D+02 |
| 50 | 0.2339D-03 | -0 2316D-03 | -0.1079D-02 | 0 1081D-02 | -0.5737D-03 | 0 5744D-03 | 0.2938D-03 | 0.1141D-02 | -0.1180D+02 | 0.7818D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 47392D-43 | 0.11157D-05 | -0.61574D-21 | 0.0 | 0.11157D-05 | 0.11157D-05 | -0.90000D+02 |
| 1 | OUTER | 0 47392D-43 | -0.10934D-05 | -0 61574D-21 | 0.0 | -0.10934D-05 | 0.0 | 0 0 |
| 2 | INNER | 0 96753D-07 | 0 17628D-06 | 0 15419D-07 | 0 96753D-07 | 0.17628D-06 | 0.17703D-06 | 0.84514D+02 |
| 2 | OUTER | -0.10631D-06 | -0.15391D-06 | 0.15419D-07 | -0 10631D-06 | -0.15391D-06 | 0.0 | 0.0 |
| 3 | INNER | -0 26475D-06 | C.28134D-05 | 0.11352D-09 | -0 26475D-06 | 0 28134D-05 | 0.28134D-05 | 0.89999D+02 |
| 3 | OUTER | 0.19245D-06 | -0.25240D-05 | 0 11352D-09 | 0.19245D-06 | -0.25240D-05 | 0.19245D-06 | 0.11972D-02 |
| 4 | INNER | -0 58631D-06 | 0.17956D-04 | -0.81300D-08 | -0 58631D-06 | 0.17956D-04 | 0.17956D-04 | -0.89987D+02 |
| 4 | OUTER | 0 47420D-06 | -0.13203D-04 | -0.81300D-08 | 0 47420D-06 | -0.13204D-04 | 0.47420D-06 | -0.17028D-01 |
| 5 | INNER | 0 82019D-06 | 0.87315D-03 | 0.17148D-06 | 0.82019D-06 | 0.87277D-03 | 0.87315D-03 | 0.89994D+02 |
| 5 | OUTER | -0 24166D-05 | -0.87317D-03 | 0 17148D-06 | -0 21167D-05 | -0.87356D-03 | 0.0 | 0.0 |
| 6 | INNER | -0 10974D-05 | 0 80084D-04 | 0 25196D-14 | -0.10974D-05 | 0 80081D-04 | 0.80084D-04 | 0.90000D+02 |
| 6 | OUTER | 0 67672D-06 | -0.68432D-04 | -0.25199D-14 | 0 67672D-06 | -0.68435D-04 | 0.67672D-06 | -0.10446D-08 |
| 7 | INNER | 0 82019D-06 | 0.87315D-03 | -0.17148D-06 | 0.82019D-06 | 0.87277D-03 | 0 87315D-03 | -0.89994D+02 |
| 7 | OUTER | -0 24166D-05 | -0.87317D-03 | -0 17148D-06 | -0.24167D-05 | -0.87356D-03 | 0.0 | 0.0 |
| 8 | INNER | -0 58631D-06 | 0 17956D-04 | 0 81300D-08 | -0.58631D-06 | 0.17956D-04 | 0.17956D-04 | 0.89987D+02 |
| 8 | OUTER | 0.47420D-06 | -0.13203D-04 | 0 81300D-08 | 0.47420D-06 | -0.13204D-04 | 0.47420D-06 | 0.17028D-01 |
| 9 | INNER | -0 26475D-06 | 0.28134D-05 | -0.11352D-09 | -0.26475D-06 | 0.28134D-05 | 0.28134D-05 | -0.89999D+02 |
| 9 | OUTER | 0 19245D-06 | -0.25240D-05 | -0.11352D-09 | 0.19245D-06 | -0.25240D-05 | 0.19245D-06 | -0.11972D-02 |
| 10 | INNER | 0 96753D-07 | 0.17628D-06 | -0.15419D-07 | 0.96753D-07 | 0.17628D-06 | 0.17703D-06 | -0.84514D+02 |
| 10 | OUTER | -0 10631D-06 | -0.15391D-06 | -0 15419D-07 | -0 10631D-06 | -0.15391D-06 | 0.0 | 0.0 |
| 11 | INNER | 0 64420D-22 | 0.11157D-05 | 0.48733D-20 | 0 0 | 0.11157D-05 | 0.11157D-05 | 0.90000D+02 |
| 11 | OUTER | -0.64420D-22 | -0.10934D-05 | 0.17071D-20 | 0.0 | -0.10934D-05 | 0.0 | 0.0 |

SYSTEM ENERGIES(IN-LB)

| | | |
|---|---|---|
| WORK INPUT TO STRUCTURE | = | 0 7522995D+03 |
| STRUCTURE KINETIC ENERGY | = | 0 4927275D+03 |
| STRUCTURE ELASTIC ENERGY | = | 0 8634123D+01 |
| STRUCTURE PLASTIC ENERGY | = | 0.2509379D+03 |
| ENERGY STORED IN ELASTIC RESTRAINTS | = | 0.0 |

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•

***** INCR. NO.= 10 TIME= 0.2500D-04 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | C.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0 0 | 0.0 |
| 5 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 14970D+01 | 0.0 | 0.0 |
| 6 | 0 37539D-03 | 0 11017D-02 | -0.53539D-03 | 0 69702D-03 | -0 68163D-02 | 0.11178D-01 | 0.37539D-03 | 0.77610D+00 | -0 53539D-03 |
| 7 | 0.17290D-03 | 0 10075D-02 | -0.43758D-03 | 0 33985D-04 | -0 53111D-02 | 0.97021D-03 | 0.37442D+00 | 0 77601D+00 | -0 43758D-03 |
| 8 | 0 32831D-17 | 0 99781D-03 | -0 43805D-03 | 0 25397D-16 | -0 51930D-02 | 0 12393D-14 | 0.74850D+00 | 0 77600D+00 | -0.43805D-03 |
| 9 | -0 17290D-03 | 0 10075D-02 | -0 43758D-03 | -0.33985D-04 | -0.53111D 02 | -0.97021D-03 | 0 11226D+01 | 0 77601D+00 | -0 43758D-03 |
| 10 | -0 37539D-03 | 0 11017D-02 | -0 53539D-03 | -0 69702D-03 | -0 68163D-02 | -0 11178D-01 | 0.14966D+01 | 0.77610D+00 | -0 53539D-03 |
| 11 | 0 51893D-03 | 0 25561D-02 | 0 29139D-02 | -0 42725D-02 | 0 60749D-02 | -0.15508D-01 | 0 51893D-03 | 0.15526D+01 | 0.29139D-02 |
| 12 | 0 22141D-03 | 0.25025D-02 | 0 22386D-02 | -0 56948D-03 | 0 35779D-02 | -0.33484D-02 | 0 37447D+00 | 0.15525D+01 | 0.22386D-02 |
| 13 | 0 23531D-16 | 0 24181D-02 | 0 21425D-02 | -0 29177D-15 | 0 28233D-02 | 0.42387D-15 | 0 74850D+00 | 0.15524D+01 | 0.21425D-02 |
| 14 | -0 22141D-03 | 0 25025D-02 | 0 22386D-02 | 0 56948D-03 | 0 35779D-02 | 0 33484D-02 | 0 11225D+01 | 0.15525D+01 | 0.22386D-02 |
| 15 | -0 51893D-03 | 0 25561D-02 | 0 29139D-02 | 0 42725D-02 | 0 60749D-02 | 0 15508D-01 | 0.14965D+01 | 0 15526D+01 | 0.29139D-02 |
| 16 | 0 62880D-03 | 0 47930D-02 | -0 12131D-01 | 0 21474D-01 | -0 18607D-02 | 0 64964D-01 | 0 62880D-03 | 0 23298D+01 | -0.12131D-01 |
| 17 | 0 37548D-03 | 0 43261D-02 | -0 83097D-02 | 0 30158D-02 | 0 86620D-02 | 0.97201D-02 | 0 37463D+00 | 0 23293D+01 | -0.83097D-02 |
| 18 | 0 22975D-16 | 0 42870D-02 | -0.80237D-02 | -0 82876D-15 | 0 98036D-02 | -0 93441D-14 | 0 74850D+00 | 0 23293D+01 | -0.80237D-02 |
| 19 | -0 37548D-03 | 0 43261D-02 | -0 83097D-02 | -0 30158D-02 | 0.86620D-02 | -0 97201D-02 | 0 11224D+01 | 0.23293D+01 | -0 83097D-02 |
| 20 | -0 62880D-03 | 0 47930D-02 | -0 12131D-01 | -0 21474D-01 | -0 18607D-02 | -0 64964D-01 | 0 14964D+01 | 0 23298D+01 | -0.12131D-01 |
| 21 | 0 24981D-02 | 0 31733D-02 | 0 85074D-01 | 0 37644D-02 | 0 22632D+00 | -0 95707D-01 | 0 24981D-02 | 0.31032D+01 | 0.85074D-01 |
| 22 | 0 97269D-03 | 0 27280D-02 | 0 85752D-01 | 0 45403D-03 | 0 20997D+00 | -0.11509D-01 | 0 37522D+00 | 0.31027D+01 | 0 85752D-01 |
| 23 | -0 88037D-16 | 0 27314D-02 | 0 85758D-01 | -0 29178D-14 | 0 20810D+00 | -0 40704D-13 | 0 74850D+00 | 0 31027D+01 | 0 85758D-01 |
| 24 | -0 97269D-03 | C 27280D-02 | 0 85752D-01 | -0.45403D-03 | 0 20997D+00 | 0.11509D-01 | 0 11218D+01 | 0 31027D+01 | 0.85752D-01 |
| 25 | -0 24981D-02 | 0 31733D-02 | 0 85074D-01 | -0 37644D-02 | 0 22632D+00 | 0 95707D-01 | 0 14945D+01 | 0.31032D+01 | 0.85074D-01 |
| 26 | 0 15211D-02 | -0 24657D-02 | 0 17265D+00 | -0 29963D-01 | 0 12132D+00 | -0.12969D-01 | 0 15211D-02 | 0 35475D+01 | 0 17265D+00 |
| 27 | 0 82648D-03 | -0 18116D-02 | 0 16775D+00 | -0 39461D-02 | 0.11680D+00 | -0 25997D-02 | 0 37508D+00 | 0 35482D+01 | 0.16775D+00 |
| 28 | -0 58625D-16 | -0 19584D-02 | 0 16733D+00 | 0 18579D-14 | 0 11707D+00 | -0 57457D-13 | 0.74850D+00 | 0 35480D+01 | 0.16733D+00 |
| 29 | -0.82648D-03 | -0 18116D-02 | 0 16775D+00 | 0 39461D-02 | 0 11680D+00 | 0 25997D-02 | 0 11219D+01 | 0.35482D+01 | 0.16775D+00 |
| 30 | -0 15211D-02 | -0 24657D-02 | 0 17265D+00 | 0 29963D-01 | 0 12132D+00 | 0 12969D-01 | 0 14955D+01 | 0.35475D+01 | 0 17265D+00 |
| 31 | 0 13316D-02 | 0 28346D-15 | 0 19546D+00 | -0 23635D-01 | 0 24744D-13 | -0 33706D-12 | 0 13316D-02 | 0 40000D+01 | 0.19546D+00 |
| 32 | 0 55316D-03 | 0 12620D-15 | 0.19089D+00 | -0 29098D-02 | 0 25951D-14 | -0 27534D-13 | 0 37480D+00 | 0 40000D+01 | 0.19089D+00 |
| 33 | 0 40732D-16 | 0 47325D-16 | 0 19075D+00 | 0 33220D-14 | 0 C6613D-15 | -0 27117D-13 | 0 74850D+00 | 0 40000D+01 | 0.19075D+00 |
| 34 | -0 55316D-03 | -0 87278D-17 | 0 19089D+00 | 0 29098D-02 | -0 80491D-15 | -0 53513D-13 | 0 11222D+01 | 0 40000D+01 | 0.19089D+00 |
| 35 | -0 13316D-02 | -0 10701D-15 | 0 19546D+00 | 0 23635D-01 | -0 14891D-13 | -0 25087D-12 | 0.14957D+01 | 0 40000D+01 | 0.19546D+00 |
| 36 | 0 15211D-02 | 0 24657D-02 | 0 17265D+00 | -0 29963D-01 | -0 12132D+00 | 0 12969D-01 | 0 15211D-02 | 0 44525D+01 | 0.17265D+00 |
| 37 | 0 82648D-03 | 0 18116D-02 | 0 16775D+00 | -0 39461D-02 | -0 11680D+00 | 0 25997D-02 | 0 37508D+00 | 0 44518D+01 | 0 16775D+00 |
| 38 | 0 15428D-15 | 0 19584D-02 | 0 16733D+00 | 0 12551D-14 | -0 11707D+00 | 0 63699D-14 | 0 74850D+00 | 0 44520D+01 | 0.16733D+00 |
| 39 | -0 82648D-03 | 0.18116D-02 | 0.16775D+00 | 0.39461D-02 | -0 11680D+00 | -0.25997D-02 | 0.11219D+01 | 0 44518D+01 | 0.16775D+00 |
| 40 | -0 15211D-02 | 0 24657D-02 | 0 17265D+00 | 0 29963D-01 | -0 12132D 00 | -0 12969D-01 | 0 14955D+01 | 0 44525D+01 | 0 17265D+00 |
| 41 | 0 24981D-02 | -0 31733D-02 | 0 85074D-01 | 0 37644D-02 | -0 22632D+00 | 0 95707D-01 | 0 24981D-02 | 0 48968D+01 | 0.85074D-01 |
| 42 | 0 97269D-03 | -0 27280D-02 | 0 85752D-01 | 0 45403D-03 | -0 20997D+00 | 0.11509D-01 | 0 37522D+00 | 0 48973D+01 | 0.85752D-01 |
| 43 | -0 10186D-15 | -0 27314D-02 | 0 85758D-01 | -0 60256D-14 | -0 20810D+00 | 0 80602D-13 | 0.74850D+00 | 0 48973D+01 | 0 85758D-01 |
| 44 | -0 97269D-03 | -0 27280D-02 | 0 85752D-01 | -0 45403D-03 | -0 20997D+00 | -0.11509D-01 | 0 11218D+01 | 0 48973D+01 | 0 85752D-01 |
| 45 | -0 24981D-02 | -0 31733D-02 | 0 85074D-01 | -0 37644D-02 | -0 22632D+00 | -0.95707D-01 | 0 14945D+01 | 0 48968D+01 | 0 85074D-01 |
| 46 | 0 62880D-03 | -0 47930D-02 | -0 12131D-01 | 0 21474D-01 | 0 18607D-02 | -0 64964D-01 | 0.62880D-03 | 0 56702D+01 | -0 12131D-01 |
| 47 | 0 37548D-03 | -0 43261D-02 | -0 83097D-02 | 0 30158D-02 | -0 86620D-02 | -0.97201D-02 | 0 37463D+00 | 0 56707D+01 | -0.83097D-02 |
| 48 | 0 41979D-17 | -0 42870D-02 | -0 80237D-02 | -0 94017D-15 | -0 96036D-02 | 0 65422D-14 | 0.74850D+00 | 0 56707D+01 | -0 80237D-02 |
| 49 | -0 37548D-03 | -0 43261D-02 | -0 83097D-02 | -0 30158D-02 | -0 86620D-02 | 0.97201D-02 | 0.11224D+01 | 0 56707D+01 | -0.83097D-02 |
| 50 | -0 62880D-03 | -0 47930D-02 | -0 12131D-01 | -0 21474D-01 | 0 18607D-02 | 0 64964D-01 | 0 14964D+01 | 0 56702D+01 | -0.12131D-01 |
| 51 | 0 51893D-03 | -0 25561D-02 | 0 29139D-02 | -0 42725D-02 | -0 60749D-02 | 0.15508D-01 | 0 51893D-03 | 0 64474D+01 | 0 29139D-02 |
| 52 | 0 22141D-03 | -0 25025D-02 | 0.22386D-02 | -0 56948D-03 | -0 35779D-02 | 0 33484D-02 | 0 37447D+00 | 0 64475D+01 | 0 22386D-02 |
| 53 | 0 38692D-17 | -0 24181D-02 | 0 21425D-02 | -0 41742D-16 | -0 28233D-02 | 0 27067D-14 | 0 74850D+00 | 0 64476D+01 | 0 21425D-02 |
| 54 | -0 22141D-03 | -0 25025D-02 | 0 22386D-02 | 0 56948D-03 | -0 35779D-02 | -0 33484D-02 | 0 11225D+01 | 0 64475D+01 | 0 22386D-02 |
| 55 | -0 51893D-03 | -0 25561D-02 | 0 29139D-02 | 0 42725D-02 | -0 60749D-02 | -0 15508D-01 | 0.14965D+01 | 0 64474D+01 | 0 29139D-02 |
| 56 | 0 37539D-03 | -0 11017D-02 | -0 53539D-03 | 0 69702D-03 | 0 68163D-02 | -0 11178D-01 | 0 37539D-03 | 0 72239D+01 | -0.53539D-03 |
| 57 | 0 17290D-03 | -0 10075D-02 | -0.43758D-03 | 0 33985D-04 | 0 53111D-02 | -0 97021D-03 | 0.37442D+00 | 0 72240D+01 | -0.43758D-03 |
| 58 | 0 34423D-17 | -0 99781D-03 | -0 43803D-03 | -0 80377D-16 | 0 51930D-02 | 0 59252D-15 | 0.74850D+00 | 0.72240D+01 | -0.43805D-03 |
| 59 | -0 17290D-03 | -0 10075D-02 | -0 43758D-03 | -0 33985D-04 | 0 53111D-02 | 0 97021D-03 | 0 11226D+01 | 0 72240D+01 | -0 43758D-03 |
| 60 | -0 37539D-03 | -0 11017D-02 | -0 53539D-03 | -0 69702D-03 | 0 68163D-02 | 0.11178D-01 | 0 14966D+01 | 0 72239D+01 | -0.53539D-03 |
| 61 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 80000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | C 0 | 0 0 | 0 0 | 0.0 | 0 37425D+00 | 0 80000D+01 | 0.0 |
| 63 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 748500+00 | 0 80000D+01 | 0.0 |
| 64 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0 80000D+01 | 0.0 |
| 65 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | -0.13544D+03 | -0.29965D+03 | -0 31238D+02 | -0.24713D+01 | -0.59216D+01 | -0 42464D+00 |
| 2 | -0.18163D+02 | -0.47127D+03 | -0.51304D+02 | -0 13785D+00 | -0.10160D+02 | -0.94223D-02 |
| 3 | 0.16698D-12 | -0.45729D+03 | -0.53312D+02 | -0 33618D-13 | -0 10316D+02 | -0 22326D-13 |
| 4 | 0.18163D+02 | -0 47127D+03 | -0.51304D+02 | 0.13785D+00 | -0 10160D+02 | 0.94223D-02 |
| 5 | 0.13544D+03 | -0 29965D+03 | -0 31238D+02 | 0 24713D+01 | -0 59216D+01 | 0 42464D+00 |
| 61 | -0.13544D+03 | 0 29965D+03 | -0.31238D+02 | -0 24713D+01 | 0 59216D+01 | 0 42464D+00 |
| 62 | -0.18163D+02 | 0 47127D+03 | -0.51304D+02 | -0 13785D+00 | 0 10160D+02 | 0 94223D-02 |
| 63 | -0 85976D-12 | 0.45729D+03 | -0 53312D+02 | 0 25752D-12 | 0 10316D+02 | -0 37319D-13 |
| 64 | 0 18163D+02 | 0 47127D+03 | -0 51304D+02 | 0.13785D+00 | 0 10160D+02 | -0 94223D-02 |
| 65 | 0 13544D+03 | 0 29965D+03 | -0 31238D+02 | 0 24713D+01 | 0.59216D+01 | -0 42464D+00 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0.1809D-03 | -0 3601D-03 | 0 9933D-03 | 0.1729D-02 | 0 1927D-03 | 0 2628D-03 | 0 1001D-02 | 0.1737D-02 | 0.8534D+02 | 0.8641D+02 |
| 2 | -0.2205D-03 | -0 2415D-03 | 0 9512D-03 | 0 1636D-02 | 0 9065D-04 | 0 1065D-03 | 0 9530D-03 | 0 1638D-02 | 0.8779D+02 | 0.8838D+02 |
| 3 | -0 2205D-03 | -0 2415D-03 | 0 9512D-03 | 0 1636D-02 | -0 9065D-04 | -0 1065D-03 | 0 9530D-03 | 0 1638D-02 | -0 8779D+02 | -0.8838D+02 |
| 4 | -0 1809D-03 | -0 3601D-03 | 0 9933D-03 | 0 1729D-02 | -0 1927D-03 | -0 2628D-03 | 0.1001D-02 | 0.1737D-02 | -0.8534D+02 | -0.8641D+02 |
| 5 | -0.7561D-03 | -0 5798D-03 | 0 2568D-02 | 0 1272D-02 | -0 3507D-03 | 0 2054D-03 | 0 2577D-02 | 0.1278D-02 | -0 8699D+02 | 0 8684D+02 |
| 6 | -0 5473D-03 | -0 5063D-03 | 0 2439D-02 | 0 1353D-02 | -0 8980D-04 | -0 9776D-04 | 0 2440D-02 | 0 1355D-02 | -0 8914D+02 | -0 8850D+02 |
| 7 | -0 5473D-03 | -0 5063D-03 | 0 2439D-02 | 0 1353D-02 | 0 8980D-04 | 0 9776D-04 | 0 2440D-02 | 0.1355D-02 | 0 8914D+02 | 0.8850D+02 |
| 8 | -0.7561D-03 | -0 5798D-03 | 0 2568D-02 | 0 1272D-02 | 0 3507D-03 | -0 2054D-03 | 0.2577D-02 | 0 1278D-02 | 0.8699D+02 | -0 8684D+02 |
| 9 | -0.8508D-03 | -0 6197D-03 | 0 3054D-02 | 0 2827D-02 | 0 1094D-02 | -0 2198D-02 | 0 3129D-02 | 0 3147D-02 | 0 8217D+02 | -0 7374D+02 |
| 10 | -0 7915D-03 | -0 8032D-03 | 0 3082D-02 | 0 2207D-02 | 0 6878D-04 | -0.1841D-03 | 0 3082D-02 | 0 2210D-02 | 0.8949D+02 | -0 8825D+02 |
| 11 | -0 7915D-03 | -0 8032D-03 | 0 3082D-02 | 0 2207D-02 | -0 6878D-04 | 0 1841D-03 | 0 3082D-02 | 0.2210D-02 | -0 8949D+02 | 0 8825D+02 |
| 12 | -0 8508D-03 | -0 6197D-03 | 0 3054D-02 | 0 2827D-02 | -0 1091D-02 | 0 2198D-02 | 0 3129D-02 | 0 3147D-02 | -0 8217D+02 | 0.7374D+02 |
| 13 | -0 5634D-02 | 0 1014D-02 | 0 1991D-01 | -0 7499D-02 | 0 7479D-03 | 0 2946D-02 | 0 1991D-01 | 0 1262D-02 | 0 8916D+02 | 0 9544D+01 |
| 14 | -0 2317D-02 | -0 1284D-02 | 0 1911D-01 | -0 7053D-02 | 0 4089D-03 | 0 4716D-03 | 0.1912D-01 | 0 0 | 0.8945D+02 | 0 0 |
| 15 | -0 2317D-02 | -0 1284D-02 | 0 1911D-01 | -0 7053D-02 | -0 4089D-03 | -0.4716D-03 | 0 1912D-01 | 0 0 | -0 8945D+02 | 0 0 |
| 16 | -0 5634D-02 | 0 1014D-02 | 0 1991D-01 | -0 7499D-02 | -0 7479D-03 | -0 2946D-02 | 0 1991D-01 | 0 1262D-02 | -0 8916D+02 | -0 9544D+01 |
| 17 | -0 8338D-03 | -0 5060D-02 | -0.3166D-02 | 0 1851D-01 | -0 5171D-02 | 0 8464D-03 | 0 8365D-03 | 0 1852D-01 | -0.3286D+02 | 0 8897D+02 |
| 18 | -0 2097D-02 | -0 2710D-02 | -0 2482D-02 | 0 1830D-01 | -0 6203D-03 | -0 3695D-03 | 0 0 | 0 1830D-01 | 0 0 | -0 8950D+02 |
| 19 | -0 2097D-02 | -0 2710D-02 | -0.2182D-02 | 0 1830D-01 | 0 6203D-03 | 0 3695D-03 | 0 0 | 0 1830D-01 | 0 0 | 0 8950D+02 |
| 20 | -0 8338D-03 | -0 5060D-02 | -0 3166D-02 | 0 1851D-01 | 0 5171D-02 | -0 8464D-03 | 0 8365D-03 | 0 1852D-01 | 0 3286D+02 | -0 8897D+02 |
| 21 | 0 1371D-02 | -0 5157D-02 | -0 7545D-02 | 0 1933D-01 | -0 4747D-04 | -0 4016D-03 | 0 1371D-02 | 0 1933D-01 | -0 1525D+00 | -0.8953D+02 |
| 22 | -0 1356D-02 | -0 2330D-02 | -0 7871D-02 | 0 1861D-01 | -0 2614D-03 | -0 7533D-03 | 0 0 | 0 1861D-01 | 0 0 | -0 8897D+02 |
| 23 | -0 1356D-02 | -0 2330D-02 | -0 7871D-02 | 0 1861D-01 | 0 2614D-03 | 0 7533D-03 | 0 0 | 0 1861D-01 | 0 0 | 0 8897D+02 |
| 24 | 0 1371D-02 | -0 5157D-02 | -0 7545D-02 | 0 1933D-01 | 0 4747D-04 | 0 4016D-03 | 0 1371D-02 | 0.1933D-01 | 0 1525D+00 | 0 8953D+02 |
| 25 | 0.1371D-02 | -0 5157D-02 | -0 7545D-02 | 0 1933D-01 | 0 4747D-04 | 0 4016D-03 | 0 1371D-02 | 0 1933D-01 | 0 1525D+00 | 0 8953D+02 |
| 26 | -0 1356D-02 | -0 2330D-02 | -0.7871D-02 | 0 1861D-01 | 0 2614D-03 | 0 7533D-03 | 0 0 | 0 1861D-01 | 0 0 | 0.8897D+02 |
| 27 | -0 1356D-02 | -0 2330D-02 | -0 7871D-02 | 0 1861D-01 | -0 2614D-03 | -0 7533D-03 | 0 0 | 0 1861D-01 | 0.0 | -0 8897D+02 |
| 28 | 0 1371D-02 | -0 5157D-02 | -0 7545D-02 | 0 1933D-01 | -0 4747D-04 | -0 4016D-03 | 0 1371D-02 | 0 1933D-01 | -0 1525D+00 | -0 8953D+02 |
| 29 | -0 8338D-03 | -0 5060D-02 | -0 3166D-02 | 0 1851D-01 | 0 5171D-02 | -0 8464D-03 | 0 8365D-03 | 0.1852D-01 | 0.3286D+02 | -0.8897D+02 |
| 30 | -0 2097D-02 | -0 2710D-02 | -0 2482D-02 | 0 1830D-01 | 0 6203D-03 | 0 3695D-03 | 0 0 | 0 1830D-01 | 0.0 | 0 8950D+02 |
| 31 | -0 2097D-02 | -0 2710D-02 | -0 2432D-02 | 0 1830D-01 | -0 6203D-03 | -0 3695D-03 | 0 0 | 0 1830D-01 | 0.0 | -0 8950D+02 |
| 32 | -0 8338D-03 | -0 5060D-02 | -0 3166D-02 | 0 1851D-01 | -0 5171D-02 | 0 8464D-03 | 0 8365D-03 | 0 1852D-01 | -0.3286D+02 | 0 8897D+02 |
| 33 | -0 5634D-02 | 0 1014D-02 | 0 1991D-01 | -0.7499D-02 | -0 7479D-03 | -0 2946D-02 | 0 1991D-01 | 0 1262D-02 | -0 8916D+02 | -0.9544D+01 |
| 34 | -0 2317D-02 | -0 1284D-02 | 0 1911D-01 | -0 7053D-02 | -0 4089D-03 | -0 4716D-03 | 0 1912D-01 | 0 0 | -0 8945D+02 | 0 0 |
| 35 | -0 2317D-02 | -0 1284D-02 | 0 1911D-01 | -0 7053D-02 | 0 4089D-03 | 0 4716D-03 | 0 1912D-01 | 0 0 | 0 8945D+02 | 0 0 |
| 36 | -0 5631D-02 | 0 1014D-02 | 0 1991D-01 | -0 7499D-02 | 0 7479D-03 | 0 2946D-02 | 0 1991D-01 | 0 1262D-02 | 0 8916D+02 | 0 9544D+01 |
| 37 | -0 8508D-03 | -0 6197D-03 | 0 3054D-02 | 0 2827D-02 | -0 1031D-02 | 0 2198D-02 | 0 3129D-02 | 0 3147D-02 | -0 8217D+02 | 0 7374D+02 |
| 38 | -0 7915D-03 | -0 8032D-03 | 0 3082D-02 | 0 2207D-02 | -0 6878D-04 | 0 1841D-03 | 0 3082D-02 | 0 2210D-02 | -0 8949D+02 | 0 8825D+02 |
| 39 | -0 7915D-03 | -0 8032D-03 | 0 3092D-02 | 0 2207D-02 | 0 6878D-04 | -0 1841D-03 | 0 3082D-02 | 0 2210D-02 | 0.8949D+02 | -0 8825D+02 |
| 40 | -0 8508D-03 | -0 6197D-03 | 0 3054D-02 | 0 2827D-02 | 0 1094D-02 | -0 2198D-02 | 0 3129D-02 | 0 3147D-02 | 0 8217D+02 | -0 7374D+02 |
| 41 | -0 7561D-03 | -0 5798D-03 | 0 2568D-02 | 0 1272D-02 | 0 3507D-03 | -0 2054D-03 | 0 2577D-02 | 0 1278D-02 | 0 8699D+02 | -0 8684D+02 |
| 42 | -0 5473D-03 | -0 5063D-03 | 0 2439D-02 | 0 1353D-02 | 0 8980D-04 | 0 9776D-04 | 0 2440D-02 | 0.1355D-02 | 0 8914D+02 | 0.8850D+02 |
| 43 | -0 5473D-03 | -0 5063D-03 | 0 2439D-02 | 0 1353D-02 | -0 8980D-04 | -0 9776D-04 | 0 2440D-02 | 0 1355D-02 | -0.8914D+02 | -0.8850D+02 |
| 44 | -0 7561D-03 | -0 5798D-03 | 0 2568D-02 | 0 1272D-02 | -0 3507D-03 | 0 2054D-03 | 0 2577D-02 | 0 1278D-02 | -0.8699D+02 | 0 8684D+02 |
| 45 | -0 1809D-03 | -0 3601D-03 | 0 9933D-03 | 0 1729D-02 | -0 1927D-03 | -0 2628D-03 | 0 1001D-02 | 0 1737D-02 | -0.8534D+02 | -0 8641D+02 |
| 46 | -0 2205D-03 | -0 2415D-03 | 0 9512D-03 | 0 1636D-02 | -0.9065D-04 | -0.1065D-03 | 0 9530D-03 | 0 1638D-02 | -0 8779D+02 | -0.8838D+02 |
| 47 | -0 2205D-03 | -0 2415D-03 | 0 9512D-03 | 0 1636D-02 | 0 9065D-04 | 0 1065D-03 | 0 9530D-03 | 0.1638D-02 | 0.8779D+02 | 0 8838D+02 |
| 48 | -0 1809D-03 | -0 3601D-03 | 0 9933D-03 | 0 1729D-02 | 0 1927D-03 | 0.2628D-03 | 0 1001D-02 | 0.1737D-02 | 0.8534D+02 | 0.8641D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0.1970D-02 | -0 9863D-03 | -0.4499D-02 | 0 1320D-01 | -0.2709D-14 | 0.2822D-14 | 0 0 | 0.1320D-01 | 0.0 | 0.9000D+02 |
| 35 | 0.1881D-02 | -0.5482D-02 | -0 1500D-02 | 0.1246D-01 | -0 2603D-13 | 0 2514D-13 | 0.1881D-02 | 0.1246D-01 | -0.2206D-09 | 0.9000D+02 |
| 38 | -0.2377D-02 | -0 2039D-02 | -0 1654D-01 | 0.2417D-01 | 0.5048D-15 | -0.7942D-15 | 0.0 | 0 2417D-01 | 0.0 | -0.9000D+02 |
| 40 | 0.5301D-02 | -0 8112D-02 | -0.1944D-01 | 0 2711D-01 | -0 4506D-02 | -0.1861D-02 | 0 5504D-02 | 0.2714D-01 | -0.5161D+01 | -0.8849D+02 |
| 48 | -0 8061D-03 | -0.1200D-02 | 0 1322D-01 | -0 1272D-01 | 0 6684D-15 | -0 6662D-15 | 0.1322D-01 | 0.0 | 0.9000D+02 | 0.0 |
| 50 | -0 4623D-02 | 0 3733D-02 | 0 1450D-01 | -0 1370D-01 | 0.6610D-02 | -0 6637D-02 | 0 1506D-01 | 0.4343D-02 | 0 8046D+02 | -0.1042D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR 1) | ELONG (DIR 2) | PRINC STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 22432D-35 | 0 17478D-02 | 0 42363D-17 | 0 0 | 0 17463D-02 | 0 17478D-02 | 0 90000D+02 |
| 1 | OUTER | 0 22432D-35 | 0.82720D-03 | 0 42363D-17 | 0 0 | 0.82686D-03 | 0.82720D-03 | 0.90000D+02 |
| 2 | INNER | -0 11776D-03 | 0.13343D-02 | -0 66803D-05 | -0 11777D-03 | 0 13334D-02 | 0 13343D-02 | -0 89868D+02 |
| 2 | OUTER | -0 12069D-03 | 0 12416D-02 | -0 66803D-05 | -0 12069D-03 | 0 12409D-02 | 0 12417D-02 | -0 89860D+02 |
| 3 | INNER | -0 51366D-03 | 0 31113D-02 | -0 83830D-04 | -0 51379D-03 | 0 31065D-02 | 0 31118D-02 | -0 89338D+02 |
| 3 | OUTER | -0 48558D-03 | 0 56125D-03 | -0 83830D-04 | -0 48570D-03 | 0.56109D-03 | 0.56292D-03 | -0 87711D+02 |
| 4 | INNER | -0 55875D-03 | 0 35660D-03 | -0.18645D-03 | -0 55891D-03 | 0 35654D-03 | 0 36600D-03 | -0 84244D+02 |
| 4 | OUTER | -0 89005D-03 | 0 48627D-02 | -0 18645D-03 | -0 89044D-03 | 0 48708D-02 | 0 48842D-02 | -0 89075D+02 |
| 5 | INNER | -0 14401D-02 | 0 17102D-01 | -0 64270D-04 | -0 14412D-02 | 0 16958D-01 | 0 17102D-01 | -0 89901D+02 |
| 5 | OUTER | -0 16989D-02 | -0 11760D-01 | -0 64270D-04 | -0 17003D-02 | -0 11830D-01 | 0 0 | 0.0 |
| 6 | INNER | -0 19698D-02 | -0 44988D-02 | -0 27244D-14 | -0 19718D-02 | -0 45090D-02 | 0 0 | 0 0 |
| 6 | OUTER | -0 98629D-03 | 0 13203D-01 | 0 28075D-14 | -0 98678D-03 | 0.13117D-01 | 0 13203D-01 | 0 90000D+02 |
| 7 | INNER | -0 14401D-02 | 0.17102D-01 | 0 64270D-04 | -0 14412D-02 | 0 16958D-01 | 0.17102D-01 | 0 89901D+02 |
| 7 | OUTER | -0 16989D-02 | -0 11760D-01 | 0 64270D-04 | -0.17003D-02 | -0 11830D-01 | 0 0 | 0 0 |
| 8 | INNER | -0 55875D-03 | 0 35660D-03 | 0 18615D-03 | -0 55891D-03 | 0 35654D-03 | 0 36600D-03 | 0 84244D+02 |
| 8 | OUTER | -0 89005D-03 | 0 48827D-02 | 0 18645D-03 | -0 89044D-03 | 0.48708D-02 | 0 48842D-02 | 0 89075D+02 |
| 9 | INNER | -0 51366D-03 | 0.31113D-02 | 0 83830D-04 | -0 51379D-03 | 0 31065D-02 | 0 31118D-02 | 0.89338D+02 |
| 9 | OUTER | -0 48558D-03 | 0 56125D-03 | 0 83830D-04 | -0 48570D-03 | 0.56109D-03 | 0 56292D-03 | 0 87711D+02 |
| 10 | INNER | -0 11776D-03 | 0.13343D-02 | 0 66803D-05 | -0 11777D-03 | 0 13334D-02 | 0.13343D-02 | 0 89868D+02 |
| 10 | OUTER | -0 12069D-03 | 0.12416D-02 | 0 66803D-05 | -0 12069D-03 | 0.12409D-02 | 0.12417D-02 | 0.89860D+02 |
| 11 | INNER | 0 29881D-20 | 0.17478D-02 | -0 44927D-17 | 0 0 | 0 17463D-02 | 0.17478D-02 | -0.90000D+02 |
| 11 | OUTER | -0 29881D-20 | 0.82720D-03 | -0 43907D-17 | 0 0 | 0.82686D-03 | 0.82720D-03 | -0.90000D+02 |

SYSTEM ENERGIES(IN-LB)

| | | |
|---|---|---|
| WORK INPUT TO STRUCTURE | = | 0 3848832D+04 |
| STRUCTURE KINETIC ENERGY | = | 0 3301723D+04 |
| STRUCTURE ELASTIC ENERGY | = | 0 1230950D+03 |
| STRUCTURE PLASTIC ENERGY | = | 0 4240136D+03 |
| ENERGY STORED IN ELASTIC RESTRAINTS | = | 0 0 |

***** INCR. NO.= 20 TIME= 0.5000D-04 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 |
| 2 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 37425D+00 | 0 0 | 0.0 |
| 3 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 74850D+00 | 0.0 | 0.0 |
| 4 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 11227D+01 | 0.0 | 0.0 |
| 5 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.14970D+01 | 0 0 | 0.0 |
| 6 | 0 28299D-02 | 0 70454D-02 | 0 38147D-02 | -0 36581D-02 | 0.25434D-01 | -0.45887D-01 | 0.28299D-02 | 0.78205D+00 | 0.38147D-02 |
| 7 | 0.13757D-02 | 0 62248D-02 | 0 29997D-02 | -0 90020D-03 | 0.18150D-01 | -0.64043D-02 | 0 37563D+00 | 0 78122D+00 | 0.29997D-02 |
| 8 | -0 22172D-16 | 0 60713D-02 | 0 28621D-02 | -0 39720D-15 | 0 17036D-01 | -0 10036D-13 | 0.74850D+00 | 0 78107D+00 | 0.28621D-02 |
| 9 | -0 13757D-02 | 0 62248D-02 | 0 29997D-02 | 0 90020D-03 | 0 18150D-01 | 0 64043D-02 | 0 11214D+01 | 0 78122D+00 | 0 29997D-02 |
| 10 | -0 28299D-02 | 0.70454D-02 | 0.38147D-02 | 0 36581D-02 | 0 25434D-01 | 0 45887D-01 | 0 14942D+01 | 0 78205D+00 | 0.38147D-02 |
| 11 | 0 37607D-02 | 0 14470D-01 | -0 13503D-01 | 0 93701D-02 | -0 68517D 01 | 0.11840D+00 | 0 37607D-02 | 0 15645D+01 | -0 13503D-01 |
| 12 | 0 17601D-02 | 0 14671D-01 | -0 10707D-01 | 0 45938D-02 | -0 45889D-01 | 0 23092D-01 | 0 37601D+00 | 0 15647D+01 | -0.10707D-01 |
| 13 | -0 10143D-15 | 0 14550D-01 | -0 98922D-02 | 0 10040D-15 | -0 42209D-01 | -0 20200D-13 | 0 74850D+00 | 0 15646D+01 | -0.98922D-02 |
| 14 | -0 17601D-02 | 0 14671D-01 | -0 10707D-01 | -0.45938D-02 | -0 45889D-01 | -0.23092D-01 | 0 11210D+01 | 0 15647D+01 | -0.10707D-01 |
| 15 | -0 37607D-02 | 0 14470D-01 | -0 13503D-01 | -0 93701D-02 | -0 68517D-01 | -0.11840D+00 | 0 14932D+01 | 0.15645D+01 | -0.13503D-01 |
| 16 | 0 40293D-02 | 0 26891D-01 | 0 12202D-01 | 0 35573D-01 | 0.19271D+00 | -0.64781D-01 | 0.40293D-02 | 0.23519D+01 | 0.12202D-01 |
| 17 | 0 18949D-02 | 0 25242D-01 | 0 20668D-01 | 0 12427D-01 | 0 18350D+00 | -0.70183D-02 | 0 37614D+00 | 0.23502D+01 | 0.20668D-01 |
| 18 | -0 19689D-15 | 0 24637D-01 | 0 22809D-01 | 0 16888D-14 | 0 18212D+00 | -0 28101D-13 | 0.74850D+00 | 0 23498D+01 | 0.22809D-01 |
| 19 | -0.18949D-02 | 0 25242D-01 | 0.20668D-01 | -0 12427D-01 | 0.18350D+00 | 0 70183D-02 | 0 11209D+01 | 0 23502D+01 | 0.20668D-01 |
| 20 | -0 40293D-02 | 0 26891D-01 | 0 12202D-01 | -0 35573D-01 | 0 19271D+00 | 0.64781D-01 | 0 14930D+01 | 0.23519D+01 | 0.12202D-01 |
| 21 | 0.61609D-02 | 0.16756D-01 | 0 21426D+00 | 0 18429D-01 | 0.37400D+00 | -0.11792D+00 | 0.61609D-02 | 0.31168D+01 | 0.21426D+00 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 22 | 0.27626D-02 | 0.14688D-01 | 0.21766D+00 | 0.37891D-02 | 0 34700D+00 | -0.34691D-01 | 0 37701D+00 | 0.31147D+01 | 0.21766D+00 |
| 23 | -0.32645D-15 | 0.14470D-01 | 0.21822D+00 | -0.39187D-14 | 0 34173D+00 | -0 96645D-13 | 0 74850D+00 | 0.31145D+01 | 0.21822D+00 |
| 24 | -0 27626D-02 | 0 14688D-01 | 0 21766D+00 | -0.37891D-02 | 0 34700D+00 | 0.34691D-01 | 0.11200D+01 | 0.31147D+01 | 0.21766D+00 |
| 25 | -0 61609D-02 | 0 16756D-01 | 0.21426D+00 | -0.18429D-01 | 0 37400D+00 | 0.11792D+00 | 0.14908D+01 | 0.31168D+01 | 0.21426D+00 |
| 26 | 0 60405D-02 | -0.89438D-02 | 0.38735D+00 | -0 43702D-01 | 0 35418D+00 | -0.19294D+00 | 0.60405D-02 | 0.35411D+01 | 0.38735D+00 |
| 27 | 0 27559D-02 | -0.63665D-02 | 0 37680D+00 | -0 14951D-01 | 0 31311D+00 | -0 42831D-01 | 0 37701D+00 | 0 35436D+01 | 0.37680D+00 |
| 28 | 0 76084D-16 | -0 62579D-02 | 0.37443D+00 | 0.18874D-14 | 0 30673D+00 | -0.18285D-12 | 0 74850D+00 | 0.35437D+01 | 0.37443D+00 |
| 29 | -0 27559D-02 | -0 63665D-02 | 0 37680D+00 | 0 14951D-01 | 0 31311D+00 | 0.42831D-01 | 0 11200D+01 | 0 35436D+01 | 0.37680D+00 |
| 30 | -0 60405D-02 | -0 89438D-02 | 0 33735D+00 | 0 43702D-01 | 0 35418D+00 | 0.19294D+00 | 0 14910D+01 | 0 35411D+01 | 0 38735D+00 |
| 31 | 0 71232D-02 | -0 10752D-14 | 0 46329D+00 | -0 98762D-01 | 0 18171D-12 | -0.18625D-11 | 0 71232D-02 | 0 40000D+01 | 0.46329D+00 |
| 32 | 0 26769D-02 | -0 37378D-15 | 0 44276D+00 | -0.22953D-01 | 0 36610D-13 | -0 14765D-12 | 0 37693D+00 | 0 40000D+01 | 0.44276D+00 |
| 33 | 0 42062D-15 | -0 67015D-15 | 0 43923D+00 | 0 11519D-13 | 0 17611D-13 | -0 14384D-12 | 0.74850D+00 | 0 40000D+01 | 0.43923D+00 |
| 34 | -0 26769D-02 | -0 87349D-15 | 0 44276D+00 | 0.22953D-01 | -0 17070D-14 | -0.26638D-12 | 0.11201D+01 | 0 40000D+01 | 0.44276D+00 |
| 35 | -0 71232D-02 | -0 26910D-15 | 0 46329D+00 | 0 98762D-01 | -0 10056D-12 | -0.12605D-11 | 0.14899D+01 | 0.40000D+01 | 0.46329D+00 |
| 36 | 0 60405D-02 | 0 89438D-02 | 0 38735D+00 | -0 43702D-01 | -0 35418D+00 | 0.19294D+00 | 0 60405D-02 | 0.44589D+01 | 0.38735D+00 |
| 37 | 0 27559D-02 | 0 63665D-02 | 0.37680D+00 | -0 14951D-01 | -0 31311D+00 | 0.42831D-01 | 0 37701D+00 | 0 44564D+01 | 0.37680D+00 |
| 38 | 0 99183D-15 | 0 62579D-02 | 0 37443D+00 | -0 23384D-14 | -0 30673D+00 | 0 26049D-13 | 0 74850D+00 | 0 44563D+01 | 0 37443D+00 |
| 39 | -0 27559D-02 | 0 63665D-02 | 0 37680D+00 | 0 14951D-01 | -0 31311D+00 | -0.42831D-01 | 0 11200D+01 | 0 44564D+01 | 0.37680D+00 |
| 40 | -0 60405D-02 | 0 89438D-02 | 0 38735D+00 | 0.43702D-01 | -0 35418D+00 | -0 19294D+00 | 0 14910D+01 | 0 44589D+01 | 0 38735D+00 |
| 41 | 0 61609D-02 | -0 16756D-01 | 0 21426D+00 | 0 18429D-01 | -0 37400D+00 | 0.11792D+00 | 0 61609D-02 | 0 48832D+01 | 0.21426D+00 |
| 42 | 0 27626D-02 | -0 14688D-01 | 0 21766D+00 | 0 37891D-02 | -0 34700D+00 | 0 34691D-01 | 0 37701D+00 | 0 48853D+01 | 0.21766D+00 |
| 43 | -0 69709D-15 | -0 14470D-01 | 0 21822D+00 | -0 16591D-13 | -0 34173D+00 | 0.17160D-12 | 0 74850D+00 | 0 48855D+01 | 0 21822D+00 |
| 44 | -0 27626D-02 | -0 14688D-01 | 0 21766D+00 | -0 37891D-02 | -0 34700D+00 | -0 34691D-01 | 0 11200D+01 | 0.48853D+01 | 0.21766D+00 |
| 45 | -0 61609D-02 | -0 16756D-01 | 0 21426D+00 | -0 18429D-01 | -0 37400D+00 | -0.11792D+00 | 0.14908D+01 | 0 48832D+01 | 0.21426D+00 |
| 46 | 0 40293D-02 | -0 26891D-01 | 0 12202D-01 | 0 35573D-01 | -0 19271D+00 | 0 64781D-01 | 0 40293D-02 | 0 56481D+01 | 0 12202D-01 |
| 47 | 0 18949D-02 | -0 25242D-01 | 0 20668D-01 | 0 12427D-01 | -0 18350D+00 | 0 70183D-02 | 0 37614D+00 | 0 56498D+01 | 0.20668D-01 |
| 48 | -0 37806D-15 | -0 24837D-01 | 0 22809D-01 | 0 21270D-14 | -0 18242D+00 | -0 18621D-13 | 0 74850D+00 | 0 56502D+01 | 0 22809D-01 |
| 49 | -0 18949D-02 | -0 25242D-01 | 0 20668D-01 | -0 12427D-01 | -0 18350D+00 | -0.70183D-02 | 0 11209D+01 | 0 56498D+01 | 0.20668D-01 |
| 50 | -0 40293D-02 | -0 26891D-01 | 0 12202D-01 | -0 35573D-01 | -0 19271D+00 | -0.64781D-01 | 0 14930D+01 | 0 56481D+01 | 0 12202D-01 |
| 51 | 0 37607D-02 | -0 14470D-01 | -0.13503D-01 | 0 93701D-02 | 0 68517D-01 | -0 11840D+00 | 0 37607D-02 | 0 64355D+01 | -0 13503D-01 |
| 52 | 0 17601D-02 | -0 14671D-01 | -0 10707D-01 | 0 45938D-02 | 0 45889D-01 | -0 23092D-01 | 0 37601D+00 | 0 64353D+01 | -0.10707D-01 |
| 53 | -0 19630D-15 | -0 14550D-01 | -0.98922D-02 | 0.78133D-15 | 0 42209D-01 | -0 10312D-13 | 0 74850D+00 | 0 64354D+01 | -0.98922D-02 |
| 54 | -0 17601D-02 | -0.14671D-01 | -0 10707D-01 | -0 45938D-02 | 0 45889D-01 | 0 23092D-01 | 0 11210D+01 | 0 64353D+01 | -0 10707D-01 |
| 55 | -0 37607D-02 | -0 14470D-01 | -0 13503D-01 | -0 93701D-02 | 0 68517D-01 | 0 11840D+00 | 0 14932D+01 | 0 64355D+01 | -0.13503D-01 |
| 56 | 0 28299D-02 | -0 70454D-02 | 0 38147D-02 | -0 36581D-02 | -0 25434D-01 | 0 45887D-01 | 0 28299D-02 | 0.72180D+01 | 0.38147D-02 |
| 57 | 0 13757D-02 | -0 62248D-02 | 0 29997D-02 | -0 90020D-03 | -0 18150D-01 | 0 64043D-02 | 0 37563D+00 | 0 72188D+01 | 0.29997D-02 |
| 58 | -0 50358D-17 | -0 60713D-02 | 0 28621D-02 | 0.40056D-15 | -0 17036D-01 | -0 27142D-14 | 0.74850D+00 | 0 72189D+01 | 0.28621D-02 |
| 59 | -0 13757D-02 | -0 62248D-02 | 0 29997D-02 | 0 90020D-03 | -0 18150D-01 | -0 64043D-02 | 0 11214D+01 | 0 72188D+01 | 0.29997D-02 |
| 60 | -0 28299D-02 | -0 70454D-02 | 0.38147D-02 | 0 36581D-02 | -0 25434D-01 | -0.45887D-01 | 0 14942D+01 | 0.72180D+01 | 0.38147D-02 |
| 61 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.80000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 37425D+00 | 0.80000D+01 | 0.0 |
| 63 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 74850D+00 | 0.80000D+01 | 0.0 |
| 64 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 11227D+01 | 0.80000D+01 | 0.0 |
| 65 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | -0.75363D+03 | -0 14544D+04 | 0 10847D+02 | 0 11578D+01 | 0.26035D+00 | 0.13400D+00 |
| 2 | -0 69600D+02 | -0 23567D+04 | 0.47066D+01 | 0 74311D-01 | -0 16905D+01 | -0 15812D-01 |
| 3 | 0 63665D-11 | -0 22951D+04 | 0.47261D+01 | 0 16972D-11 | -0.19659D+01 | 0 35318D-12 |
| 4 | 0.69600D+02 | -0 23567D+04 | 0 47066D+01 | -0 74311D-01 | -0 16905D+01 | 0.15812D-01 |
| 5 | 0.75363D+03 | -0 14544D+04 | 0 10847D+02 | -0.11578D+01 | 0.26035D+00 | -0.13400D+00 |
| 61 | -0 75363D+03 | 0 14544D+04 | 0 10847D+02 | 0.11578D+01 | -0.26035D+00 | -0 13400D+00 |
| 62 | -0 69600D+02 | 0 23567D+04 | 0 47066D+01 | 0 74311D-01 | 0.16905D+01 | 0 15812D-01 |
| 63 | -0 60254D-11 | 0 22951D+04 | 0 47261D+01 | 0 30383D-12 | 0.19659D+01 | -0 74107D-13 |
| 64 | 0 69600D+02 | 0 23567D+04 | 0 47066D+01 | -0 74311D-01 | 0 16905D+01 | -0.15812D-01 |
| 65 | 0 75363D+03 | 0 14544D+04 | 0 10847D+02 | -0.11578D+01 | -0.26035D+00 | 0.13400D+00 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0.2274D-02 | -0 1608D-02 | 0 9877D-02 | 0 7252D-02 | 0 1609D-02 | 0 1626D-02 | 0 9930D-02 | 0 7326D-02 | 0.8623D+02 | 0.8480D+02 |
| 2 | -0.1861D-02 | -0 1815D-02 | 0 9072D-02 | 0 6796D-02 | 0 6911D-03 | 0 6740D-03 | 0.9083D-02 | 0 6809D-02 | 0.8819D+02 | 0.8776D+02 |
| 3 | -0.1861D-02 | -0.1815D-02 | 0.9072D-02 | 0 6796D-02 | -0.6911D-03 | -0.6740D-03 | 0.9083D-02 | 0.6809D-02 | -0.8819D+02 | -0.8776D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | -0.2274D-02 | -0.1608D-02 | 0.9877D-02 | 0.7252D-02 | -0.1609D-02 | -0.1626D-02 | 0.9930D-02 | 0.7326D-02 | -0.8623D+02 | -0.8480D+02 |
| 5 | -0.2965D-02 | -0 6249D-02 | 0.5680D-02 | 0 1525D-01 | 0.1051D-02 | -0.8359D-03 | 0 5711D-02 | 0.1525D-01 | 0.8653D+02 | -0.8889D+02 |
| 6 | -0 4052D-02 | -0 4327D-02 | 0 7128D-02 | 0.1506D-01 | 0.2132D-03 | -0.4405D-03 | 0.7129D-02 | 0.1506D-01 | 0.8945D+02 | -0.8935D+02 |
| 7 | -0 4052D-02 | -0 4327D-02 | 0 7128D-02 | 0 1506D-01 | -0.2132D-03 | 0 4405D-03 | 0.7129D-02 | 0.1506D-01 | -0.8945D+02 | 0 8935D+02 |
| 8 | -0.2965D-02 | -0 6249D-02 | 0 5680D-02 | 0 1525D-01 | -0.1051D-02 | 0 8359D-03 | 0.5711D-02 | 0 1525D-01 | -0.8653D+02 | 0.8889D+02 |
| 9 | -0 9204D-02 | -0 1358D-02 | 0 3079D-01 | -0 5557D-03 | 0.7283D-03 | -0 3031D-02 | 0 3079D-01 | 0 6107D-03 | 0.8948D+02 | -0.5241D+02 |
| 10 | -0 6428D-02 | -0 3314D-02 | 0 2869D-01 | -0 1003D-02 | 0 1990D-04 | -0 9786D-03 | 0 2869D-01 | 0 0 | 0 8998D+02 | 0 0 |
| 11 | -0 6428D-02 | -0 3314D-02 | 0 2869D-01 | -0 1003D-02 | -0 1990D-04 | 0 9786D-03 | 0 2869D-01 | 0 0 | -0 8998D+02 | 0 0 |
| 12 | -0 9204D-02 | -0 1358D-02 | 0 3079D-01 | -0 5557D-03 | -0 7283D-03 | 0 3031D-02 | 0 3079D-01 | 0 6107D-03 | -0 8948D+02 | 0 5241D+02 |
| 13 | -0 1010D-01 | -0 4276D-02 | 0 2915D-01 | 0 6616D-02 | 0 1492D-02 | 0 2421D-02 | 0 2916D-01 | 0 6749D-02 | 0.8891D+02 | 0 8374D+02 |
| 14 | -0 7683D-02 | -0 4743D-02 | 0 2770D-01 | 0 6623D-02 | 0 3634D-03 | 0 1241D-02 | 0 2770D-01 | 0 6657D-02 | 0 8971D+02 | 0 8688D+02 |
| 15 | -0.7683D-02 | -0 4743D-02 | 0 2770D-01 | 0 6623D-02 | -0.3634D-03 | -0 1244D-02 | 0 2770D-01 | 0 6657D-02 | -0 8971D+02 | -0.8688D+02 |
| 16 | -0 1010D-01 | -0 4276D-02 | 0 2915D-01 | 0 6616D-02 | -0 1492D-02 | -0 2421D-02 | 0 2916D-01 | 0 6749D-02 | -0 8891D+02 | -0 8374D+02 |
| 17 | -0 8449D-02 | -0.9347D-02 | 0 1540D-01 | 0 2220D-01 | -0 1004D-01 | 0 5273D-02 | 0 1642D-01 | 0 2242D-01 | -0 7858D+02 | 0 8526D+02 |
| 18 | -0 6672D-02 | -0 8069D-02 | 0 1443D-01 | 0 2232D-01 | -0 2620D-02 | 0 8049D-03 | 0.1451D-01 | 0 2233D-01 | -0 8646D+02 | 0 8924D+02 |
| 19 | -0 6672D-02 | -0 8069D-02 | 0.1443D-01 | 0 2232D-01 | 0 2620D-02 | -0 8049D-03 | 0 1451D-01 | 0 2233D-01 | 0 8646D+02 | -0 8924D+02 |
| 20 | -0 8449D-02 | -0 9347D-02 | 0 1540D-01 | 0 2220D-01 | 0 1004D-01 | -0 5273D-02 | 0 1642D-01 | 0 2242D-01 | 0 7858D+02 | -0 8526D+02 |
| 21 | -0 1009D-02 | -0 1756D-01 | -0 9126D-02 | 0 6491D-01 | -0 7724D-02 | 0 3362D-02 | 0 5347D-03 | 0 6494D-01 | -0 2179D+02 | 0.8883D+02 |
| 22 | -0 4316D-02 | -0 1014D-01 | -0 1111D-01 | 0 5868D-01 | -0 1581D-02 | -0 5242D-03 | 0.0 | 0 5868D-01 | 0 0 | -0 8978D+02 |
| 23 | -0 4316D-02 | -0.1014D-01 | -0 1111D-01 | 0.5868D-01 | 0 1581D-02 | 0 5242D-03 | 0 0 | 0 5868D-01 | 0 0 | 0 8978D+02 |
| 24 | -0 1009D-02 | -0.1756D-01 | -0 9126D-02 | 0 6491D-01 | 0 7724D-02 | -0 3362D-02 | 0 5347D-03 | 0 6494D-01 | 0.2179D+02 | -0 8883D+02 |
| 25 | -0 1009D-02 | -0 1756D-01 | -0 9126D-02 | 0 6491D-01 | 0.7724D-02 | -0 3362D-02 | 0 5347D-03 | 0 6494D-01 | 0 2179D+02 | -0 8883D+02 |
| 26 | -0 4316D-02 | -0 1014D-01 | -0 1111D-01 | 0 5868D-01 | 0 1581D-02 | 0 5242D-03 | 0 0 | 0 5868D-01 | 0 0 | 0 8978D+02 |
| 27 | -0.4316D-02 | -0 1014D-01 | -0.1111D-01 | 0 5968D-01 | -0 1581D-02 | -0 5242D-03 | 0 0 | 0 5868D-01 | 0 0 | -0 8978D+02 |
| 28 | -0 1009D-02 | -0 1756D-01 | -0 9126D-02 | 0 6491D-01 | -0 7724D-02 | 0 3362D-02 | 0 5317D-03 | 0 6494D-01 | -0 2179D+02 | 0 8883D+02 |
| 29 | -0 8449D-02 | -0 9347D-02 | 0 1540D-01 | 0 2220D-01 | 0 1004D-01 | -0 5273D-02 | 0 1642D-01 | 0.2242D-01 | 0 7858D+02 | -0 8526D+02 |
| 30 | -0 6672D-02 | -0 8069D-02 | 0.1443D-01 | 0 2232D-01 | 0 2620D-02 | -0 8049D-03 | 0 1451D-01 | 0 2233D-01 | 0 8646D+02 | -0 8924D+02 |
| 31 | -0 6672D-02 | -0 8069D-02 | 0 1443D-01 | 0 2232D-01 | -0 2620D-02 | 0 8049D-03 | 0 1451D-01 | 0 2233D-01 | -0 8646D+02 | 0 8924D+02 |
| 32 | -0 8449D-02 | -0 9347D-02 | 0 1540D-01 | 0 2220D-01 | -0 1004D-01 | 0 5273D-02 | 0.1642D-01 | 0 2242D-01 | -0.7858D+02 | 0 8526D+02 |
| 33 | -0 1010D-01 | -0 4276D-02 | 0 2915D-01 | 0 6616D-02 | -0 1492D-02 | -0 2421D-02 | 0 2916D-01 | 0 6749D-02 | -0.8891D+02 | -0 8374D+02 |
| 34 | -0 7683D-02 | -0 4743D-02 | 0 2770D-01 | 0 6623D-02 | -0 3631D-03 | -0 1244D-02 | 0 2770D-01 | 0 6657D-02 | -0 8971D+02 | -0 8688D+02 |
| 35 | -0 7683D-02 | -0 4743D-02 | 0 2770D-01 | 0.6623D-02 | 0 3634D-03 | 0 1244D-02 | 0 2770D-01 | 0 6657D-02 | 0 8971D+02 | 0 8688D+02 |
| 36 | -0 1010D-01 | -0 4276D-02 | 0 2915D-01 | 0 6616D-02 | 0.1492D-02 | 0 2421D-02 | 0.2916D-01 | 0 6749D-02 | 0.8891D+02 | 0 8374D+02 |
| 37 | -0 9204D-02 | -0 1358D-02 | 0 3079D-01 | -0 5557D-03 | -0 7283D-03 | 0 3031D-02 | 0.3079D-01 | 0 6107D-03 | -0 8948D+02 | 0.5241D+02 |
| 38 | -0.6428D-02 | -0 3314D-02 | 0 2869D-01 | -0 1003D-02 | -0.1990D-04 | 0.9786D-03 | 0 2869D-01 | 0 0 | -0 8998D+02 | 0 0 |
| 39 | -0 6128D-02 | -0.3314D-02 | 0 2869D-01 | -0 1003D-02 | 0 1990D-04 | -0 9786D-03 | 0.2869D-01 | 0 0 | 0 8998D+02 | 0 0 |
| 40 | -0 9204D-02 | -0 1358D-02 | 0 3079D-01 | -0 5557D-03 | 0.7283D-03 | -0 3031D-02 | 0 3079D-01 | 0 6107D-03 | 0 8948D+02 | -0.5241D+02 |
| 41 | -0 2965D-02 | -0 6249D-02 | 0 5680D-02 | 0 1525D-01 | -0 1051D-02 | 0 8359D-03 | 0 5711D-02 | 0 1525D-01 | -0 8653D+02 | 0 8889D+02 |
| 42 | -0.4052D-02 | -0 4327D-02 | 0 7128D-02 | 0 1506D-01 | -0 2132D-03 | 0 4405D-03 | 0 7129D-02 | 0 1506D-01 | -0 8945D+02 | 0.8935D+02 |
| 43 | -0 4052D-02 | -0 4327D-02 | 0 7128D-02 | 0 1506D-01 | 0.2132D-03 | -0 4405D-03 | 0 7129D-02 | 0.1506D-01 | 0.8945D+02 | -0 8935D+02 |
| 44 | -0 2965D-02 | -0 6249D-02 | 0 5680D-02 | 0 1525D-01 | 0 1051D-02 | -0 8359D-03 | 0 5711D-02 | 0.1525D-01 | 0.8653D+02 | -0 8889D+02 |
| 45 | -0 2274D-02 | -0 1608D-02 | 0.9877D-02 | 0 7252D-02 | -0 1609D-02 | -0 1626D-02 | 0 9930D-02 | 0.7326D-02 | -0 8623D+02 | -0 8480D+02 |
| 46 | -0 1861D-02 | -0 1815D-02 | 0 9072D-02 | 0 6796D-02 | -0 6911D-03 | -0 6740D-03 | 0.9083D-02 | 0 6809D-02 | -0 8819D+02 | -0.8776D+02 |
| 47 | -0 1861D-02 | -0 1815D-02 | 0 9072D-02 | 0 6796D-02 | 0 6911D-03 | 0 6740D-03 | 0 9083D-02 | 0 6809D-02 | 0 8819D+02 | 0 8776D+02 |
| 48 | -0 2274D-02 | -0 1608D-02 | 0 9877D-02 | 0 7252D-02 | 0 1609D-02 | 0 1626D-02 | 0 9930D-02 | 0 7326D-02 | 0.8623D+02 | 0.8480D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0 5677D-02 | -0 8628D-02 | -0 1448D-01 | 0 4230D-01 | -0 1433D-13 | 0.1502D-13 | 0 0 | 0 4230D-01 | 0 0 | 0 9000D+02 |
| 35 | 0.8233D-02 | -0 2224D-01 | -0.1460D-01 | 0 5435D-01 | -0 1364D-12 | 0 1207D-12 | 0.8233D-02 | 0.5435D-01 | -0.1711D-09 | 0.9000D+02 |
| 38 | -0 6253D-02 | -0 8474D-02 | 0 6246D-03 | 0 6130D-01 | 0 2146D-14 | -0 3172D-14 | 0 6246D-03 | 0 6130D-01 | 0 9000D+02 | -0 9000D+02 |
| 40 | -0 2975D-02 | -0 1266D-01 | 0 1309D-01 | 0 7513D-01 | -0 2720D-01 | 0 1216D-01 | 0.2085D-01 | 0 7555D-01 | -0 6028D+02 | 0 8606D+02 |
| 48 | -0 6351D-02 | -0 3772D-02 | 0 3277D-01 | 0 4054D-03 | -0.1918D-14 | 0 1850D-14 | 0 3277D-01 | 0 4054D-03 | -0.9000D+02 | 0.9000D+02 |
| 50 | -0 9348D-02 | -0 7632D-03 | 0 3585D-01 | 0 4247D-02 | -0 2509D-02 | 0 1061D-01 | 0 3589D-01 | 0 7601D-02 | -0.8835D+02 | 0.5769D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG.(DIR.1) | ELONG (DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 10231D-33 | 0.70499D-02 | -0.28609D-16 | 0.0 | 0.70252D-02 | 0.70499D-02 | -0 90000D+02 |
| 1 | OUTER | 0 10231D-33 | 0.86179D-02 | -0 28609D-16 | 0 0 | 0.85811D-02 | 0 86179D-02 | -0.90000D+02 |
| 2 | INNER | -0 96584D-03 | 0.80338D-02 | -0 10588D-03 | -0 96310D-03 | 0.80018D-02 | 0 80341D-02 | -0 89663D+02 |
| 2 | OUTER | -0 93142D-03 | 0.76353D-02 | -0 10508D-03 | -0 93165D-03 | 0.76064D-02 | 0 76356D-02 | -0.89646D+02 |
| 3 | INNER | -0 38790D-02 | 0 64132D-02 | -0 38477D-03 | -0 38865D-02 | 0.63927D-02 | 0.64168D-02 | -0.88930D+02 |
| 3 | OUTER | -0.40693D-02 | 0.15492D-01 | -0.38477D-03 | -0.40776D-02 | 0.15374D-01 | 0.15494D-01 | -0.89437D+02 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | INNER | -0 57591D-02 | 0.24165D-01 | -0.56794D-03 | -0.5,758D-02 | 0.23880D-01 | 0.24168D-01 | -0.89456D+02 |
| 4 | OUTER | -0 38793D-02 | 0.24215D-02 | -0 56794D-03 | -0.38869D-02 | 0.24186D-02 | 0.24343D-02 | -0.87425D+02 |
| 5 | INNER | -0 69399D-02 | 0 21350D-01 | -0.90536D-03 | -0.69642D-02 | 0.21127D-01 | 0.21357D-01 | -0.89084D+02 |
| 5 | OUTER | -0 48318D-02 | 0.26603D-02 | -0 90536D-03 | -0 48435D-02 | 0 26568D-02 | 0.26876D-02 | -0.86555D+02 |
| 6 | INNER | -0 56772D-02 | -0.14483D-01 | -0 14195D-13 | -0 56934D-02 | -0.14589D-01 | 0.0 | 0.0 |
| 6 | OUTER | -0 86280D-02 | 0.42295D-01 | 0 15149D-13 | -0 86656D-02 | 0.41437D-01 | 0.42295D-01 | 0.90000D+02 |
| 7 | INNER | -0 69399D-02 | 0 21350D-01 | 0 90536D-03 | -0 69642D-02 | 0.21127D-01 | 0.21357D-01 | 0.89084D+02 |
| 7 | OUTER | -0 48318D-02 | 0.26603D-02 | 0.90536D-03 | -0.48435D-02 | 0.26568D-02 | 0.26876D-02 | 0.86555D+02 |
| 8 | INNER | -0.57591D-02 | 0.24165D-01 | 0 56794D-03 | -0 57758D-02 | 0.23880D-01 | 0 24168D-01 | 0.89456D+02 |
| 8 | OUTER | -0 38793D-02 | 0.24215D-02 | 0.56794D-03 | -0.38869D-02 | 0.24186D-02 | 0 24343D-02 | 0.87425D+02 |
| 9 | INNER | -0 38790D-02 | 0.64132D-02 | 0.38477D-03 | -0.38865D-02 | 0.63927D-02 | 0.64168D-02 | 0.88930D+02 |
| 9 | OUTER | -0 40693D-02 | 0.15492D-01 | 0 38477D-03 | -0.40776D-02 | 0.15374D-01 | 0.15494D-01 | 0.89437D+02 |
| 10 | INNER | -0 96584D-03 | 0.80338D-02 | 0 10588D-03 | -0.96631D-03 | 0.80018D-02 | 0.80341D-02 | 0.89663D+02 |
| 10 | OUTER | -0 93142D-03 | 0.76353D-02 | 0 10588D-03 | -0.93185D-03 | 0.76064D-02 | 0.76356D-02 | 0.89646D+02 |
| 11 | INNER | 0 13844D-18 | 0.70499D-02 | 0.69117D-17 | 0.0 | 0.70252D-02 | 0.70499D-02 | 0.90000D+02 |
| 11 | OUTER | -0 13844D-18 | 0.86179D-02 | 0.62386D-17 | 0.0 | 0.85811D-02 | 0.86179D-02 | 0.90000D+02 |

SYSTEM ENERGIES(IN-LB)

WORK INPUT TO STRUCTURE = 0.3848832D+04
STRUCTURE KINETIC ENERGY = 0.2381437D+04
STRUCTURE ELASTIC ENERGY = 0.2736041D+03
STRUCTURE PLASTIC ENERGY = 0 1193791D+04
ENERGY STORED IN ELASTIC RESTRAINTS= 0.0

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***** INCR. NO.= 140 TIME= 0.3500D-03 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.11227D+01 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.0 | 0.0 |
| 6 | 0.11168D-01 | -0.16063D-01 | 0.22456D+00 | -0.38388D-02 | 0.35040D+00 | 0.20650D-01 | 0.11168D-01 | 0.75894D+00 | 0.22456D+00 |
| 7 | 0.53935D-02 | -0.16882D-01 | 0.22373D+00 | -0.14533D-02 | 0.36099D+00 | 0.13046D-01 | 0.37964D+00 | 0.75812D+00 | 0.22373D+00 |
| 8 | 0.39218D-12 | -0.16938D-01 | 0.22341D+00 | -0.54197D-12 | 0.36261D+00 | -0.21121D-11 | 0.74850D+00 | 0.75806D+00 | 0.22341D+00 |
| 9 | -0.53935D-02 | -0.16882D-01 | 0.22373D+00 | 0.14533D-02 | 0.36099D+00 | -0.13046D-01 | 0.11174D+01 | 0.75812D+00 | 0.22373D+00 |
| 10 | -0.11168D-01 | -0.16063D-01 | 0.22456D+00 | 0.38388D-02 | 0.35040D+00 | -0.20650D-01 | 0.14858D+01 | 0.75894D+00 | 0.22456D+00 |
| 11 | 0.10482D-01 | -0.47561D-01 | 0.51225D+00 | -0.98638D-02 | 0.39266D+00 | -0.62600D-01 | 0.10482D-01 | 0.15024D+01 | 0.51225D+00 |
| 12 | 0.53509D-02 | -0.46237D-01 | 0.51097D+00 | -0.23590D-02 | 0.38957D+00 | 0.81612D-03 | 0.37960D+00 | 0.15038D+01 | 0.51097D+00 |
| 13 | 0.10598D-11 | -0.45974D-01 | 0.51063D+00 | -0.22515D-11 | 0.38947D+00 | -0.31344D-11 | 0.74850D+00 | 0.15040D+01 | 0.51063D+00 |
| 14 | -0.53509D-02 | -0.46237D-01 | 0.51097D+00 | 0.23590D-02 | 0.38957D+00 | -0.81612D-03 | 0.11174D+01 | 0.15038D+01 | 0.51097D+00 |
| 15 | -0.10482D-01 | -0.47561D-01 | 0.51225D+00 | 0.98638D-02 | 0.39266D+00 | 0.62600D-01 | 0.14865D+01 | 0.15024D+01 | 0.51225D+00 |
| 16 | 0.99764D-02 | -0.49729D-01 | 0.75111D+00 | -0.20050D-01 | 0.33755D+00 | -0.13326D+00 | 0.99764D-02 | 0.22753D+01 | 0.75111D+00 |
| 17 | 0.48527D-02 | -0.47760D-01 | 0.74555D+00 | -0.99619D-02 | 0.32412D+00 | -0.64496D-02 | 0.37910D+00 | 0.22772D+01 | 0.74555D+00 |
| 18 | 0.12465D-11 | -0.46977D-01 | 0.74362D+00 | 0.27322D-11 | 0.32257D+00 | -0.95271D-11 | 0.74850D+00 | 0.22780D+01 | 0.74362D+00 |
| 19 | -0.48527D-02 | -0.47700D-01 | 0.74555D+00 | 0.99619D-02 | 0.32412D+00 | 0.64496D-02 | 0.11179D+01 | 0.22772D+01 | 0.74555D+00 |
| 20 | -0.99764D-02 | -0.49729D-01 | 0.75111D+00 | 0.20050D-01 | 0.33755D+00 | 0.13326D+00 | 0.14870D+01 | 0.22753D+01 | 0.75111D+00 |
| 21 | 0.82092D-02 | -0.31763D-01 | 0.89283D+00 | -0.62094D-02 | 0.24566D+00 | -0.17556D+00 | 0.82092D-02 | 0.30682D+01 | 0.89283D+00 |
| 22 | 0.38831D-02 | -0.31307D-01 | 0.88973D+00 | -0.82074D-02 | 0.21480D+00 | -0.36351D-01 | 0.37813D+00 | 0.30687D+01 | 0.88973D+00 |
| 23 | 0.90110D-12 | -0.30898D-01 | 0.88810D+00 | 0.69554D-12 | 0.20897D+00 | -0.42541D-12 | 0.74850D+00 | 0.30691D+01 | 0.88810D+00 |
| 24 | -0.38831D-02 | -0.31307D-01 | 0.88973D+00 | 0.82074D-02 | 0.21480D+00 | 0.36351D-01 | 0.11189D+01 | 0.30687D+01 | 0.88973D+00 |
| 25 | -0.82092D-02 | -0.31763D-01 | 0.89283D+00 | 0.62094D-02 | 0.24566D+00 | 0.17556D+00 | 0.14888D+01 | 0.30682D+01 | 0.89283D+00 |
| 26 | 0.85445D-02 | -0.21497D-01 | 0.95626D+00 | -0.30929D-01 | 0.13937D+00 | -0.14155D+00 | 0.85445D-02 | 0.35285D+01 | 0.95626D+00 |
| 27 | 0.43943D-02 | -0.21507D-01 | 0.94816D+00 | -0.12569D-01 | 0.11210D+00 | -0.22280D-01 | 0.37864D+00 | 0.35285D+01 | 0.94816D+00 |
| 28 | 0.47549D-12 | -0.21314D-01 | 0.94563D+00 | -0.95775D-12 | 0.10868D+00 | -0.92713D-12 | 0.74850D+00 | 0.35287D+01 | 0.94563D+00 |
| 29 | -0.43943D-02 | -0.21507D-01 | 0.94816D+00 | 0.12569D-01 | 0.11210D+00 | 0.22280D-01 | 0.11184D+01 | 0.35285D+01 | 0.94816D+00 |
| 30 | -0.85445D-02 | -0.21497D-01 | 0.95626D+00 | 0.30929D-01 | 0.13937D+00 | 0.14155D+00 | 0.14885D+01 | 0.35285D+01 | 0.95626D+00 |
| 31 | 0.12252D-01 | -0.12828D-11 | 0.96231D+00 | -0.17867D-01 | 0.92588D-10 | -0.54499D-09 | 0.12252D-01 | 0.40000D+01 | 0.96231D+00 |
| 32 | 0.46056D-02 | 0.19815D-11 | 0.95365D+00 | -0.20740D-01 | 0.19522D-10 | 0.19140D-09 | 0.37886D+00 | 0.40000D+01 | 0.95365D+00 |
| 33 | 0.18158D-12 | 0.97387D-11 | 0.95040D+00 | -0.28519D-12 | -0.19881D-10 | -0.26675D-10 | 0.74850D+00 | 0.40000D+01 | 0.95040D+00 |
| 34 | -0.46056D-02 | 0.19362D-11 | 0.95365D+00 | 0.20740D-01 | 0.12620D-10 | -0.18811D-09 | 0.11181D+01 | 0.40000D+01 | 0.95365D+00 |
| 35 | -0.12252D-01 | -0.26007D-12 | 0.96231D+00 | 0.17867D-01 | 0.66454D-10 | 0.39031D-09 | 0.14847D+01 | 0.40000D+01 | 0.96231D+00 |
| 36 | 0.85445D-02 | 0.21497D-01 | 0.95626D+00 | -0.30929D-01 | -0.13937D+00 | 0.14155D+00 | 0.85445D-02 | 0.44715D+01 | 0.95626D+00 |
| 37 | 0.43943D-02 | 0.21507D-01 | 0.94816D+00 | -0.12569D-01 | -0.11210D+00 | 0.22280D-01 | 0.37864D+00 | 0.44715D+01 | 0.94816D+00 |
| 38 | -0.13822D-12 | 0.21314D-01 | 0.94563D+00 | 0.11766D-11 | -0.10868D+00 | 0.95883D-12 | 0.74850D+00 | 0.44713D+01 | 0.94563D+00 |
| 39 | -0.43943D-02 | 0.21507D-01 | 0.94816D+00 | 0.12569D-01 | -0.11210D+00 | -0.22280D-01 | 0.11184D+01 | 0.44715D+01 | 0.94816D+00 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 40 | -0 85445D-02 | 0 21497D-01 | 0 95626D+00 | 0.30929D-01 | -0 13937D+00 | -0.14155D+00 | 0 14885D+01 | 0.44715D+01 | 0.95626D+00 |
| 41 | 0.82092D-02 | 0 31763D-01 | 0.89283D+00 | -0.62094D-02 | -0 24566D+00 | 0.17556D+00 | 0.82092D-02 | 0.49318D+01 | 0.89283D+00 |
| 42 | 0.38831D-02 | 0 31307D-01 | 0.88973D+00 | -0 82074D-02 | -0 21480D+00 | 0.36351D-01 | 0.37813D+00 | 0.49313D+01 | 0.88973D+00 |
| 43 | -0 78789D-12 | 0 30898D-01 | 0 88810D+00 | 0 38751D-12 | -0 20897D+00 | 0.48970D-11 | 0 74850D+00 | 0.49309D+01 | 0.88810D+00 |
| 44 | -0.38831D-02 | 0.31307D-01 | 0 88973D+00 | 0 82074D-02 | -0 21480D+00 | -0 36351D-01 | 0 11189D+01 | 0.49313D+01 | 0.88973D+00 |
| 45 | -0 82092D-02 | 0 31763D-01 | 0 89283D+00 | 0 62094D-02 | -0 24566D+00 | -0.17556D+00 | 0 14888D+01 | 0 49318D+01 | 0.89283D+00 |
| 46 | 0 99764D-02 | 0 49729D-01 | 0 75111D+00 | -0 20050D-01 | -0 33755D+00 | 0 13326D+00 | 0.99764D-02 | 0 57247D+01 | 0.75111D+00 |
| 47 | 0 48527D-02 | 0 47760D-01 | 0 74555D+00 | -0 99619D-02 | -0 32412D+00 | 0 64496D-02 | 0.37910D+00 | 0.57228D+01 | 0 74555D+00 |
| 48 | -0 17127D-11 | 0 46977D-01 | 0 74362D+00 | -0 63437D-12 | -0 32257D+00 | -0 22217D-10 | 0 74850D+00 | 0 57220D+01 | 0.74362D+00 |
| 49 | -0 48527D-02 | 0.47760D-01 | 0.74555D+00 | 0 99619D-02 | -0 32412D+00 | -0 64496D-02 | 0 11179D+01 | 0 57228D+01 | 0.74555D+00 |
| 50 | -0 99764D-02 | 0 49729D-01 | 0 75111D+00 | 0 20050D-01 | -0 33755D+00 | -0 13326D+00 | 0 14870D+01 | 0 57247D+01 | 0.75111D+00 |
| 51 | 0 10482D-01 | 0 47561D-01 | 0 51225D+00 | -0 98638D-02 | -0 39266D+00 | 0 62600D-01 | 0 10482D-01 | 0 64976D+01 | 0 51225D+00 |
| 52 | 0 53509D-02 | 0 46237D-01 | 0 51097D+00 | -0 23590D-02 | -0 38957D+00 | -0 81612D-03 | 0 37960D+00 | 0 64962D+01 | 0.51097D+00 |
| 53 | -0 16977D-11 | 0 45974D-01 | 0.51063D+00 | 0 37265D-11 | -0 38947D+00 | -0 69404D-11 | 0 74850D+00 | 0.64960D+01 | 0.51063D+00 |
| 54 | -0.53509D-02 | 0 46237D-01 | 0 51097D+00 | 0 23590D-02 | -0 38957D+00 | 0.81612D-03 | 0.11174D+01 | 0 64962D+01 | 0.51097D+00 |
| 55 | -0 10482D-01 | 0 47561D-01 | 0 51225D+00 | 0 98638D-02 | -0 39266D+00 | -0 62600D-01 | 0 14865D+01 | 0.64976D+01 | 0 51225D+00 |
| 56 | 0 11168D-01 | 0 16063D-01 | 0 22456D+00 | -0.38388D-02 | -0 35040D+00 | -0 20650D-01 | 0 11168D-01 | 0 72411D+01 | 0.22456D+00 |
| 57 | 0 53935D-02 | 0 16882D-01 | 0 22373D+00 | -0 14533D-02 | -0.36099D+00 | -0 13046D-01 | 0.37964D+00 | 0.72419D+01 | 0.22373D+00 |
| 58 | -0 49691D-12 | 0 16936D-01 | 0 22341D+00 | 0 27186D-12 | -0 36261D+00 | -0.39224D-11 | 0 74850D+00 | 0 72419D+01 | 0 22341D+00 |
| 59 | -0 53935D-02 | 0 16882D-01 | 0 22373D+00 | 0 14533D-02 | -0 36099D+00 | 0.13046D-01 | 0 11174D+01 | 0.72419D+01 | 0 22373D+00 |
| 60 | -0 11168D-01 | 0 16063D-01 | 0 22456D+00 | 0.38388D-02 | -0 35040D+00 | 0.20650D-01 | 0 14858D+01 | 0.72411D+01 | 0.22456D+00 |
| 61 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.80000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 37425D+00 | 0.80000D+01 | 0.0 |
| 63 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 74850D+00 | 0 80000D+01 | 0.0 |
| 64 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0 80000D+01 | 0.0 |
| 65 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | -0 20851D+03 | -0.55717D+03 | -0 19668D+03 | -0.17373D+02 | -0 39141D+02 | -0 25709D+01 |
| 2 | -0 58213D+02 | -0 67276D+03 | -0 35976D+03 | 0 19649D+00 | -0 67213D+02 | -0.39230D-01 |
| 3 | -0 11835D-07 | -0 65654D+03 | -0 35882D+03 | 0 20692D-09 | -0.68003D+02 | 0 97145D-10 |
| 4 | 0 58213D+02 | -0 67278D+03 | -0 35976D+03 | -0.19649D+00 | -0.67213D+02 | 0 39230D-01 |
| 5 | 0 20851D+03 | -0 55717D+03 | -0.19668D+03 | 0.17373D+02 | -0.39141D+02 | 0 25709D+01 |
| 61 | -0 20851D+03 | 0 55717D+03 | -0 19668D+03 | -0.17373D+02 | 0 39141D+02 | 0.25709D+01 |
| 62 | -0 58213D+02 | 0 67278D+03 | -0 35976D+03 | 0 19649D+00 | 0.67213D+02 | 0 39230D-01 |
| 63 | 0 14240D-08 | 0 65654D+03 | -0 35882D+03 | 0 97057D-09 | 0 68003D+02 | -0 23037D-10 |
| 64 | 0 58213D+02 | 0 67276D+03 | -0 35976D+03 | -0 19649D+00 | 0.67213D+02 | -0 39230D-01 |
| 65 | 0 20851D+03 | 0.55717D+03 | -0 19668D+03 | 0.17373D+02 | 0 39141D+02 | -0.25709D+01 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0 7425D-02 | -0 7951D-02 | 0 6157D-01 | 0 1471D-01 | 0 6811D-02 | 0 9373D-02 | 0 6174D-01 | 0 1564D-01 | 0.8717D+02 | 0.7876D+02 |
| 2 | -0 6933D-02 | -0 7475D-02 | 0.6051D-01 | 0 1281D-01 | 0 2883D-02 | 0.3404D-02 | 0 6054D-01 | 0 1295D-01 | 0.8878D+02 | 0.8524D+02 |
| 3 | -0 6933D-02 | -0 7475D-02 | 0 6051D-01 | 0.1281D-01 | -0 2883D-02 | -0 3404D-02 | 0 6054D-01 | 0.1295D-01 | -0 8878D+02 | -0.8524D+02 |
| 4 | -0 7425D-02 | -0 7951D-02 | 0 6157D-01 | 0 1471D-01 | -0 6841D-02 | -0 9373D-02 | 0 6174D-01 | 0 1564D-01 | -0.8717D+02 | -0.7876D+02 |
| 5 | -0 1483D-01 | -0 1430D-01 | 0 3126D-01 | 0 2703D-01 | -0 3966D-03 | 0 1515D-02 | 0.3126D-01 | 0 2705D-01 | -0.8975D+02 | 0.8895D+02 |
| 6 | -0 1426D-01 | -0 1445D-01 | 0 3178D-01 | 0 2820D-01 | 0 2995D-04 | 0.1187D-03 | 0 3178D-01 | 0 2820D-01 | 0 8998D+02 | 0 8992D+02 |
| 7 | -0 1426D-01 | -0 1115D-01 | 0 3178D-01 | 0 2820D-01 | -0 2935D-04 | -0 1187D-03 | 0 3178D-01 | 0 2820D-01 | -0 8998D+02 | -0 8992D+02 |
| 8 | -0.1483D-01 | -0 1430D-01 | 0 3126D-01 | 0 2703D-01 | 0 3966D-03 | -0 1515D-02 | 0 3126D-01 | 0 2705D-01 | 0 8975D+02 | -0 8895D+02 |
| 9 | -0 1331D-01 | -0 1404D-01 | 0 3257D-01 | 0 4089D-01 | -0 1241D-03 | 0.4098D-02 | 0 3257D-01 | 0 4097D-01 | -0 8992D+02 | 0 8787D+02 |
| 10 | -0 1288D-01 | -0 1337D-01 | 0.3113D-01 | 0 4019D-01 | -0 3872D-03 | 0 1104D-02 | 0.3143D-01 | 0 4019D-01 | -0 8975D+02 | 0 8942D+02 |
| 11 | -0 1288D-01 | -0 1437D-01 | 0 3143D-01 | 0 4019D-01 | 0 3872D-03 | -0 1104D-02 | 0 3143D-01 | 0.4019D-01 | 0 8975D+02 | -0 8942D+02 |
| 12 | -0 1331D-01 | -0 1404D-01 | 0 3257D-01 | 0 4089D-01 | 0 1241D-03 | -0 4098D-02 | 0 3257D-01 | 0 4097D-01 | 0 8992D+02 | -0 8787D+02 |
| 13 | -0 1221D-01 | -0 1299D-01 | 0 2514D-01 | 0 3875D-01 | 0 3975D-02 | -0 2944D-02 | 0 2554D-01 | 0 3880D-01 | 0 8699D+02 | -0 8837D+02 |
| 14 | -0 1082D-01 | -0.1251D-01 | 0 2425D-01 | 0 3910D-01 | 0 1030D-02 | -0 2392D-03 | 0.2426D-01 | 0 3910D-01 | 0 8916D+02 | -0 8987D+02 |
| 15 | -0 1082D-01 | -0 1251D-01 | 0 2425D-01 | 0 3910D-01 | -0 1030D-02 | 0 2392D-03 | 0 2426D-01 | 0 3910D-01 | -0 8916D+02 | 0 8987D+02 |
| 16 | -0 1221D-01 | -0 1299D-01 | 0 2544D-01 | 0 3875D-01 | -0 3975D-02 | 0 2944D-02 | 0 2554D-01 | 0 3880D-01 | -0 8699D+02 | 0.8837D+02 |
| 17 | -0 9939D-02 | -0 1217D-01 | 0 1705D-01 | 0 4053D-01 | -0 1002D-02 | 0 5740D-03 | 0 1706D-01 | 0 4053D-01 | -0.8894D+02 | 0 8969D+02 |
| 18 | -0 9517D-02 | -0 1256D-01 | 0 1646D-01 | 0 3932D-01 | 0 3879D-03 | 0 9845D-03 | 0.1646D-01 | 0 3932D-01 | 0.8957D+02 | 0 8946D+02 |
| 19 | -0 9517D-02 | -0 1256D-01 | 0.1646D-01 | 0 3932D-01 | -0.3879D-03 | -0 9845D-03 | 0 1646D-01 | 0.3932D-01 | -0 8957D+02 | -0 8946D+02 |
| 20 | -0 9939D-02 | -0 1247D-01 | 0 1705D-01 | 0 4053D-01 | 0.1002D-02 | -0.5740D-03 | 0.1706D-01 | 0.4053D-01 | 0.8894D+02 | -0 8969D+02 |
| 21 | -0 1342D-01 | -0.1736D-01 | 0.3420D-01 | 0 6143D-01 | 0 5116D-02 | 0 3974D-02 | 0 3433D-01 | 0 6148D-01 | 0.8693D+02 | 0.8856D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 22 | -0.9552D-02 | -0.1443D-01 | 0.3525D-01 | 0 6004D-01 | 0 5013D-03 | 0 6654D-03 | 0 3525D-01 | 0 6004D-01 | 0.8968D+02 | 0.8974D+02 |
| 23 | -0 9552D-02 | -0.1443D-01 | 0.3525D-01 | 0 6004D-01 | -0.5013D-03 | -0 6654D-03 | 0 3525D-01 | 0.6004D-01 | -0.8968D+02 | -0.8974D+02 |
| 24 | -0.1342D-01 | -0.1736D-01 | 0 3420D-01 | 0 6143D-01 | -0.5116D-02 | -0.3974D-02 | 0 3433D-01 | 0.6148D-01 | -0 8693D+02 | -0.8856D+02 |
| 25 | -0 1342D-01 | -0 1736D-01 | 0 3420D-01 | 0 6143D-01 | -0 5116D-02 | -0 3974D-02 | 0,3433D-01 | 0.6148D-01 | -0 8693D+02 | -0 8856D+02 |
| 26 | -0 9552D-02 | -0 1443D-01 | 0 3525D-01 | 0 6004D-01 | -0 5013D-03 | -0 6654D-03 | 0.3525D-01 | 0 6004D-01 | -0.8968D+02 | -0.8974D+02 |
| 27 | -0 9552D-02 | -0.1413D-01 | 0 3525D-01 | 0 6004D-01 | 0 5013D-03 | 0 6654D-03 | 0 3525D-01 | 0 6004D-01 | 0 8968D+02 | 0.8974D+02 |
| 28 | -0.1342D-01 | -0 1736D-01 | 0 3420D-01 | 0 6143D-01 | 0 5116D-02 | 0 3974D-02 | 0.3433D-01 | 0 6148D-01 | 0 8693D+02 | 0.8856D+02 |
| 29 | -0 9939D-02 | -0 1247D-01 | 0 1705D-01 | 0 4053D-01 | 0 1002D-02 | -0 5740D-03 | 0 1706D-01 | 0 4053D-01 | 0 8894D+02 | -0.8969D+02 |
| 30 | -0 9517D-02 | -0 1256D-01 | 0 1646D-01 | 0 3932D-01 | -0 3879D-03 | -0 9845D-03 | 0 1646D-01 | 0 3932D-01 | -0 8957D+02 | -0 8946D+02 |
| 31 | -0 9517D-02 | -0 1256D-01 | 0 1646D-01 | 0 3932D-01 | 0 3879D-03 | 0 9845D-03 | 0 1646D-01 | 0 3932D-01 | 0 8957D+02 | 0 8946D+02 |
| 32 | -0 9939D-02 | -0 1247D-01 | 0 1705D-01 | 0 4053D-01 | -0 1002D-02 | 0 5740D-03 | 0 1706D-01 | 0 4053D-01 | -0 8894D+02 | 0 8969D+02 |
| 33 | -0 1221D-01 | -0 1299D-01 | 0 2544D-01 | 0 3875D-01 | -0.3975D-02 | 0 2944D-02 | 0 2554D-01 | 0 3880D-01 | -0 8699D+02 | 0 8837D+02 |
| 34 | -0 1082D-01 | -0 1251D-01 | 0 2425D-01 | 0 3910D-01 | -0.1030D-02 | 0 2392D-03 | 0 2426D-01 | 0 3910D-01 | -0 8916D+02 | 0 8987D+02 |
| 35 | -0 1082D-01 | -0 1251D-01 | 0 2425D-01 | 0 3910D-01 | 0 1030D-02 | -0 2392D-03 | 0 2426D-01 | 0 3910D-01 | 0 8916D+02 | -0 8987D+02 |
| 36 | -0 1221D-01 | -0 1299D-01 | 0 2544D-01 | 0 3875D-01 | 0 3975D-02 | -0 2944D-02 | 0 2554D-01 | 0 3880D-01 | 0 8699D+02 | -0.8837D+02 |
| 37 | -0 1331D-01 | -0 1404D-01 | 0 3257D-01 | 0 4089D-01 | 0 1241D-03 | -0 4098D-02 | 0 3257D-01 | 0 4097D-01 | 0 8992D+02 | -0 8787D+02 |
| 38 | -0 1288D-01 | -0 1437D-01 | 0 3143D-01 | 0 4019D-01 | 0 3872D-03 | -0 1104D-02 | 0 3143D-01 | 0 4019D-01 | 0 8975D+02 | -0 8942D+02 |
| 39 | -0 1288D-01 | -0 1437D-01 | 0 3143D-01 | 0 4019D-01 | -0 3872D-03 | 0 1104D-02 | 0 3143D-01 | 0 4019D-01 | -0.8975D+02 | 0 8942D+02 |
| 40 | -0 1331D-01 | -0 1404D-01 | 0 3257D-01 | 0 4089D-01 | -0 1241D-03 | 0 4098D-02 | 0 3257D-01 | 0 4097D-01 | -0 8992D+02 | 0.8787D+02 |
| 41 | -0 1133D-01 | -0 1430D-01 | 0 3126D-01 | 0 2703D-01 | 0 3366D-03 | -0 1515D-02 | 0 3126D-01 | 0 2705D-01 | 0 8975D+02 | -0.8895D+02 |
| 42 | -0 1426D-01 | -0 1415D-01 | 0 3178D-01 | 0 2820D-01 | -0 2995D-04 | -0 1187D-03 | 0 3178D-01 | 0 2820D-01 | -0 8998D+02 | -0.8992D+02 |
| 43 | -0 1426D-01 | -0 1445D-01 | 0 3178D-01 | 0.2820D-01 | 0 2995D-04 | 0 1187D-03 | 0 3178D-01 | 0 2820D-01 | 0 8998D+02 | 0 8992D+02 |
| 44 | -0 1483D-01 | -0 1430D-01 | 0 3126D-01 | 0 2703D-01 | -0 3966D-03 | 0 1515D-02 | 0 3126D-01 | 0.2705D-01 | -0 8975D+02 | 0.8895D+02 |
| 45 | -0 7425D-02 | -0 7951D-02 | 0 6157D-01 | 0 1471D-01 | -0 6841D-02 | -0 9373D-02 | 0 6174D-01 | 0 1564D-01 | -0 8717D+02 | -0 7876D+02 |
| 46 | -0 6933D-02 | -0 7475D-02 | 0 6051D-01 | 0 1281D-01 | -0 2883D-02 | -0 3404D-02 | 0 6054D-01 | 0 1295D-01 | -0 8878D+02 | -0.8524D+02 |
| 47 | -0 6933D-02 | -0 7475D-02 | 0 6051D-01 | 0 1281D-01 | 0 2883D-02 | 0 3404D-02 | 0 6054D-01 | 0.1295D-01 | 0 8878D+02 | 0.8524D+02 |
| 48 | -0 7425D-02 | -0 7951D-02 | 0 6157D-01 | 0.1471D-01 | 0 6841D-02 | 0 9373D-02 | 0 6174D-01 | 0 1564D-01 | 0 8717D+02 | 0.7876D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0.1085D-01 | -0.1377D-01 | 0 6479D-01 | 0 2994D-01 | -0.3463D-11 | 0 1978D-11 | 0 6479D-01 | 0 2994D-01 | -0 9000D+02 | 0.9000D+02 |
| 35 | -0 2379D-01 | -0 1674D-01 | 0.7023D-01 | 0 2533D-01 | 0.3841D-10 | -0 4121D-10 | 0 7023D-01 | 0.2533D-01 | 0.9000D+02 | -0.9000D+02 |
| 38 | -0.9632D-02 | -0 1385D-01 | 0 2406D-01 | 0 5641D-01 | -0 2482D-12 | -0 4438D-12 | 0 2406D-01 | 0 5641D-01 | -0 9000D+02 | -0.9000D+02 |
| 40 | -0 8025D-02 | -0 1319D-01 | 0 2949D-01 | 0 6052D-01 | -0 1428D-01 | 0 1459D-01 | 0 3081D-01 | 0.6124D-01 | -0.7958D+02 | 0.8440D+02 |
| 48 | -0 1147D-01 | -0.1446D-01 | 0 5108D-01 | 0 7242D-01 | -0 8344D-12 | 0 3698D-11 | 0 5108D-01 | 0.7242D-01 | -0 9000D+02 | 0.9000D+02 |
| 50 | -0.1200D-01 | -0 1497D-01 | 0 5210D-01 | 0 8224D-01 | -0 1657D-01 | 0 1062D-01 | 0 5315D-01 | 0 8253D-01 | -0 8275D+02 | 0.8688D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG (DIR 2) | PRINC STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 32010D-25 | 0 44241D-01 | 0 50604D-12 | 0 0 | 0.43303D-01 | 0 44241D-01 | 0 90000D+02 |
| 1 | OUTER | 0 32010D-25 | -0 87952D-01 | 0.50604D-12 | 0 0 | -0.92202D-01 | 0 0 | 0 0 |
| 2 | INNER | -0 36713D-02 | 0 45870D-01 | -0.38776D-04 | -0 36780D-02 | 0.44864D-01 | 0 45870D-01 | -0.89978D+02 |
| 2 | OUTER | -0 37669D-02 | -0 42726D-01 | -0 38776D-04 | -0 37740D-02 | -0 43680D-01 | 0 0 | 0 0 |
| 3 | INNER | -0 14141D-01 | 0 29537D-01 | 0 97338D-04 | -0 14242D-01 | 0 29113D-01 | 0 29537D-01 | 0 89936D+02 |
| 3 | OUTER | -0 14616D-01 | 0 27802D-01 | 0 97336D-04 | -0 14724D-01 | 0 27426D-01 | 0 27802D-01 | 0 89934D+02 |
| 4 | INNER | -0 13460D-01 | 0 30134D-01 | 0 11509D-02 | -0.13551D-01 | 0.29693D-01 | 0 30141D-01 | 0 89244D+02 |
| 4 | OUTER | -0 11277D-01 | 0 54159D-01 | 0 1.509D-02 | -0 11380D-01 | 0 53052D-01 | 0 54464D-01 | 0 89520D+02 |
| 5 | INNER | -0 10596D-01 | 0 19191D-01 | 0 17374D-02 | -0 10653D-01 | 0.19016D-01 | 0 19216D-01 | 0 88331D+02 |
| 5 | OUTER | -0 13497D-01 | 0 52331D-01 | 0 17374D-02 | -0 13590D-01 | 0 51029D-01 | 0 52342D-01 | 0 89244D+02 |
| 6 | INNER | -0 10847D-01 | 0.64793D-01 | 0 17296D-10 | -0 10906D-01 | 0.62820D-01 | 0 64793D-01 | 0.90000D+02 |
| 6 | OUTER | -0 13766D-01 | 0 29937D-01 | 0 22737D-10 | -0 13862D-01 | 0 29501D-01 | 0 29937D-01 | 0 90000D+02 |
| 7 | INNER | -0 10596D-01 | 0.19191D-01 | -0.17374D-02 | -0 10653D-01 | 0 19010D-01 | 0.19216D-01 | -0 88331D+02 |
| 7 | OUTER | -0 13497D-01 | 0 52331D-01 | -0 17374D-02 | -0 13590D-01 | 0.51029D-01 | 0 52342D-01 | -0 89244D+02 |
| 8 | INNER | -0 13460D-01 | 0 30134D-01 | -0 11509D-02 | -0 13551D-01 | 0 29693D-01 | 0.30141D-01 | -0 89244D+02 |
| 8 | OUTER | -0 14277D-01 | 0 54459D-01 | -0 11503D-02 | -0 11380D-01 | 0 53052D-01 | 0 54464D-01 | -0 89520D+02 |
| 9 | INNER | -0 14141D-01 | 0 29537D-01 | -0 97336D-04 | -0.14242D-01 | 0.29113D-01 | 0 29537D-01 | -0 89936D+02 |
| 9 | OUTER | -0.14616D-01 | 0.27802D-01 | -0.97338D-04 | -0 14724D-01 | 0 27426D-01 | 0.27802D-01 | -0 89934D+02 |
| 10 | INNER | -0 36713D-02 | 0 45870D-01 | 0 38776D-04 | -0 36780D-02 | 0.44864D-01 | 0 45870D-01 | 0 89978D+02 |
| 10 | OUTER | -0 37669D-02 | -0 42726D-01 | 0.38776D-04 | -0 37740D-02 | -0.43680D-01 | 0.0 | 0.0 |
| 11 | INNER | 0 35898D-18 | 0 44241D-01 | 0.64117D-12 | 0 0 | 0.43303D-01 | 0.44241D-01 | 0.90000D+02 |
| 11 | OUTER | -0 35898D-18 | -0.87952D-01 | 0 64117D-12 | 0 0 | -0.92202D-01 | 0.0 | 0.0 |

SYSTEM ENERGIES(IN-LB)

WORK INPUT TO STRUCTURE = 0.3848832D+04
STRUCTURE KINETIC ENERGY = 0.8794630D+02
STRUCTURE ELASTIC ENERGY = 0.6741908D+02
STRUCTURE PLASTIC ENERGY = 0.3693466D+04
ENERGY STORED IN ELASTIC RESTRAINTS= 0.0

***** INCR. NO.= 150 TIME= 0.3750D-03 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.37425D+00 | 0.0 | 0.0 |
| 3 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.74850D+00 | 0.0 | 0.0 |
| 4 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.11227D+01 | 0 0 | 0.0 |
| 5 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.0 | 0.0 |
| 6 | 0.10577D-01 | -0 15472D-01 | 0 21855D+00 | -0.62583D-02 | 0.33756D+00 | 0 15768D-01 | 0.10577D-01 | 0.75953D+00 | 0.21855D+00 |
| 7 | 0 51089D-02 | -0.15957D-01 | 0 21709D+00 | -0.26210D-02 | 0 34761D+00 | 0.13124D-01 | 0.37936D+00 | 0.75904D+00 | 0.21709D+00 |
| 8 | 0 17067D-12 | -0 15916D-01 | 0 21655D+00 | -0 85720D-12 | 0 34926D+00 | -0 29345D-11 | 0.74850D+00 | 0.75908D+00 | 0.21655D+00 |
| 9 | -0 51089D-02 | -0 15957D-01 | 0 21709D+00 | 0 26210D-02 | 0 34761D+00 | -0.13124D-01 | 0.11176D+01 | 0.75904D+00 | 0.21709D+00 |
| 10 | -0 10577D-01 | -0 15472D-01 | 0 21855D+00 | 0 62583D-02 | 0 33756D+00 | -0.15768D-01 | 0 14864D+01 | 0.75953D+00 | 0 21855D+00 |
| 11 | 0 99164D-02 | -0 42556D-01 | 0 49051D+00 | -0.98516D-02 | 0 37709D+00 | -0.66297D-01 | 0.99164D-02 | 0.15074D+01 | 0.49051D+00 |
| 12 | 0 50632D-02 | -0 41245D-01 | 0 48922D+00 | -0 23659D-02 | 0 37439D+00 | 0.24323D-02 | 0.37931D+00 | 0 15088D+01 | 0 48922D+00 |
| 13 | 0 80972D-12 | -0 40976D-01 | 0 48887D+00 | -0 23143D-11 | 0.37460D+00 | -0.23026D-11 | 0.74850D+00 | 0.15090D+01 | 0 48887D+00 |
| 14 | -0 50632D-02 | -0 41245D-01 | 0 48922D+00 | 0 23659D-02 | 0 37439D+00 | -0.24323D-02 | 0.11177D+01 | 0 15088D+01 | 0 48922D+00 |
| 15 | -0 99164D-02 | -0 42556D-01 | 0 49051D+00 | 0 98516D-02 | 0.37709D+00 | 0.66297D-01 | 0 14871D+01 | 0 15074D+01 | 0.49051D+00 |
| 16 | 0 94630D-02 | -0 42452D-01 | 0 71707D+00 | -0.15207D-01 | 0 33367D+00 | -0.13143D+00 | 0.94630D-02 | 0 22825D+01 | 0.71707D+00 |
| 17 | 0 46101D-02 | -0 40797D-01 | 0.71243D+00 | -0 88388D-02 | 0 32049D+00 | -0.58165D-02 | 0 37886D+00 | 0 22842D+01 | 0.71243D+00 |
| 18 | 0 10040D-11 | -0 40075D-01 | 0 71067D+00 | 0 23317D-11 | 0 31906D+00 | -0.10914D-10 | 0 74850D+00 | 0 22849D+01 | 0.71067D+00 |
| 19 | -0 46101D-02 | -0 40797D-01 | 0.71243D+00 | 0 88388D-02 | 0 32049D+00 | 0 58165D-02 | 0 11181D+01 | 0.22842D+01 | 0.71243D+00 |
| 20 | -0 94630D-02 | -0 42452D-01 | 0 71707D+00 | 0 15207D-01 | 0 33367D+00 | 0 13143D+00 | 0.14875D+01 | 0 22825D+01 | 0.71707D+00 |
| 21 | 0 78979D-02 | -0 28001D-01 | 0.87038D+00 | -0 57092D-02 | 0 27058D+00 | -0 17812D+00 | 0 78979D-02 | 0 30720D+01 | 0.87038D+00 |
| 22 | 0 37051D-02 | -0 27512D-01 | 0 86704D+00 | -0 91965D-02 | 0 23784D+00 | -0 39752D-01 | 0.37796D+00 | 0 30725D+01 | 0.86704D+00 |
| 23 | 0 74222D-12 | -0 27076D-01 | 0 86519D+00 | -0 77961D-12 | 0 23135D+00 | 0 39382D-11 | 0.74850D+00 | 0 30729D+01 | 0.86519D+00 |
| 24 | -0 37051D-02 | -0 27512D-01 | 0 86704D+00 | 0 91965D-02 | 0 23784D+00 | 0.39752D-01 | 0 11190D+01 | 0 30725D+01 | 0 86704D+00 |
| 25 | -0 78979D-02 | -0 28001D-01 | 0.87038D+00 | 0 57092D-02 | 0 27058D+00 | 0.17812D+00 | 0 14891D+01 | 0 30720D+01 | 0 87038D+00 |
| 26 | 0 82257D-02 | -0 20683D-01 | 0 94859D+00 | -0 38349D-01 | 0 17072D+00 | -0.16177D+00 | 0 82257D-02 | 0 35293D+01 | 0 94859D+00 |
| 27 | 0 41950D-02 | -0 20351D-01 | 0 93843D+00 | -0 16392D-01 | 0 13905D+00 | -0 28622D-01 | 0 37844D+00 | 0 35296D+01 | 0 93843D+00 |
| 28 | 0 33151D-12 | -0 20112D-01 | 0 93517D+00 | -0 28075D-12 | 0 13443D+00 | 0 20816D-11 | 0 74850D+00 | 0 35299D+01 | 0.93517D+00 |
| 29 | -0 41950D-02 | -0 20351D-01 | 0 93843D+00 | 0 16392D-01 | 0 13905D+00 | 0 28622D-01 | 0 11186D+01 | 0 35296D+01 | 0.93843D+00 |
| 30 | -0 82257D-02 | -0 20683D-01 | 0 94859D+00 | 0 38349D-01 | 0 17072D+00 | 0 16177D+00 | 0.14888D+01 | 0 35293D+01 | 0 94859D+00 |
| 31 | 0 11871D-01 | 0 47311D-13 | 0 96197D+00 | -0 29993D-01 | 0 12727D-09 | -0.59200D-09 | 0.11871D-01 | 0 40000D+01 | 0.96197D+00 |
| 32 | 0 43607D-02 | 0 29948D-11 | 0 95008D+00 | -0 26395D-01 | 0 57128D-10 | 0.18801D-09 | 0 37861D+00 | 0 40000D+01 | 0.95008D+00 |
| 33 | 0 16244D-12 | 0 10268D-10 | 0.94575D+00 | -0 42258D-13 | 0 24093D-10 | -0 27419D-10 | 0 74850D+00 | 0.40000D+01 | 0 94575D+00 |
| 34 | -0 43607D-02 | 0 19735D-11 | 0 95008D+00 | 0 26395D-01 | 0 48763D-10 | -0 18904D-09 | 0 11184D+01 | 0 40000D+01 | 0.95008D+00 |
| 35 | -0 11871D-01 | 0 70815D-12 | 0 96197D+00 | 0 29999D-01 | 0 98154D-10 | 0 42941D-09 | 0 14851D+01 | 0 40000D+01 | 0 96197D+00 |
| 36 | 0 82257D-02 | 0 20683D-01 | 0 94859D+00 | -0 38349D-01 | -0 17072D+00 | 0 16177D+00 | 0 82257D-02 | 0 44707D+01 | 0.94859D+00 |
| 37 | 0 41950D-02 | 0 20351D-01 | 0 93843D+00 | -0 16392D-01 | -0 13905D+00 | 0 28622D-01 | 0 37844D+00 | 0.44704D+01 | 0 93843D+00 |
| 38 | 0 89324D-14 | 0 20112D-01 | 0 93517D+00 | 0 27328D-11 | -0 13443D+00 | 0 95934D-11 | 0 74850D+00 | 0.44701D+01 | 0 93517D+00 |
| 39 | -0 41950D-02 | 0 20351D-01 | 0 93843D+00 | 0 16392D-01 | -0 13905D+00 | -0 28622D-01 | 0 11196D+01 | 0 44704D+01 | 0.93843D+00 |
| 40 | -0 82257D-02 | 0 20683D-01 | 0 94859D+00 | 0 38349D-01 | -0 17072D+00 | -0 16177D+00 | 0 14888D+01 | 0 44707D+01 | 0.94859D+00 |
| 41 | 0 78979D-02 | 0 28001D-01 | 0 87038D+00 | -0 57092D-02 | -0 27058D+00 | 0 17812D+00 | 0 78979D-02 | 0 49280D+01 | 0.87038D+00 |
| 42 | 0 37051D-02 | 0 27512D-01 | 0 86704D+00 | -0 91965D-02 | -0 23784D+00 | 0 39752D-01 | 0 37796D+00 | 0.49275D+01 | 0 86704D+00 |
| 43 | -0 41837D-12 | 0 27076D-01 | 0 86519D+00 | 0 60115D-11 | -0 23135D+00 | 0.10183D-10 | 0 74850D+00 | 0 49271D+01 | 0.86519D+00 |
| 44 | -0 37051D-02 | 0 27512D-01 | 0 86704D+00 | 0 91965D-02 | -0 23784D+00 | -0 39752D-01 | 0 11190D+01 | 0 49275D+01 | 0.86704D+00 |
| 45 | -0 78979D-02 | 0 28001D-01 | 0 87038D+00 | 0 57092D-02 | -0 27058D+00 | -0 17812D+00 | 0 14891D+01 | 0 49280D+01 | 0 87038D+00 |
| 46 | 0 94630D-02 | 0 42452D-01 | 0 71707D+00 | -0 15207D-01 | -0 33367D+00 | 0.13143D+00 | 0 94630D-02 | 0.57175D+01 | 0.71707D+00 |
| 47 | 0 46101D-02 | 0 40797D-01 | 0 71243D+00 | -0 88388D-02 | -0 32049D+00 | 0 58165D-02 | 0 37886D+00 | 0 57158D+01 | 0.71243D+00 |
| 48 | -0 10255D-11 | 0 40075D-01 | 0 71067D+00 | 0 26425D-11 | -0 31906D+00 | -0 23868D-10 | 0 74850D+00 | 0 57151D+01 | 0.71067D+00 |
| 49 | -0 46101D-02 | 0 40797D-01 | 0 71243D+00 | 0 88388D-02 | -0 32049D+00 | -0 58165D-02 | 0.11181D+01 | 0 57158D+01 | 0.71243D+00 |
| 50 | -0 94630D-02 | 0 42452D-01 | 0 71707D+00 | 0 15207D-01 | -0 33367D+00 | -0 13143D+00 | 0 14875D+01 | 0 57175D+01 | 0.71707D+00 |
| 51 | 0 99164D-02 | 0 42556D-01 | 0 49051D+00 | -0 98516D-02 | -0 37709D+00 | 0 66297D-01 | 0.99164D-02 | 0 64926D+01 | 0.49051D+00 |
| 52 | 0.50632D-02 | 0 41245D-01 | 0 48922D+00 | -0 23659D-02 | -0 37439D+00 | -0.24323D-02 | 0 37931D+00 | 0 64912D+01 | 0.48922D+00 |
| 53 | -0 11043D-11 | 0.40976D-01 | 0 48887D+00 | 0 63194D-11 | -0 37460D+00 | -0 81592D-11 | 0.74850D+00 | 0 64910D+01 | 0.48887D+00 |
| 54 | -0 50632D-02 | 0 41245D-01 | 0 48922D+00 | 0 23659D-02 | -0.37439D+00 | 0.24323D-02 | 0.11177D+01 | 0.64912D+01 | 0.48922D+00 |
| 55 | -0.99164D-02 | 0 42556D-01 | 0 49051D+00 | 0 98516D-02 | -0 37709D+00 | -0 66297D-01 | 0.14871D+01 | 0.64926D+01 | 0.49051D+00 |
| 56 | 0 10577D-01 | 0.15472D-01 | 0 21855D+00 | -0 62583D-02 | -0 33756D+00 | -0.15768D-01 | 0 10577D-01 | 0.72405D+01 | 0.21855D+00 |
| 57 | 0 51089D-02 | 0.15957D-01 | 0 21709D+00 | -0 26210D-02 | -0 34761D+00 | -0.13124D-01 | 0.37936D+00 | 0.72410D+01 | 0.21709D+00 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 58 | -0.17301D-12 | 0.15916D-01 | 0.21655D+00 | 0.13874D-11 | -0 34926D+00 | -0.65051D-11 | 0.74850D+00 | 0.72409D+01 | 0.21655D+00 |
| 59 | -0.51089D-02 | 0.15957D-01 | 0 21709D+00 | 0 26210D-02 | -0 34761D+00 | 0.13124D-01 | 0.11176D+01 | 0.72410D+01 | 0.21709D+00 |
| 60 | -0.10577D-01 | 0.15472D-01 | 0.21855D+00 | 0 62583D-02 | -0.33756D+00 | 0.15768D-01 | 0.14864D+01 | 0.72405D+01 | 0.21855D+00 |
| 61 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.80000D+01 | 0.0 |
| 62 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.37425D+00 | 0.80000D+01 | 0.0 |
| 63 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.74850D+00 | 0.80000D+01 | 0.0 |
| 64 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 11227D+01 | 0 80000D+01 | 0.0 |
| 65 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.14970D+01 | 0.80000D+01 | 0.0 |

REACTION FORCES AT CONSTRAINED NODES

| NODE | RX(LBS) | RY(LBS) | RZ(LBS) | MX(LBS-IN) | MY(LBS-IN) | MXY(LBS-IN-IN) |
|---|---|---|---|---|---|---|
| 1 | -0.17901D+03 | -0 14156D+03 | -0.41818D+02 | -0.59745D+01 | -0.66443D+01 | -0 38651D+00 |
| 2 | -0 30362D+02 | -0 30566D+02 | -0.75548D+02 | 0 28669D-01 | -0 43400D+01 | -0 17978D-01 |
| 3 | 0.44715D-08 | -0 44600D+02 | -0 75675D+02 | -0 60652D-09 | -0 43263D+01 | 0.22240D-11 |
| 4 | 0.30362D+02 | -0 30566D+02 | -0 75548D+02 | -0.28669D-01 | -0 43400D+01 | 0.17978D-01 |
| 5 | 0 17901D+03 | -0 14156D+03 | -0.41818D+02 | 0 59745D+01 | -0 66443D+01 | 0 38651D+00 |
| 61 | -0 17901D+03 | 0 14156D+03 | -0 41818D+02 | -0 59745D+01 | 0 66443D+01 | 0 38651D+00 |
| 62 | -0 30362D+02 | 0 30566D+02 | -0 75548D+02 | 0 28669D-01 | 0 43400D+01 | 0 17978D-01 |
| 63 | -0 16765D-07 | 0 44600D+02 | -0 75675D+02 | 0 14277D-08 | 0.43263D+01 | -0 40298D-10 |
| 64 | 0 30362D+02 | 0.30566D+02 | -0 75548D+02 | -0.28669D-01 | 0 43400D+01 | -0 17978D-01 |
| 65 | 0.17901D+03 | 0 14156D+03 | -0 41818D+02 | 0 59745D+01 | 0 66443D+01 | -0.38651D+00 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION AT CENTROID OF EACH ELEMENT

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| ELEM. | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 1 | -0 6995D-02 | -0 7561D-02 | 0 5860D-01 | 0.1349D-01 | 0 6227D-02 | 0 9348D-02 | 0 5874D-01 | 0 1448D-01 | 0.8729D+02 | 0.7803D+02 |
| 2 | -0 6472D-02 | -0.7175D-02 | 0 5757D-01 | 0 1163D-01 | 0 2618D-02 | 0 3375D-02 | 0 5760D-01 | 0.1178D-01 | 0.8883D+02 | 0.8491D+02 |
| 3 | -0 6472D-02 | -0 7175D-02 | 0 5757D-01 | 0 1163D-01 | -0 2618D-02 | -0 3375D-02 | 0 5760D-01 | 0.1178D-01 | -0.8883D+02 | -0.8491D+02 |
| 4 | -0 6995D-02 | -0 7561D-02 | 0.5860D-01 | 0 1349D-01 | -0.6227D-02 | -0 9348D-02 | 0 5874D-01 | 0 1448D-01 | -0.8729D+02 | -0.7803D+02 |
| 5 | -0.1397D-01 | -0 1360D-01 | 0 2851D-01 | 0 2459D-01 | -0 7614D-04 | 0 1263D-02 | 0 2851D-01 | 0 2460D-01 | -0.8995D+02 | 0 8905D+02 |
| 6 | -0.1339D-01 | -0 1379D-01 | 0 2906D-01 | 0 2569D-01 | 0 1305D-03 | 0 4685D-04 | 0 2906D-01 | 0 2569D-01 | 0.8991D+02 | 0.8997D+02 |
| 7 | -0 1339D-01 | -0 1379D-01 | 0 2906D-01 | 0 2569D-01 | -0.1305D-03 | -0 4685D-04 | 0 2906D-01 | 0 2569D-01 | -0.8991D+02 | -0 8997D+02 |
| 8 | -0.1397D-01 | -0 1360D-01 | 0 2851D-01 | 0.2459D-01 | 0 7614D-04 | -0 1263D-02 | 0 2851D-01 | 0 2460D-01 | 0 8995D+02 | -0 8905D+02 |
| 9 | -0 1276D-01 | -0 1315D-01 | 0 3129D-01 | 0 3804D-01 | 0 3292D-03 | 0 3787D-02 | 0 3129D-01 | 0 3811D-01 | 0.8979D+02 | 0.8788D+02 |
| 10 | -0 1227D-01 | -0.1357D-01 | 0 3015D-01 | 0 3740D-01 | -0 2749D-03 | 0 1111D-02 | 0 3015D-01 | 0 3741D-01 | -0 8981D+02 | 0.8938D+02 |
| 11 | -0 1227D-01 | -0 1357D-01 | 0 3015D-01 | 0 3740D-01 | 0 2749D-03 | -0 1111D-02 | 0 3015D-01 | 0 3741D-01 | 0 8981D+02 | -0 8938D+02 |
| 12 | -0 1276D-01 | -0 1315D-01 | 0 3129D-01 | 0 3804D-01 | -0 3292D-03 | -0 3787D-02 | 0 3129D-01 | 0 3811D-01 | -0 8979D+02 | -0 8788D+02 |
| 13 | -0 1204D-01 | -0 1210D-01 | 0.2533D-01 | 0 3500D-01 | 0 3120D-02 | -0 2592D-02 | 0 2541D-01 | 0 3504D-01 | 0 8739D+02 | -0 8842D+02 |
| 14 | -0 1032D-01 | -0 1188D-01 | 0 2422D-01 | 0 3564D-01 | 0 8124D-03 | -0 1199D-03 | 0 2422D-01 | 0 3564D-01 | 0 8933D+02 | -0 8993D+02 |
| 15 | -0 1032D-01 | -0 1188D-01 | 0 2122D-01 | 0 3564D-01 | -0 8124D-03 | 0 1199D-03 | 0 2422D-01 | 0.3564D-01 | -0 8933D+02 | 0 8993D+02 |
| 16 | -0 1204D-01 | -0 1210D-01 | 0 2533D-01 | 0 3500D-01 | -0 3420D-02 | 0 2592D-02 | 0 2541D-01 | 0 3504D-01 | -0 8739D+02 | 0 8842D+02 |
| 17 | -0 9523D-02 | -0 1212D-01 | 0 1556D-01 | 0 3801D-01 | -0 2579D-02 | 0 1267D-02 | 0 1562D-01 | 0 3802D-01 | -0 8706D+02 | 0 8928D+02 |
| 18 | -0 8699D-02 | -0.1236D-01 | 0 1491D-01 | 0 3687D-01 | -0 2135D-03 | 0 1026D-02 | 0 1481D-01 | 0 3688D-01 | -0.8974D+02 | 0.8940D+02 |
| 19 | -0 8699D-02 | -0 1236D-01 | 0 1491D-01 | 0 3687D-01 | 0 2135D-03 | -0 1026D-02 | 0 1481D-01 | 0 3688D-01 | 0 8974D+02 | -0 8940D+02 |
| 20 | -0 9523D-02 | -0 1212D-01 | 0 1556D-01 | 0 3801D-01 | 0 2579D-02 | -0 1267D-02 | 0 1562D-01 | 0 3802D-01 | 0 8706D+02 | -0 8928D+02 |
| 21 | -0 1205D-01 | -0 1758D-01 | 0 2876D-01 | 0 6247D-01 | 0 4224D-02 | 0 4614D-02 | 0 2888D-01 | 0.6254D-01 | 0 8705D+02 | 0 8835D+02 |
| 22 | -0 8241D-02 | -0 1451D-01 | 0 2961D-01 | 0 6031D-01 | 0 1799D-03 | 0 7939D-03 | 0 2962D-01 | 0 6031D-01 | 0 8986D+02 | 0 8970D+02 |
| 23 | -0 8241D-02 | -0 1451D-01 | 0 2961D-01 | 0 6031D-01 | -0 1799D-03 | -0 7939D-03 | 0 2962D-01 | 0 6031D-01 | -0 8986D+02 | -0 8970D+02 |
| 24 | -0 1205D-01 | -0 1758D-01 | 0 2876D-01 | 0 6247D-01 | -0 4224D-02 | -0 4614D-02 | 0 2888D-01 | 0 6254D-01 | -0 8705D+02 | -0 8835D+02 |
| 25 | -0 1205D-01 | -0 1758D-01 | 0 2878D-01 | 0 6247D-01 | -0.4221D-02 | -0 4614D-02 | 0 2888D-01 | 0 6254D-01 | -0 8705D+02 | -0 8835D+02 |
| 26 | -0 8241D-02 | -0 1451D-01 | 0 2961D-01 | 0 6031D-01 | -0 1799D-03 | -0 7939D-03 | 0 2962D-01 | 0 6031D-01 | -0 8986D+02 | -0 8970D+02 |
| 27 | -0 8241D-02 | -0 1451D-01 | 0 2961D-01 | 0 6031D-01 | 0 1799D-03 | 0 7939D-03 | 0 2962D-01 | 0 6031D-01 | 0 8986D+02 | 0 8970D+02 |
| 28 | -0 1205D-01 | -0 1758D-01 | 0 2878D-01 | 0 6247D-01 | 0 4224D-02 | 0 4614D-02 | 0 2888D-01 | 0 6254D-01 | 0.8705D+02 | 0 8835D+02 |
| 29 | -0 9523D-02 | -0 1212D-01 | 0.1556D-01 | 0 3801D-01 | 0 2579D-02 | -0 1267D-02 | 0 1562D-01 | 0 3802D-01 | 0 8706D+02 | -0 8928D+02 |
| 30 | -0 8699D-02 | -0 1236D-01 | 0 1431D-01 | 0 3687D-01 | 0 2135D-03 | -0 1026D-02 | 0 1481D-01 | 0 3688D-01 | 0 8974D+02 | -0 8940D+02 |
| 31 | -0 8699D-02 | -0 1236D-01 | 0 1431D-01 | 0 3687D-01 | -0 2135D-03 | 0.1026D-02 | 0 1481D-01 | 0 3688D-01 | -0 8974D+02 | 0 8940D+02 |
| 32 | -0 9523D-02 | -0 1212D-01 | 0 1556D-01 | 0 3801D-01 | -0 2579D-02 | 0 1267D-02 | 0 1562D-01 | 0 3802D-01 | -0 8706D+02 | 0 8928D+02 |
| 33 | -0.1204D-01 | -0 1210D-01 | 0 2533D-01 | 0 3500D-01 | -0 3120D-02 | 0 2592D-02 | 0 2541D-01 | 0 3504D-01 | -0 8739D+02 | 0 8842D+02 |
| 34 | -0 1032D-01 | -0 1188D-01 | 0 2422D-01 | 0 3564D-01 | -0 8124D-03 | 0 1199D-03 | 0 2422D-01 | 0.3564D-01 | -0 8933D+02 | 0 8993D+02 |
| 35 | -0.1032D-01 | -0 1188D-01 | 0 2422D-01 | 0 3564D-01 | 0 8124D-03 | -0 1199D-03 | 0.2422D-01 | 0.3564D-01 | 0 8933D+02 | -0.8993D+02 |
| 36 | -0 1204D-01 | -0 1210D-01 | 0 2533D-01 | 0 3500D-01 | 0 3120D-02 | -0 2592D-02 | 0 2541D-01 | 0 3504D-01 | 0 8739D+02 | -0.8842D+02 |
| 37 | -0 1276D-01 | -0 1315D-01 | 0 3129D-01 | 0 3804D-01 | -0 3292D-03 | 0 3787D-02 | 0 3129D-01 | 0 3811D-01 | -0.8979D+02 | -0 8788D+02 |
| 38 | -0 1227D-01 | -0 1357D-01 | 0 3015D-01 | 0 3740D-01 | 0 2749D-03 | -0 1111D-02 | 0 3015D-01 | 0 3741D-01 | 0.8981D+02 | -0.8938D+02 |
| 39 | -0 1227D-01 | -0 1357D-01 | 0 3015D-01 | 0 3740D-01 | -0 2749D-03 | 0.1111D-02 | 0.3015D-01 | 0.3741D-01 | -0.8981D+02 | 0.8938D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 40 | -0.1276D-01 | -0.1315D-01 | 0.3129D-01 | 0.3804D-01 | 0.3292D-03 | 0.2787D-02 | 0.3129D-01 | 0.3811D-01 | 0.8979D+02 | 0.8788D+02 |
| 41 | -0.1397D-01 | -0.1360D-01 | 0.2851D-01 | 0.2459D-01 | 0.7614D-04 | -0.1263D-02 | 0.2851D-01 | 0.2460D-01 | 0.8995D+02 | -0.8905D+02 |
| 42 | -0.1339D-01 | -0.1379D-01 | 0.2906D-01 | 0.2569D-01 | -0.1305D-03 | -0.4685D-04 | 0.2906D-01 | 0.2569D-01 | -0.8991D+02 | -0.8997D+02 |
| 43 | -0.1339D-01 | -0.1379D-01 | 0.2906D-01 | 0.2569D-01 | 0.1305D-03 | 0.4685D-04 | 0.2906D-01 | 0.2569D-01 | 0.8991D+02 | 0.8997D+02 |
| 44 | -0.1397D-01 | -0.1360D-01 | 0.2851D-01 | 0.2459D-01 | -0.7614D-04 | 0.1263D-02 | 0.2851D-01 | 0.2460D-01 | -0.8995D+02 | 0.8905D+02 |
| 45 | -0.6995D-02 | -0.7561D-02 | 0.5860D-01 | 0.1349D-01 | -0.6227D-02 | -0.9348D-02 | 0.5874D-01 | 0.1448D-01 | -0.8729D+02 | -0.7803D+02 |
| 46 | -0.6472D-02 | -0.7175D-02 | 0.5757D-01 | 0.1163D-01 | -0.2618D-02 | -0.3375D-02 | 0.5760D-01 | 0.1178D-01 | -0.8883D+02 | -0.8491D+02 |
| 47 | -0.6472D-02 | -0.7175D-02 | 0.5757D-01 | 0.1163D-01 | 0.2618D-02 | 0.3375D-02 | 0.5760D-01 | 0.1178D-01 | 0.8883D+02 | 0.8491D+02 |
| 48 | -0.6995D-02 | -0.7561D-02 | 0.5860D-01 | 0.1349D-01 | 0.6227D-02 | 0.9348D-02 | 0.5874D-01 | 0.1448D-01 | 0.8729D+02 | 0.7803D+02 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 33 | -0.9395D-02 | -0.1391D-01 | 0.5916D-01 | 0.3022D-01 | -0.3371D-11 | 0.2223D-11 | 0.5916D-01 | 0.3022D-01 | -0.9000D+02 | 0.9000D+02 |
| 35 | -0.2203D-01 | -0.1719D-01 | 0.6445D-01 | 0.2749D-01 | 0.4495D-10 | -0.4265D-10 | 0.6445D-01 | 0.2749D-01 | 0.9000D+02 | -0.9000D+02 |
| 38 | -0.8546D-02 | -0.1387D-01 | 0.2047D-01 | 0.5777D-01 | 0.8271D-12 | -0.1130D-11 | 0.2047D-01 | 0.5777D-01 | 0.9000D+02 | -0.9000D+02 |
| 40 | -0.6847D-02 | -0.1322D-01 | 0.2737D-01 | 0.6400D-01 | -0.1775D-01 | 0.1525D-01 | 0.2954D-01 | 0.6475D-01 | -0.7629D+02 | 0.8441D+02 |
| 48 | -0.1088D-01 | -0.1375D-01 | 0.5216D-01 | 0.6758D-01 | -0.1291D-11 | 0.3782D-11 | 0.5216D-01 | 0.6758D-01 | -0.9000D+02 | 0.9000D+02 |
| 50 | -0.1228D-01 | -0.1342D-01 | 0.5311D-01 | 0.7671D-01 | -0.1536D-01 | 0.1145D-01 | 0.5430D-01 | 0.7707D-01 | -0.8342D+02 | 0.8638D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG.(DIR.1) | ELONG.(DIR 2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0.60618D-26 | 0.43823D-01 | 0.22021D-12 | 0.0 | 0.42903D-01 | 0.43823D-01 | 0.90000D+02 |
| 1 | OUTER | 0.60618D-26 | -0.84898D-01 | 0.22021D-12 | 0.0 | -0.88844D-01 | 0.0 | 0.0 |
| 2 | INNER | -0.34469D-02 | 0.44627D-01 | 0.28111D-04 | -0.31529D-02 | 0.43673D-01 | 0.44627D-01 | 0.89983D+02 |
| 2 | OUTER | -0.35987D-02 | -0.41383D-01 | 0.28111D-04 | -0.36052D-02 | -0.42276D-01 | 0.0 | 0.0 |
| 3 | INNER | -0.13241D-01 | 0.26606D-01 | 0.28584D-03 | -0.11330D-01 | 0.26262D-01 | 0.26607D-01 | 0.89795D+02 |
| 3 | OUTER | -0.13990D-01 | 0.26765D-01 | 0.28584D-03 | -0.14090D-01 | 0.26416D-01 | 0.26765D-01 | 0.89799D+02 |
| 4 | INNER | -0.12756D-01 | 0.27426D-01 | 0.11090D-02 | -0.12839D-01 | 0.27060D-01 | 0.27434D-01 | 0.89210D+02 |
| 4 | OUTER | -0.13520D-01 | 0.51728D-01 | 0.11090D-02 | -0.13613D-01 | 0.50455D-01 | 0.51733D-01 | 0.89513D+02 |
| 5 | INNER | -0.10031D-01 | 0.19200D-01 | 0.16585D-02 | -0.10082D-01 | 0.19019D-01 | 0.19223D-01 | 0.88376D+02 |
| 5 | OUTER | -0.12889D-01 | 0.48125D-01 | 0.16585D-02 | -0.12973D-01 | 0.47020D-01 | 0.48137D-01 | 0.89221D+02 |
| 6 | INNER | -0.93955D-02 | 0.59164D-01 | 0.18701D-10 | -0.94400D-02 | 0.57511D-01 | 0.59164D-01 | 0.90000D+02 |
| 6 | OUTER | -0.13908D-01 | 0.30221D-01 | 0.24294D-10 | -0.11006D-01 | 0.29778D-01 | 0.30221D-01 | 0.90000D+02 |
| 7 | INNER | -0.10031D-01 | 0.19200D-01 | -0.16585D-02 | -0.10082D-01 | 0.19019D-01 | 0.19223D-01 | -0.88376D+02 |
| 7 | OUTER | -0.12889D-01 | 0.48125D-01 | -0.16585D-02 | -0.12973D-01 | 0.47020D-01 | 0.48137D-01 | -0.89221D+02 |
| 8 | INNER | -0.12756D-01 | 0.27426D-01 | -0.11090D-02 | -0.12839D-01 | 0.27060D-01 | 0.27434D-01 | -0.89210D+02 |
| 8 | OUTER | -0.13520D-01 | 0.51728D-01 | -0.11090D-02 | -0.13613D-01 | 0.50455D-01 | 0.51733D-01 | -0.89513D+02 |
| 9 | INNER | -0.13241D-01 | 0.26606D-01 | -0.28584D-03 | -0.11330D-01 | 0.26262D-01 | 0.26607D-01 | -0.89795D+02 |
| 9 | OUTER | -0.13990D-01 | 0.26765D-01 | -0.28584D-03 | -0.14090D-01 | 0.26416D-01 | 0.26765D-01 | -0.89799D+02 |
| 10 | INNER | -0.34469D-02 | 0.44627D-01 | -0.28111D-04 | -0.34529D-02 | 0.43673D-01 | 0.44627D-01 | -0.89983D+02 |
| 10 | OUTER | -0.35987D-02 | -0.41383D-01 | -0.28111D-04 | -0.36052D-02 | -0.42276D-01 | 0.0 | 0.0 |
| 11 | INNER | 0.59735D-18 | 0.43823D-01 | 0.22324D-12 | 0.0 | 0.42903D-01 | 0.43823D-01 | 0.90000D+02 |
| 11 | OUTER | -0.59735D-18 | -0.84898D-01 | 0.22324D-12 | 0.0 | -0.88844D-01 | 0.0 | 0.0 |

SYSTEM ENERGIES(IN-LB)

```
     WORK INPUT TO STRUCTURE             =  0.3848832D+04
     STRUCTURE KINETIC ENERGY            =  0.1135526D+03
     STRUCTURE ELASTIC ENERGY            =  0.4064170D+02
     STRUCTURE PLASTIC ENERGY            =  0.3694637D+04
     ENERGY STORED IN ELASTIC RESTRAINTS=  0.0
```

## SECTION 8

## CIVM-PLATE 1 CODE EXAMPLE: FRAGMENT-IMPACTED NARROW RECTANGULAR PLATE

To illustrate the use of the CIVM-PLATE 1 computer program, a narrow rectangular plate with both ends ideally clamped and its two sides restrained by only z-direction line elastic springs is analyzed.* The plate is subjected only to impact by a single non-deformable spherical "fragment". Two alternate finite-element modelings of the plate are described, and the associated input data for each are shown. However, for conciseness, solution output data are given for only one of these modelings; these results can be employed by the user to check the adaptation of CIVM-PLATE 1 to his particular computing facilities.

### 8.1 Problem Description

A narrow rectangular uniform-thickness aluminum plate (0.200 x 1.60 x 6.80 in) with an aluminum stiffener of 0.1 by 0.2 in cross section "integrally attached", located at the plate's midwidth upper-surface station, and extending for the full 6.80-in span -- as depicted in Fig. 18 -- is the example structure. Its two ends are ideally clamped. Along its other two sides there are only z-direction line springs of $k_z$ = 3,000 lb/(in-in of span); hence, $k_x = k_y = k_\theta = k_\psi = 0$.

This plate is impacted by a solid (non-deformable) steel sphere of 1-inch diameter; the initial point of impact is near the midwidth-midspan station of the plate. The pre-impact line of flight of the sphere is in the y,z "symmetry" plane and is approaching along a line at 60° from the y (spanwise) axis as shown in Fig. 18. Prior to impact the sphere is assumed to have a translational velocity of 2794 in/sec and to be rotating about the x axis with $\omega_{fx}$ = 1650 rad/sec; hence, initially $\omega_{fy} = \omega_{fz} = 0$. Also, it is assumed that a fixed value of the coefficient of friction $\mu = 0.20$ applies

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*Additional examples and transient response predictions obtained by using the CIVM-PLATE 1 program are described in Subsection 7.6 of Ref. 14 as the "small strain" calculations.

during impact-interaction between the fragment and the plate. Fragment-plate impact is assumed to be perfectly-elastic; hence, e = 1 is used for the coefficient of restitution. Hence, the pertinent fragment data are:

$$V_{fx} = 0 \qquad \omega_{fx} = 1650 \text{ rad/sec}$$

$$V_{fy} = 1397.0 \text{ in/sec} \qquad \omega_{fy} = 0$$

$$V_{fz} = 2419.675 \text{ in/sec} \qquad \omega_{fz} = 0$$

$$m_f = 0.3815 \times 10^{-3} \frac{\text{lb-sec}^2}{\text{in}} \qquad I_f = 0.3816 \times 10^{-4} \text{ in-lb-sec}^2$$

Also, during an impact, the value of the impact-affected "circle radius" $L_{eff}$ is assumed to be $L_{eff} = 0.42$ inch which is slightly greater than twice the thickness (h = 0.20 in) of the uniform plate.

Both the plate and the stiffener are assumed to consist of the same type of aluminum. Its mass per unit volume is assumed to be $0.25384 \times 10^{-3}$ $(\text{lb-sec}^2)/\text{in}^4$. The uniaxial stress-strain $(\sigma,\varepsilon)$ properties of this material are represented by a 3-sublayer version of the mechanical-sublayer material model, defined by $\sigma_1,\varepsilon_1$ = 44,200 psi, 0.00442 in/in; $\sigma_2,\varepsilon_2$ = 49,200 psi, 0.0760; and $\sigma_3,\varepsilon_3$ = 76,400 psi, 0.615. For this example, the material is assumed to be insensitive to strain rate; hence, it is specified that D = p = 0. The material's elastic modulus and Poisson ratio are assumed to be, respectively, $10 \times 10^6$ psi and 1/3.

## 8.2 Modeling and Input Data

For illustrative purposes, two different structural models of the stiffened-plate specimen depicted in Fig. 18 are described in the following. First, the entire plate is modeled by 56 LLC plate elements as shown in Fig. 19. Next, since this problem involves symmetry only about the y,z plane, a 28-element half-plate model with symmetry invoked along the plate's midwidth station is used. The two models otherwise employ the same basic finite-element layout. For each of these models, the selected numbering of elements and nodes is shown in Fig. 19.

Pilot calculations showed that the largest linear-system natural frequency of the full-plate structural model is $\omega_{max} = 0.1748 \times 10^7$ rad/sec.

Hence, if the timewise central-difference model were used to predict the linear transient response of this model, a $\Delta t$ not exceeding $\Delta t = 2/\omega_{max} = 1.14$ $\mu$sec would be required. However, since the CIVM-PLATE 1 program uses the Houbolt (and a modified Houbolt) operator, a reasonable choice is $\Delta t = 2.0$ $\mu$sec, as discussed in Subsection 5.5.2.

Solution output data will be requested at the end of every 10 time steps or cycles; however, for conciseness, these data will be included here only at the end of time cycles (or time steps) 0, 1, 2, 3, 4, 140, and 150 (last).+ The desired solution output data are indicated as the following print options (Card 51 of Subsection 4.2):

| | |
|---|---|
| IOP1 | Displacements and current (global) X,Y,Z coordinates of each node+. |
| IOP4 | Strain (Green) components, principal (tensile) strain and direction, and elongation (in user-specified directions) at additional spanwise locations (evaluated at both upper and lower surfaces at each spanwise location) as specified by the user. |
| IOP7 | System energies (i.e., work input to structure, structure kinetic energy, structure elastic energy, structure plastic energy, energy stored in elastic restoring springs, and fragment translational and rotational kinetic energies) |
| IOP8 | Strain (Green) components, principal (tensile) strain and direction at the upper and lower plate surfaces at user-specified nodes (by nodal averaging). |

Print options IOP2, IOP3, and IOP5 are not desired; hence, set these quantities equal to zero on Card 51 (of Subsection 4.2). Summarized concisely as follows are the affected element or node numbers for each of these two finite-element models (see Fig. 19):

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+Automatically at each printout time, one obtains the global X,Y,Z location of the fragment as well as its translational and rotational velocity components.

| Output Option | Full-Plate Model | Half-Plate Model |
|---|---|---|
| IOP1 | All Nodes | All Nodes |
| IOP4 | Elements 30,31,34,35 | Elements 15 and 17 |
| IOP7 | Entire System | Entire System |
| IOP8 | Nodes 38,39,43,44, 48,49,53,54 | Nodes 22,23,25,26 28,29,31,32 |

The associated input data on each card are described for the full-plate and the half-plate model in Subsections 8.2.1 and 8.2.2, respectively. However, solution output data are presented in Subsection 8.3 only for the full-plate model.

### 8.2.1 Input Data for the Full-Plate Model

The values to be punched on the data cards are as follows (see Fig. 19a):

Card 1 8I10

MAXEL = 56
MNSL = 3
MNXST = 1
MNYST = 28
NBE = 1
MBWE = 1
MNC = 16335
NIAN = 75

Card 2 20I4

IMESH = 0
IMCONT = 0
IPUNCH = 0

Card 3 20I4

NEAD = 4
NECD = 14

Card 4a 5D16.7

| | | |
|---|---|---|
| EPAN | = | 0.1D8 |
| ANUP | = | .3333333D0 |
| DENSP | = | 0.25384D-3 |
| TH | = | 0.200D0 |
| XDIST | = | 1.600D0 |

Card 4b 5D16.7

| | | |
|---|---|---|
| YDIST | = | 6.8D0 |
| ZPOS | = | 0.0D0 |

Card 5 5D16.7

| | | |
|---|---|---|
| XG(1) | = | 0.0D0 |
| XG(2) | = | .400D0 |
| XG(3) | = | .800D0 |
| XG(4) | = | 1.200D0 |
| XG(5) | = | 1.600D0 |

Card 6a 5D16.7

| | | |
|---|---|---|
| YG(1) | = | 0.0D0 |
| YG(2) | = | 0.40D0 |
| YG(3) | = | 0.80D0 |
| YG(4) | = | 1.40D0 |
| YG(5) | = | 2.0D0 |

Card 6b 5D16.7

| | | |
|---|---|---|
| YG(6) | = | 2.60D0 |
| YG(7) | = | 3.0D0 |
| YG(8) | = | 3.4D0 |
| YG(9) | = | 3.8D0 |
| YG(10) | = | 4.2D0 |

Card 6c 5D16.7

| | | |
|---|---|---|
| YG(11) | = | 4.80D0 |
| YG(12) | = | 5.40D0 |
| YG(13) | = | 6.0D0 |
| YG(14) | = | 6.4D0 |
| YG(15) | = | 6.8D0 |

| | | |
|---|---|---|
| Card 7 | | 20I4 |
| NCSB(1) | = 2 | |
| NCSB(2) | = 0 | |
| NCSB(3) | = 2 | |
| NCSB(4) | = 0 | |
| Card 8 | | 20I4 |
| NAST | = 0 | |
| NCST | = 2 | |
| Card 17* | | 5D16.7 |
| APCST(1) | = 0.775D0 | |
| APCST(2) | = 0.825D0 | |
| Card 18 | | 5D16.7 |
| RHC(1) | = 0.2D0 | |
| RHC(2) | = 0.2D0 | |
| Card 19 | | 5D16.7 |
| RWC(1) | = 0.05D0 | |
| RWC(2) | = 0.05D0 | |
| Card 20 | | 5D16.7 |
| ECS(1) | = 0.1D8 | |
| ECS(2) | = 0.1D8 | |
| Card 21 | | 5D16.7 |
| ANUCS(1) | = 0.3333333D0 | |
| ANUCS(2) | = 0.3333333D0 | |
| Card 22 | | 5D16.7 |
| DENCS(1) | = 0.25384D-3 | |
| DENCS(2) | = 0.25384D-3 | |
| Card 23 | | 5D16.7 |
| XFCST(1) | = -0.2D0 | |
| XFCST(2) | = -0.2D0 | |

*For omitted cards see page 308.

Card 24 — 20I4

MATCS(1) = 1

MATCS(2) = 1

Card 25 — 20I4

NAXS = 0

NAYS = 0

Card 44* — 20I4

NMAT = 1

Card 45 — 20I4

NSUB = 3

Card 46 — 5D16.7

SIG(1) = 44.2D3

SIG(2) = 49.2D3

SIG(3) = 76.4D3

Card 47 — 5D16.7

EPS(1) = .00442D0

EPS(2) = .0760D0

EPS(3) = 0.615D0

Card 48 — 5D16.7

DSR(1) = 0.0D0

PSR(1) = 0.0D0

Card 49 — 20I4

ITIMEF = 150

INCRT = 1

IOUT = 1

Card 50 — 5D16.7

DELTAT = 2.0D-06

TIMEF = 3.00D-04

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*For omitted cards see page 308.

Card 51 2014

| | | |
|---|---|---|
| IOP1 | = 1 | |
| IOP2 | = 0 | |
| IOP3 | = 0 | |
| IOP4 | = 1 | |
| IOP5 | = 0 | |
| IOP6 | = 0 | |
| IOP7 | = 1 | |
| IOP8 | = 1 | |

Card 54* 20I4

| | |
|---|---|
| NASP | = 4 |

Card 55a I4,2D16.7

| | |
|---|---|
| LNASP(1) | = 30 |
| SLASP(1) | = 0.5D0 |
| ELASP(1) | = 0.5D0 |

Card 55b I4,2D16.7

| | |
|---|---|
| LNASP(2) | = 31 |
| SLASP(2) | = 0.5D0 |
| ELASP(2) | = 0.5D0 |

Card 55c I4,2D16.7

| | |
|---|---|
| LNASP(3) | = 34 |
| SLASP(3) | = 0.5D0 |
| ELASP(3) | = 0.5D0 |

Card 55d I4,2D16.7

| | |
|---|---|
| LNASP(4) | = 35 |
| SLASP(4) | = 0.5D0 |
| ELASP(4) | = 0.5D0 |

Card 56 20I4

| | |
|---|---|
| NASPE | = 2 |

---

*For omitted cards see page 308.

Card 57a I4,2D16.7

| | | |
|---|---|---|
| NSP | = 1 | |
| DIR1 | = 45.0D0 | |
| DIR2 | = 135.0D0 | |

Card 57b I4,2D16.7

| | | |
|---|---|---|
| NSP | = 2 | |
| DIR1 | = 45.0D0 | |
| DIR2 | = 135.0D0 | |

Card 61* 20I4

| | |
|---|---|
| NNSA | = 8 |

Card 62 20I4

| | |
|---|---|
| NVSA(1) | = 33 |
| NVSA(2) | = 35 |
| NVSA(3) | = 38 |
| NVSA(4) | = 40 |
| NVSA(5) | = 48 |
| NVSA(6) | = 50 |

Card 63 20I4

| | |
|---|---|
| NELES | = 28 |

Card 64a 20I4

| | |
|---|---|
| LNUM | = 4 |
| ISIDE | = 2 |

Card 65a 5D16.7

| | |
|---|---|
| SS(1) | = 0.0D0 |
| SS(2) | = 0.0D0 |
| SS(3) | = 0.3000D+04 |
| SS(4) | = 0.0D0 |
| SS(5) | = 0.0D0 |

Card 64b 20I4

| | |
|---|---|
| LNUM | = 8 |
| ISIDE | = 2 |

---

*For omitted cards see page 308.

| | | |
|---|---|---|
| Card 65b | | 5D16.7 |
| SS(1) | = 0.0D0 | |
| SS(2) | = 0.0D0 | |
| SS(3) | = 0.3000D+04 | |
| SS(4) | = 0.0D0 | |
| SS(5) | = 0.0D0 | |
| Card 64c | | 20I4 |
| LNUM | = 12 | |
| ISIDE | = 2 | |
| Card 65c | | 5D16.7 |
| SS(1) | = 0.0D0 | |
| SS(2) | = 0.0D0 | |
| SS(3) | = 0.3000D+04 | |
| SS(4) | = 0.0D0 | |
| SS(5) | = 0.0D0 | |
| Card 64d | | 20I4 |
| LNUM | = 16 | |
| ISIDE | = 2 | |
| Card 65d | | 5D16.7 |
| SS(1) | = 0.0D0 | |
| SS(2) | = 0.0D0 | |
| SS(3) | = 0.3000D+04 | |
| SS(4) | = 0.0D0 | |
| SS(5) | = 0.0D0 | |
| Card 64e | | 20I4 |
| LNUM | = 20 | |
| ISIDE | = 2 | |
| Card 65e | | 5D16.7 |
| SS(1) | = 0.0D0 | |
| SS(2) | = 0.0D0 | |
| SS(3) | = 0.3000D+04 | |
| SS(4) | = 0.0D0 | |
| SS(5) | = 0.0D0 | |

Card 64f 20I4

LNUM = 24

ISIDE = 2

Card 65f 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64g 20I4

LNUM = 28

ISIDE = 2

Card 65g 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64h 20I4

LNUM = 32

ISIDE = 2

Card 65h 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64i 20I4

LNUM = 36

ISIDE = 2

Card 65i 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64j 20I4

LNUM = 40

ISIDE = 2

Card 65j 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64k 20I4

LNUM = 44

ISIDE = 2

Card 65k 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64ℓ 20I4

LNUM = 48

ISIDE = 2

Card 65ℓ 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64m 20I4

LNUM = 52

ISIDE = 2

Card 65m 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64n 20I4

LNUM = 56

ISIDE = 2

Card 65n 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64o 20I4

LNUM = 1

ISIDE = 4

Card 65o 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64p 20I4

LNUM = 5

ISIDE = 4

Card 65p 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64q 20I4

LNUM = 9

ISIDE = 4

Card 65q 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64r 20I4

LNUM = 13

ISIDE = 4

Card 65r 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64s 20I4

LNUM = 17

ISIDE = 4

Card 65s 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64t 20I4

LNUM = 21

ISIDE = 4

Card 65t 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64u 20I4

LNUM = 25

ISIDE = 4

Card 65u 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64v 20I4

LNUM = 29

ISIDE = 4

Card 65v 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64w 20I4

LNUM = 33

ISIDE = 4

Card 65w 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+4

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64x 20I4

LNUM = 37

ISIDE = 4

Card 65x 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+4

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64y 20I4

LNUM = 41

ISIDE = 4

Card 65y 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+4

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64z 20I4

LNUM = 45

ISIDE = 4

Card 65z 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+4

SS(4) = 0.0D0

SS(5) = 0.0D0

| | | | |
|---|---|---|---|
| Card 64aa | | | 20I4 |
| | LNUM | = 49 | |
| | ISIDE | = 4 | |
| Card 65aa | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+4 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 64bb | | | 20I4 |
| | LNUM | = 53 | |
| | ISIDE | = 4 | |
| Card 65bb | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+4 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 66 | | | 5D16.7 |
| | XF | = 0.8D0 | |
| | YF | = 3.394412D+00 | |
| | ZF | = -0.6098787D+0 | |
| Card 67 | | | 5D16.7 |
| | VF(1) | = 0.0D0 | |
| | VF(2) | = 1397.0D+00 | |
| | VF(3) | = 2419.675D+00 | |
| Card 68 | | | 5D16.7 |
| | OMEGF(1) | = 1650.0D0 | |
| | OMEGF(2) | = 0.0D0 | |
| | OMEGF(3) | = 0.0D0 | |

Card 69 5D16.7

RF = 0.5D0

FMASS = 0.38153D-03

FMOI = 0.38160D-04

Card 70 5D16.7

COEFR = 1.0D0

FRNC = 0.2D0

Card 71 20I4

NSYM = 0

IPSS = 0

ICUT = 0

IUSEF = 1

Card 73* 5D16.7

EFFL = 0.42D0

Cards 9 through 16 are omitted because NAST on Card 8 is zero (that is, there are no complete X-direction stiffeners).

Cards 26 through 34 are omitted because NAXS on Card 25 is zero (that is, there are no additional X-direction stiffeners).

Cards 35 through 43 are omitted because NAYS on Card 25 is zero (that is, there are no additional Y-direction stiffeners).

Cards 52 and 53 are omitted because IOP3 on Card 51 is zero.

Cards 58 through 60 are omitted because IOP6 on Card 51 is zero.

Card 72 is omitted because ICUT on Card 71 is zero.

The following is the computer input deck for this example:

---

*For omitted cards see page 308.

```
      56          3          1         28          1          1     16335          75
  0   0   0
  4  14
          0.1D8       .3333333D0        .25384D-3           .200D0          1.600D0
          6.8D0            0 0D0
          0.0D0          .40000D0            .800D0           1.2D0            1.6D0
          0 0D0            0.4D0             0.8D0           1.4D0            2.0D0
          2.6D0            3.0D0             3.4D0           3.8D0            4.2D0
          4.8D0            5 4D0             6.0D0           6.4D0            6.8D0
  2   0   2   0
  0   2
        .775D0           .825D0
           2D0             .2D0
         .05D0            .05D0
         0.1D8            0.1D8
0.3333333    D00.33333333    D0
   0.25384  D-3 0.25384     D-3
        -0.2D0           -0.2D0
  1   1
  0   0
  1
  3
       44 2D3            49 2D3            76.4D3
     .00442D0          .0760D0           0.615D0
         0.0D0            0.0D0
150   1   1
      2.0D-06          3.0D-04
  1   0   0   1   0   0   1   1
  4
 30            0.5D0           0.5D0
 31            0.5D0           0.5D0
 34            0.5D0           0.5D0
 35            0 5D0           0.5D0
  2
  1           45.0D0         135.0D0
  2           45 0D0         135.0D0
  8
 38  39  43  44  48  49  53  54
 28
  4   2
          0.0D0            0.0D0     0.3000 D+04            0.0D0            0.0D0
  8   2
          0.0D0            0 0D0     0.3000 D+04            0.0D0            0.0D0
 12   2
          0.0D0            0.0D0     0.3000 D+04            0.0D0            0.0D0
 16   2
          0.0D0            0.0D0     0.3000 D+04            0.0D0            0.0D0
 20   2
          0.0D0            0.0D0     0.3000 D+04            0.0D0            0.0D0
 24   2
          0.0D0            0.0D0     0.3000 D+04            0.0D0            0.0D0
```

```
 28   2
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 32   2
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 36   2
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 40   2
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 44   2
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 48   2
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 52   2
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 56   2
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
  1   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
  5   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
  9   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 13   4
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 17   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 21   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 25   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 29   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 33   4
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 37   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 41   4
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
 45   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 49   4
           0.0D0             0.0D0       0.3000 D+04           0.0D0             0.0D0
 53   4
           0.0D0             0 0D0       0.3000 D+04           0.0D0             0.0D0
           0 8D0   3.394412 D+00 -0.6098737  D+0
           0.0D0       1397 0D+00 2419.675   D+00
  1650.0    D+00             0 0D0             0.0D0
           0.5D0  0.381513  D-03   0.38160  D-04
           1.0D0             0.2D0
  0    0   0   1
          0.42D0
```

### 8.2.2 Input Data for the Half-Plate Model*

The values to be punched on the data cards are as follows (see Fig. 19b):

| | | |
|---|---|---|
| Card 1 | | 8I10 |
| MAXEL | = 28 | |
| MNSL | = 3 | |
| MNXST | = 1 | |
| MNYST | = 14 | |
| NBE | = 1 | |
| MBWE | = 1 | |
| MNC | = 6561 | |
| NIAN | = 45 | |
| Card 2 | | 20I4 |
| IMESH | = 0 | |
| IMCONT | = 0 | |
| IPUNCH | = 0 | |
| Card 3 | | 20I4 |
| NEAD | = 2 | |
| NECD | = 14 | |
| Card 4a | | 5D16.7 |
| EDAN | = 0.1D8 | |
| ANUP | = .3333333D0 | |
| DENSP | = 0.25384D-3 | |
| TH | = 0.200D0 | |
| XDIST | = 0.800D0 | |
| Card 4b | | 5D16.7 |
| YDIST | = 6.8D0 | |
| ZPOS | = 0.0D0 | |
| Card 5 | | 5D16.7 |
| XG(1) | = 0.0D0 | |
| XG(2) | = .40000D0 | |
| XG(3) | = .800D0 | |

*Certain input cards are omitted; see page 308.

Card 6a 5D16.7

YG(1) = 0.0D0
YG(2) = 0.4D0
YG(3) = 0.8D0
YG(4) = 1.4D0
YG(5) = 2.0D0

Card 6b 5D16.7

YG(6) = 2.6D0
YG(7) = 3.0D0
YG(8) = 3.4D0
YG(9) = 3.8D0
YG(10) = 4.2D0

Card 6c 5D16.7

YG(11) = 4.80D0
YG(12) = 5.40D0
YG(13) = 6.0D0
YG(14) = 6.4D0
YG(15) = 6.8D0

Card 7 20I4

NCSB(1) = 2
NCSB(2) = 0
NCSB(3) = 2
NCSB(4) = 1

Card 8 20I4

NAST = 0
NCST = 1

Card 17 5D16.7

APCST(1) = 0.025D0

Card 18 5D16.7

RHC(1) = 0.2D0

Card 19 5D16.7

RWC(1) = 0.05D0

| | | |
|---|---|---|
| Card 20 | | 5D16.7 |
| ECS(1) | = 0.1D8 | |
| Card 21 | | 5D16.7 |
| ANUCS(1) | = 0.3333333D0 | |
| Card 22 | | 5D16.7 |
| DENCS(1) | = 0.25384D-3 | |
| Card 23 | | 5D16.7 |
| XFCST(1) | = -0.2D0 | |
| Card 24 | | 20I4 |
| MATCS(1) | = 1 | |
| Card 25 | | 20I4 |
| NAXS | = 0 | |
| NAYS | = 0 | |
| Card 44 | | 20I4 |
| NMAT | = 1 | |
| Card 45 | | 20I4 |
| NSUB | = 3 | |
| Card 46 | | 5D16.7 |
| SIG(1) | = 44.2D3 | |
| SIG(2) | = 49.2D3 | |
| SIG(3) | = 76.4D3 | |
| Card 47 | | 5D16.7 |
| EPS(1) | = .00442D0 | |
| EPS(2) | = .0760D0 | |
| EPS(3) | = 0.615D0 | |
| Card 48 | | 5D16.7 |
| DSR(1) | = 0.0D0 | |
| PSR(1) | = 0.0D0 | |
| | | 20I4 |

| | | | |
|---|---|---|---|
| Card 49 | | | 20I4 |
| | ITIMEF | = 150 | |
| | INCRT | = 1 | |
| | IOUT | = 1 | |
| Card 50 | | | 5D16.7 |
| | DELTAT | = 2.0D-06 | |
| | TIMEF | = 3.0D-04 | |
| Card 51 | | | 20I4 |
| | IOP1 | = 1 | |
| | IOP2 | = 0 | |
| | IOP3 | = 0 | |
| | IOP4 | = 1 | |
| | IOP5 | = 0 | |
| | IOP6 | = 0 | |
| | IOP7 | = 1 | |
| | IOP8 | = 1 | |
| Card 54 | | | 20I4 |
| | NASP | = 2 | |
| Card 55a | | | I4,2D16.7 |
| | LNASP(1) | = 15 | |
| | SLASP(1) | = 0.5D0 | |
| | ELASP(1) | = 0.5D0 | |
| Card 55b | | | I4,2D16.7 |
| | LNASP(2) | = 17 | |
| | SLASP(2) | = 0.5D0 | |
| | ELASP(2) | = 0.5D0 | |
| Card 56 | | | 20I4 |
| | NASPE | = 1 | |
| Card 57 | | | I4,2D16.7 |
| | NSP | = 1 | |
| | DIR1 | = 45.0D0 | |
| | DIR2 | = 135.0D0 | |

| | | | |
|---|---|---|---|
| Card 61 | | | 20I4 |
| | NNSA | = 8 | |
| Card 62 | | | 20I4 |
| | NVSA(1) | = 22 | |
| | NVSA(2) | = 23 | |
| | NVSA(3) | = 25 | |
| | NVSA(4) | = 26 | |
| | NVSA(5) | = 28 | |
| | NVSA(6) | = 29 | |
| | NVSA(7) | = 31 | |
| | NVSA(8) | = 32 | |
| Card 63 | | | 20I4 |
| | NELES | = 14 | |
| Card 64a | | | 20I4 |
| | LNUM | = 2 | |
| | ISIDE | = 2 | |
| Card 65a | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+04 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 64b | | | 20I4 |
| | LNUM | = 4 | |
| | ISIDE | = 2 | |
| Card 65b | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+04 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |

| | | | |
|---|---|---|---|
| Card 64c | | | 20I4 |
| | LNUM | = 6 | |
| | ISIDE | = 2 | |
| Card 65c | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+04 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 64d | | | 20I4 |
| | LNUM | = 8 | |
| | ISIDE | = 2 | |
| Card 65d | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+04 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 64e | | | 20I4 |
| | LNUM | = 10 | |
| | ISIDE | = 2 | |
| Card 65e | | | 5D16.7 |
| | SS(1) | = 0.0D0 | |
| | SS(2) | = 0.0D0 | |
| | SS(3) | = 0.3000D+04 | |
| | SS(4) | = 0.0D0 | |
| | SS(5) | = 0.0D0 | |
| Card 64f | | | 20I4 |
| | LNUM | = 12 | |
| | ISIDE | = 2 | |

Card 65f 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64g 20I4

LNUM = 14

ISIDE = 2

Card 65g 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64h 20I4

LNUM = 16

ISIDE = 2

Card 65h 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64i 20I4

LNUM = 18

ISIDE = 2

Card 65i 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64j 20I4

LNUM = 20

ISIDE = 2

Card 65j 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64k 20I4

LNUM = 22

ISIDE = 2

Card 65k 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64ℓ 20I4

LNUM = 24

ISIDE = 2

Card 65ℓ 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64m 20I4

LNUM = 26

ISIDE = 2

Card 65m 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 64n 20I4

LNUM = 28

ISIDE = 2

Card 65n 5D16.7

SS(1) = 0.0D0

SS(2) = 0.0D0

SS(3) = 0.3000D+04

SS(4) = 0.0D0

SS(5) = 0.0D0

Card 66 5D16.7

XF = 0.8D0

YF = 3.399412D+00

ZF = -0.6098787D+0

Card 67 5D16.7

VF(1) = 0.0D0

VF(2) = 1397.0D+00

VF(3) = 2419.675D+00

Card 68 5D16.7

OMEGF(1) = 1650.0D0

OMEGF(2) = 0.0D0

OMEGF(3) = 0.0D0

Card 69 5D16.7

RF = 0.5D0

FMASS = 0.38153D-03

FMOI = 0.38160D-04

| | | |
|---|---|---|
| Card 70 | | 5D16.7 |
| COEFR | = 1.0D0 | |
| FRNC | = 0.2D0 | |
| Card 71 | | 20I4 |
| NSYM | = 0 | |
| IPSS | = 0 | |
| ICUT | = 0 | |
| IUSEF | = 1 | |
| Card 73 | | 5D16.7 |
| EFFL | = 0.42D0 | |

The following is the complete input deck for this example:

```
      28          3          1         14          1          1       6561         45
 0    0    0
 2   14
           0.1D8      .33333333D0       .25384D-3           .200D0          0.800D0
           6.8D0           0 0D0
           0.0D0        .4000000D0          .800D0
           0.0D0           0.4D0           0.8D0           1.4D0           2.0D0
           2.6D0           3 0D0           3.4D0           3.8D0           4.2D0
           4.8D0           5.4D0           6.0D0           6.4D0           6.8D0
 2    0    2    1
 0    1
          .025D0
           .20D0
           .05D0
           0.1D8
0.3333333     D0
   0.25384  D-3
          -0.2D0
 1
 0    0
 1
 3
          44.2D3          49.2D3          76.4D3
        .00442D0        .0760D0          0.615D0
           0.0D0           0 0D0
150    1    1
        2.0D-06         3.0D-04
 1    0    0    1    0    0    1    1
 2
15              0.5D0           0.5D0
17              0 5D0           0.5D0
 1
 1             45.0D0         135.0D0
 8
22   23   25   26   28   29   31   32
14
 2    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
 4    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
 6    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
 8    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
10    2
           0 0D0           0 0D0    0.3000 D+04           0.0D0           0.0D0
12    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
14    2
           0.0D0           0.0D0    0.3000 D+04           0.0D0           0.0D0
16    2
```

```
                0.0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 18    2
                0.0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 20    2
                0.0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 22    2
                0.0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 24    2
                0.0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 26    2
                0 0D0               0.0D0      0.3000 D+04            0.0D0            0.0D0
 28    2
                0.0D0               0 0D0      0.3000 D+04            0.0D0            0.0D0
                0 0D0    3.394412 D+00 -0.6098787  D+0
                0 0D0       1397.0D+00 2419.675    D+00
  1650.0        D+00                0 0D0              0.0D0
                0.5D0   0.381513  D-03   0.38160  D-04
                1.0D0               0.2D0
  1    4        0    1
               0.42D0
```

The FORTRAN listing of the main program for the full CIVM plate example is given on pages 232-234. For the half-plate model, the associated main program is the same except for the following DIMENSION statement:

```
      DIMENSION DELD(450),DIS(450),DISP(450),DISM1(450),DISM2(450),      IMAN0063
     2FLN(450),FLVA(450),FLVM(450),FLVP(450),VEL(450),ICOL(450),         IMAN0064
     3INUM(450),KROW(450),NDEX(450),STF(16335),AMASS(450),               IMAN0065
     4NP(4,56),NODE(1344),TAUSS(56,12,9),TAUSE(56,12,9),TAUEE(56,12,9),  IMAN0066
     5EPSSI(9,56),EPSSO(9,56),EPEEI(9,56),EPEEO(9,56),EPSEI(9,56),       IMAN0067
     6EPSEO(9,56),NBC(60),BC(60),RFM(1,1),ILAST(1),UCF1(1),UCF2(1),      IMAN0068
     7XG(75),YG(75),ZG(75),XGI(75),YGI(75),TAGSS(1,12,3),LNXS(1),        IMAN0069
     8XSPROP(7,1),MATXS(1),TSGEE(28,12,3),LNYS(28),YSPROP(7,28),MATYS(28IMAN0070
     9),LNRS(28),ISRS(28),SC(5,28),NVSA(8),NCON(4,8),PMASS(75),VN(3,75),IMAN0071
     A VNB(3,75),SI(75),NEFF(75),ALPHA(75)                               IMAN0072
```

For the half-plate example, replace the original above DIMENSION statement with this DIMENSION statement:

```
      DIMENSION DELD(270),DIS(270),DISP(270),DISM1(270),DISM2(270),      IMAN1064
     2FLN(270),FLVA(270),FLVM(270),FLVP(270),VEL(270),ICOL(270)          IMAN1065
     3,INUM(270),KROW(270),NDEX(270),STF(6561),AMASS(270),               IMAN1066
     4NP(4,28),NODE(672),TAUSS(28,12,9),TAUSE(28,12,9),TAUEE(28,12,9),   IMAN1067
     5EPSSI(9,28),EPSSO(9,28),EPEEI(9,28),EPEEO(9,28),EPSEI(9,28),       IMAN1068
     6EPSEO(9,28),NBC(81),BC(81),RFM(1,1),ILAST(1),UCF1(1),UCF2(1),      IMAN1069
     7XG(45),YG(45),ZG(45),XGI(45),YGI(45),TAGSS(1,12,3),LNXS(1),        IMAN1070
     8XSPROP(7,1),MATXS(1),TSGEE(14,12,3),LNYS(14),YSPROP(7,14),MATYS(14IMAN1071
     9),LNRS(14),ISRS(14),SC(5,14),NVSA(8),NCON(4,8),PMASS(45),VN(3,45),IMAN1072
     A VNB(3,45),SI(45),NEFF(45),ALPHA(45)                               IMAN1073
```

## 8.3 Solution Output Data for the Full-Plate Model

The following is the output for 150 cycles (300 μsec) for the fragment-impacted narrow plate specimen depicted in Fig. 18; the associated finite-element model is shown in Fig. 19.

The first segment of the output describes the initial geometry and material properties of the basic plate and the integrally attached stiffener. The element numbers and global node numbers of the finite-element model of the structure are shown. The boundary conditions employed and the associated constrained degrees of freedom are identified. Next, pertinent geometric and material properties for the stiffener are listed. Then, the properties for each sublayer of the mechanical-sublayer material model are given: (a) stress, (b) strain, (c) yield stress, and (d) sublayer weighting factor. Finally, the time-step size used and the intended calculation run time and periodic printout times are identified.

The next segment describes the output to be given at every printout cycle, the properties of all elastic restoring springs, all pre-impact fragment data, and the selected coefficient of restitution and friction coefficient for impact. This output consists of (a) the generalized displacements at and the global location of each node, (b) the global location of as well as the components of the translational and rotational velocities of the fragment, (c) strain components as well as principal (tensile) strain and direction at 8 specified nodes, (d) strain components, principal (tensile) strain and direction, and relative elongations at 4 specified additional stations, and (e) a tabulation of the various energies of the system. Also noted are the number of impacts which have occurred and the current effective impact location.

In the interest of conciseness, only a portion of the called-for output is shown. Included are: (1) all input-verification information and (2) scheduled output at the end of time cycles 0, 1, 2, 3, 140, and 150 (last)+.
This output listing is intended for use in verifying the (successful) adaptation of the CIVM-PLATE 1 program to a user's computer facility.

+An auxiliary computer run was made with printout every cycle to give the output shown at the end of cycles 1, 2, 3, and 4.

CIVM-PLATE 1 COMPUTER CODE (SMALL STRAIN THEORY) : USER INPUT FOR ARRAY DIMENSIONS

| | | |
|---|---|---|
| MAXEL | = | 56 |
| MNSL | = | 3 |
| MNXST | = | 1 |
| MNYST | = | 28 |
| NBE | = | 1 |
| MBWE | = | 1 |
| MNC | = | 16335 |
| NIAN | = | 75 |

AUTO-GENERATED FINITE-ELEMENT MESH INFORMATION FOR STIFFENED OR UNSTIFFENED FLAT-PLATE PROBLEM

| | | |
|---|---|---|
| X-LENGTH(IN) | = | 0.1600000D+01 |
| Y-LENGTH(IN) | = | 0.6800000D+01 |
| Z-POSITION(IN) | = | 0.0 |
| THICKNESS(IN) | = | 0.2000000D+00 |
| YOUNGS MODULUS(PSI) | = | 0.1000000D+08 |
| POISSONS RATIO | = | 0.3333333D+00 |
| DENSITY(LB-SEC**2/IN**4) | = | 0.2538400D-03 |
| NO. OF ELEM. IN X-DIRECTION | = | 4 |
| NO OF ELEM IN Y-DIRECTION | = | 14 |
| NO OF D O F /ELEM. | = | 24 |
| TOTAL NO OF ELEMENTS | = | 56 |
| TOTAL NO OF D.O F | = | 450 |
| NO OF X-DIR STIFFENERS | = | 0 |
| NO OF Y-DIR STIFFENERS | = | 28 |

THE GLOBAL NODE NUMBERS ASSOCIATED WITH EACH ELEMENT ARE AS FOLLOWS :

| ELEMENT | GLOBAL NODE NUMBERS | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 7 | 6 |
| 2 | 2 | 3 | 8 | 7 |
| 3 | 3 | 4 | 9 | 8 |
| 4 | 4 | 5 | 10 | 9 |
| 5 | 6 | 7 | 12 | 11 |
| 6 | 7 | 8 | 13 | 12 |
| 7 | 8 | 9 | 14 | 13 |
| 8 | 9 | 10 | 15 | 14 |
| 9 | 11 | 12 | 17 | 16 |
| 10 | 12 | 13 | 18 | 17 |
| 11 | 13 | 14 | 19 | 18 |
| 12 | 14 | 15 | 20 | 19 |
| 13 | 16 | 17 | 22 | 21 |
| 14 | 17 | 18 | 23 | 22 |
| 15 | 18 | 19 | 24 | 23 |
| 16 | 19 | 20 | 25 | 24 |
| 17 | 21 | 22 | 27 | 26 |
| 18 | 22 | 23 | 28 | 27 |
| 19 | 23 | 24 | 29 | 28 |
| 20 | 24 | 25 | 30 | 29 |
| 21 | 26 | 27 | 32 | 31 |
| 22 | 27 | 28 | 33 | 32 |
| 23 | 28 | 29 | 34 | 33 |
| 24 | 29 | 30 | 35 | 34 |
| 25 | 31 | 32 | 37 | 36 |
| 26 | 32 | 33 | 38 | 37 |
| 27 | 33 | 34 | 39 | 38 |
| 28 | 34 | 35 | 40 | 39 |
| 29 | 36 | 37 | 42 | 41 |
| 30 | 37 | 38 | 43 | 42 |
| 31 | 38 | 39 | 44 | 43 |
| 32 | 39 | 40 | 45 | 44 |
| 33 | 41 | 42 | 47 | 46 |

| | | | | |
|---|---|---|---|---|
| 34 | 42 | 43 | 48 | 47 |
| 35 | 43 | 44 | 49 | 48 |
| 36 | 44 | 45 | 50 | 49 |
| 37 | 46 | 47 | 52 | 51 |
| 38 | 47 | 48 | 53 | 52 |
| 39 | 48 | 49 | 54 | 53 |
| 40 | 49 | 50 | 55 | 54 |
| 41 | 51 | 52 | 57 | 56 |
| 42 | 52 | 53 | 58 | 57 |
| 43 | 53 | 54 | 59 | 58 |
| 44 | 54 | 55 | 60 | 59 |
| 45 | 56 | 57 | 62 | 61 |
| 46 | 57 | 58 | 63 | 62 |
| 47 | 58 | 59 | 64 | 63 |
| 48 | 59 | 60 | 65 | 64 |
| 49 | 61 | 62 | 67 | 66 |
| 50 | 62 | 63 | 68 | 67 |
| 51 | 63 | 64 | 69 | 68 |
| 52 | 64 | 65 | 70 | 69 |
| 53 | 66 | 67 | 72 | 71 |
| 54 | 67 | 68 | 73 | 72 |
| 55 | 68 | 69 | 74 | 73 |
| 56 | 69 | 70 | 75 | 74 |

BOUNDARY CONDITIONS (SEE WRITEUP FOR CONVENTION FOR SIDE NUMBER AND BOUNDARY CONDITION)

| SIDE NUMBER | BOUNDARY CONDITION |
|---|---|
| 1 | IDEALLY CLAMPED |
| 2 | FREE |
| 3 | IDEALLY CLAMPED |
| 4 | FREE |

THE FOLLOWING 60 DEGREES OF FREEDOM ARE CONSTRAINED

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 421 | 422 | 423 | 424 | 425 | 426 | 427 | 428 | 429 | 430 |
| 431 | 432 | 433 | 434 | 435 | 436 | 437 | 438 | 439 | 440 | 441 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 449 | 450 |

INFORMATION FOR Y-DIRECTION STIFFENERS

| STIFF. NO. | ON ELEM. | MAT. NO. | THICKNESS | WIDTH | YOUNGS MOD. | POISSON RATIO | DENSITY | PSI-LOC. | OFFSET,ZF |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 2 | 6 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 3 | 10 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 4 | 14 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 5 | 18 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 6 | 22 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 7 | 26 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 8 | 30 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 9 | 34 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 10 | 38 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 11 | 42 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 12 | 46 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 13 | 50 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 14 | 54 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.93750D+00 | -0.20000D+00 |
| 15 | 3 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 16 | 7 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 17 | 11 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 18 | 15 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 19 | 19 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 20 | 23 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 21 | 27 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 22 | 31 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 23 | 35 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 24 | 39 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 25 | 43 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 26 | 47 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 27 | 51 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |
| 28 | 55 | 1 | 0.20000D+00 | 0.50000D-01 | 0.10000D+08 | 0.33333D+00 | 0.25384D-03 | 0.62500D-01 | -0.20000D+00 |

MAX SIZE IS 16335

PROPERTIES FOR USER-SPECIFIED MATERIALS

PROPERTIES FOR MATERIAL NUMBER 1

| SUBLAYER | STRESS POINT | STRAIN POINT | YIELD STRESS | WEIGHTING FACTOR |
|---|---|---|---|---|
| 1 | 0.4420000D+05 | 0.4420000D-02 | 0.4420000D+05 | 0.9937861D+00 |
| 2 | 0.4920000D+05 | 0.7600000D-01 | 0.8488500D+06 | 0.1725692D-02 |
| 3 | 0 7640000D+05 | 0.6150000D+00 | 0.6909200D+07 | 0 4488190D-02 |

STRAIN RATE PARAMETERS. D= 0 0 P= 0.0

TIMEWISE SOLUTION PARAMETERS
TIME-STEP SIZE= 0 2000000D-05 SEC.
RUN WILL TERMINATE AT TIME = 0 3000000D-03 SEC. OR AT CYCLE NUMBER 150
REGULAR PRINTOUT WILL BE GIVEN EVERY 10 CYCLES

***** OUTPUT CONTROL INFORMATION *****

THE FOLLOWING RESULTS WILL BE GIVEN AT EVERY REGULAR PRINTOUT CYCLE.
NODAL DISPLACEMENTS AND LOCATION
STRAIN COMPONENTS, PRINCIPAL STRAIN AND DIRECTION AT THE FOLLOWING 8 NODES (OBTAINED BY NODAL AVERAGING):
38 39 43 44 48 49 53 54
STRAIN COMPONENTS, ELONGATION IN SPECIFIED DIRECTIONS, PRINCIPAL STRAIN AND DIRECTION AT THE FOLLOWING 4 ADDTL. POINTS:

| ADDTL. POINT | ON ELEM | PSI-LOCATION | ETA-LOCATION | ELONG DIR.-1(DEG) | ELONG. DIR.-2(DEG) |
|---|---|---|---|---|---|
| 1 | 30 | 0 5000000D+00 | 0.5000000D+00 | 0.4500000D+02 | 0.1350000D+03 |
| 2 | 31 | 0.5000000D+00 | 0 5000000D+00 | 0 4500000D+02 | 0.1350000D+03 |
| 3 | 34 | 0 5000000D+00 | 0 5000000D+00 | 0 0 | 0.9000000D+02 |
| 4 | 35 | 0 5000000D+00 | 0.5000000D+00 | 0.0 | 0 9000000D+02 |

SYSTEM ENERGIES



LOCATION AND PROPERTIES OF LINE TRANSLATIONAL AND ROTATIONAL LINEAR RESTORING SPRINGS

| ELEM. NO. | ELEM. SIDE | KX(LB/IN/IN) | KY(LB/IN/IN) | KZ(LB/IN/IN) | K-THETA(IN-LB/RAD/IN) | K-PSI(IN-LB/RAD/IN) |
|---|---|---|---|---|---|---|
| 4 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0.0 | 0.0 |
| 8 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0.0 | 0.0 |
| 12 | 2 | 0.0 | 0 0 | 0.3000000D+04 | 0.0 | 0.0 |
| 16 | 2 | 0 0 | 0 0 | 0.3000000D+04 | 0 0 | 0.0 |
| 20 | 2 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0 0 |
| 24 | 2 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 28 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 32 | 2 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 36 | 2 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 40 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0.0 | 0.0 |
| 44 | 2 | 0 0 | 0.0 | 0 3000000D+04 | 0 0 | 0 0 |
| 48 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 52 | 2 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0 0 |
| 56 | 2 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 1 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 5 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0.0 | 0.0 |
| 9 | 4 | 0.0 | 0 0 | 0.3000000D+04 | 0 0 | 0.0 |
| 13 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 17 | 4 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0 0 |
| 21 | 4 | 0 0 | 0.0 | 0 3000000D+04 | 0 0 | 0 0 |
| 25 | 4 | 0 0 | 0.0 | 0 3000000D+04 | 0.0 | 0.0 |
| 29 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 33 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0 0 |
| 37 | 4 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 41 | 4 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 45 | 4 | 0 0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 49 | 4 | 0.0 | 0 0 | 0 3000000D+04 | 0 0 | 0.0 |
| 53 | 4 | 0.0 | 0 0 | 0 3000000D+04 | 0.0 | 0.0 |

HIGHEST NATURAL FREQUENCY(RAD/SEC)= 0.1748652D+07

HIGHEST EIGENVECTOR NORMALIZED BY LARGEST VALUE

| NODE | U | V | W | PSIX | PSIY | TWIST |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 |
| 2 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 4 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 |
| 5 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 6 | -0.35139D-08 | 0.44238D-07 | 0 60626D-02 | 0 35444D+00 | 0.40451D-01 | 0.0 |
| 7 | -0.13576D-06 | 0 21164D-05 | -0 19616D-02 | 0 18590D+00 | -0 58777D-01 | 0.0 |
| 8 | 0 37885D-08 | 0 42080D-04 | -0 54413D-03 | 0 17067D-01 | -0 10436D-01 | 0 0 |
| 9 | 0.13600D-06 | 0 21375D-05 | 0 95879D-03 | 0 97152D-01 | 0 25437D-01 | 0.0 |
| 10 | -0 62293D-08 | 0 58175D-07 | -0 31307D-02 | 0 18244D+00 | -0 21666D-01 | 0 0 |
| 11 | 0 29133D-07 | -0 52983D-07 | 0 11999D-01 | 0 78012D+00 | 0 29044D+00 | 0 0 |
| 12 | 0 61814D-06 | -0 26364D-06 | -0 36638D-02 | 0.44270D+00 | -0 11644D+00 | 0 0 |
| 13 | -0 15888D-08 | 0.79465D-05 | -0 55001D-03 | 0 28999D-01 | -0.90483D-02 | 0 0 |
| 14 | -0 61811D-06 | -0.53482D-06 | 0 18047D-02 | 0 2215CD+00 | 0 54841D-01 | 0.0 |
| 15 | -0 25110D-07 | -0.59877D-07 | -0 59563D-02 | 0 38763D+00 | -0 14442D+00 | 0.0 |
| 16 | 0 12572D-06 | 0 11263D-06 | 0 13334D-01 | 0 88038D+00 | 0 79973D-02 | 0 0 |
| 17 | -0 37265D-06 | -0 69987D-06 | -0 43368D-02 | 0 50825D+00 | 0 61180D-02 | 0 0 |
| 18 | -0 96771D-09 | -0 32113D-05 | -0 87799D-03 | 0 37141D-01 | -0 20746D-02 | 0 0 |
| 19 | 0 37263D-06 | -0 53580D-06 | 0 20607D-02 | 0 24945D+00 | -0 47981D-02 | 0.0 |
| 20 | -0 12320D-06 | 0 12205D-06 | -0 64837D-02 | 0 42792D+00 | -0 16486D-03 | 0 0 |
| 21 | -0 10405D-06 | -0 31909D-06 | 0 14576D-01 | 0 96134D+00 | 0 29907D-01 | 0.0 |
| 22 | -0 20969D-05 | -0 10832D-05 | -0 47362D-02 | 0.55606D+00 | -0 22095D-01 | 0 0 |
| 23 | -0 19145D-08 | 0 36091D-04 | -0 89218D-03 | 0 39095D-01 | 0 37938D-02 | 0 0 |
| 24 | 0 20968D-05 | -0 12562D-05 | 0 21689D-02 | 0 26115D+00 | 0 11402D-01 | 0.0 |
| 25 | 0 10894D-06 | -0 33092D-06 | -0 66873D-02 | 0 44325D+00 | -0 25239D-02 | 0.0 |
| 26 | -0 35573D-06 | 0 13432D-06 | 0 15394D-01 | 0 10000D+01 | -0 32675D+00 | 0 0 |
| 27 | 0 31068D-05 | 0.10414D-04 | -0 44592D-02 | 0 56636D+00 | 0.14421D+00 | 0 0 |
| 28 | 0 96182D-09 | 0.18426D-03 | 0 63389D-03 | 0 38458D-01 | 0 36017D-01 | 0 0 |
| 29 | -0 31065D-05 | 0.10817D-04 | 0 26023D-02 | 0 26271D+00 | -0 14911D-01 | 0 0 |
| 30 | 0 35321D-06 | 0 15118D-06 | -0 58154D-02 | 0 41730D+00 | 0.20574D+00 | 0 0 |
| 31 | -0 54857D-06 | -0 52391D-06 | 0 13555D-01 | 0 79349D+00 | 0 65229D-01 | 0 0 |
| 32 | 0 75411D-05 | -0.91065D-05 | -0 33260D-02 | 0 40792D+00 | 0.13333D+00 | 0 0 |
| 33 | 0 51831D-08 | -0 18189D-03 | 0 12623D-02 | 0.36112D-01 | 0.12127D+00 | 0 0 |
| 34 | -0 75407D-05 | -0.93680D-05 | 0 25690D-02 | 0 23776D+00 | 0 12502D+00 | 0 0 |
| 35 | 0 53533D-06 | -0.54911D-06 | -0 56597D-02 | 0 38302D+00 | 0 20605D+00 | 0.0 |
| 36 | 0 13481D-07 | 0.13990D-05 | 0 87834D-02 | 0 55614D+00 | 0 20780D+00 | 0 0 |
| 37 | -0 96768D-07 | -0 36104D-05 | -0 26440D-02 | 0 31527D+00 | 0 16864D+00 | 0 0 |
| 38 | -0 72257D-08 | -0 23212D-03 | -0 75318D-04 | 0 32260D-01 | 0 15559D+00 | 0 0 |
| 39 | 0 95768D-07 | -0 36104D-05 | 0 25080D-02 | 0 30066D+00 | 0.16864D+00 | 0 0 |
| 40 | 0 51072D-08 | 0.13990D-05 | -0 83747D-02 | 0 53003D+00 | 0 20780D+00 | 0 0 |
| 41 | 0 54475D-06 | -0 55216D-06 | 0 61393D-02 | 0 41242D+00 | 0 20946D+00 | 0 0 |
| 42 | -0 75734D-05 | -0 91164D-05 | -0 27249D-02 | 0 25357D+00 | 0.12418D+00 | 0.0 |
| 43 | 0 51831D-08 | -0 17655D-03 | -0 14078D-02 | 0 36112D-01 | 0.11933D+00 | 0.0 |
| 44 | 0 75738D-05 | -0 88550D-05 | 0 31701D-02 | 0 39211D+00 | 0 13249D+00 | 0 0 |
| 45 | -0 55799D-06 | -0 52695D-06 | -0 13075D-01 | 0 76108D+00 | 0 68644D-01 | 0 0 |
| 46 | 0 35159D-06 | 0 14332D-06 | 0 63782D-02 | 0 45481D+00 | 0 21977D+00 | 0.0 |
| 47 | -0 30172D-05 | 0.10805D-04 | -0 27916D-02 | 0 28134D+00 | -0 19530D-01 | 0 0 |
| 48 | 0 96182D-09 | 0.18597D-03 | -0 75519D-03 | 0 38458D-01 | 0 34692D-01 | 0 0 |
| 49 | 0 30175D-05 | 0 10432D-04 | 0 42699D-02 | 0 54473D+00 | 0.13962D+00 | 0 0 |
| 50 | -0.35411D-06 | 0 12676D-06 | -0 14831D-01 | 0 96249D+00 | -0 31272D+00 | 0 0 |
| 51 | 0 10163D-06 | -0 32913D-06 | 0 72557D-02 | 0 48092D+00 | -0 33111D-02 | 0 0 |
| 52 | 0 21079D-05 | -0 13216D-05 | -0 23594D-02 | 0 28316D+00 | 0 12210D-01 | 0 0 |
| 53 | -0 19145D-08 | 0.35034D-04 | 0 74468D-03 | 0 39395D-01 | 0 35741D-02 | 0 0 |
| 54 | -0 21081D-05 | -0 11517D-05 | 0 45457D-02 | 0 53115D+00 | -0 21286D-01 | 0 0 |
| 55 | -0 96737D-07 | -0.31730D-06 | -0 14003D-01 | 0 92667D+00 | 0 29120D-0. | 0 0 |
| 56 | -0 13250D-06 | 0 11861D-06 | 0 70205D-02 | 0 46342D+00 | -0 25612D-03 | 0 0 |
| 57 | 0 37901D-06 | -0.47404D-06 | -0 22397D-02 | 0 26976D+00 | -0 50868D-02 | 0 0 |
| 58 | -0 96771D-09 | -0 21922D-05 | 0 73955D-03 | 0.37141D-01 | -0 17478D-02 | 0 0 |
| 59 | -0.37904D-06 | -0 63810D-06 | 0 41578D-02 | 0 48794D+00 | 0 58293D-02 | 0 0 |
| 60 | 0 13502D-06 | 0.10918D-06 | -0 12797D-01 | 0 84486D+00 | 0 79060D-02 | 0 0 |
| 61 | -0 27360D-07 | -0.47416D-07 | 0 64486D-02 | 0 41963D+00 | -0 15622D+00 | 0 0 |
| 62 | -0.50393D-06 | -0.46773D-06 | -0 19562D-02 | 0 23958D+00 | 0 59828D-01 | 0.0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 63 | -0.15888D-08 | 0.67428D-05 | 0 46102D-03 | 0.28999D-01 | -0 75916D-02 | 0.0 |
| 64 | 0.50397D-06 | -0 19655D-06 | 0 35123D-02 | 0 42468D+00 | -0.11145D+00 | 0 0 |
| 65 | 0.31384D-07 | -0.40523D-07 | -0 11506D-01 | 0.74813D+00 | 0 27864D+00 | 0.0 |
| 66 | -0.68892D-08 | 0 49644D-07 | 0 33772D-02 | 0.19690D+00 | -0.23186D-01 | 0.0 |
| 67 | 0.11571D-06 | 0.17770D-05 | -0 10425D-02 | 0 10461D+00 | 0 28185D-01 | 0.0 |
| 68 | 0 37885D-08 | 0.35011D-04 | 0 45399D-03 | 0 17067D-01 | -0 87211D-02 | 0.0 |
| 69 | -0 11548D-06 | 0 17560D-05 | 0 18779D-02 | 0.17844D+00 | -0 56029D-01 | 0 0 |
| 70 | -0.28540D-08 | 0.35707D-07 | -0 58161D-02 | 0 33998D+00 | 0 38932D-01 | 0 0 |
| 71 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 |
| 72 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 |
| 73 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |
| 74 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 |
| 75 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 |

AFTER 158 ITERATIONS, CONVERGENCE RATIO= 0 96230890-07
CENTRAL-DIFFERENCE BOUND ON TIME-STEP(SEC) FOR LINEAR SYSTEM= 0.1 43738D-05
ACTUAL TIME-STEP BOUND (SEC) WOULD BE= 0.9149905D-06

FRAGMENT LOCATION, VELOCITY, AND PROPERTIES AT TIME=0.0 SEC.

| | | |
|---|---|---|
| X-LOCATION(IN) | = | 0 8000000D+00 |
| Y-LOCATION(IN) | = | 0 3394412D+01 |
| Z-LOCATION(IN) | = | -0 6098787D+00 |
| X-VELOCITY(IN/SEC) | = | 0 0 |
| Y-VELOCITY(IN/SEC) | = | 0 1397000D+04 |
| Z-VELOCITY(IN/SEC) | = | 0 2419675D+04 |
| OMEGA-X(RAD/SEC) | = | 0 1650000D+04 |
| OMEGA-Y(RAD/SEC) | = | 0 0 |
| OMEGA-Z(RAD/SEC) | = | 0.0 |
| FRAGMENT RADIUS(IN) | = | 0 5000000D+00 |
| FRAGMENT MASS(LB-SEC*SEC/IN) | = | 0 3815130D-03 |
| FRAG MOMENT OF INERTIA(IN**4) | = | 0.3816000D-04 |
| KINETIC ENERGY(IN-LB) | = | 0.1541074D+04 |



CONTACT SURFACE PROPERTIES
COEF OF RESTITUTION= 0.1000000D+01
FRICTION COEFFICIENT= 0 2000000D+00
NO. OF SYMMETRIES = 0

USER SPECIFIED EFFECTIVE LENGTH= 0 4200000D+00

ROUNDING ERROR PARAMETER IN FACTORING ROW 390 = 0.52440101     NO OF NEGATIVE DIAG= 0

***** INCR. NO.= 0 TIME= 0.0 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.0 | 0.0 |
| 5 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.16000D+01 | 0 0 | 0.0 |
| 6 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.400000D+00 | 0.0 |
| 7 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0.40000D+00 | 0 0 |
| 8 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.80000D+00 | 0.40000D+00 | 0.0 |
| 9 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 12000D+01 | 0 40000D+00 | 0 0 |
| 10 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0.40000D+00 | 0.0 |
| 11 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 80000D+00 | 0.0 |
| 12 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 80000D+00 | 0.0 |
| 13 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 80000D+00 | 0.0 |
| 14 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0 80000D+00 | 0.0 |
| 15 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.16000D+01 | 0.80000D+00 | 0.0 |
| 16 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.14000D+01 | 0.0 |
| 17 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 14000D+01 | 0.0 |
| 18 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 14000D+01 | 0.0 |
| 19 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.14000D+01 | 0.0 |
| 20 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.16000D+01 | 0 14000D+01 | 0.0 |
| 21 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.20000D+01 | 0.0 |
| 22 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.40000D+00 | 0.20000D+01 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.80000D+00 | 0.20000D+01 | 0.0 |
| 24 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.12000D+01 | 0.20000D+01 | 0.0 |
| 25 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.16000D+01 | 0.20000D+01 | 0.0 |
| 26 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.26000D+01 | 0.0 |
| 27 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.40000D+00 | 0.26000D+01 | 0.0 |
| 28 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 80000D+00 | 0.26000D+01 | 0.0 |
| 29 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.12000D+01 | 0.26000D+01 | 0.0 |
| 30 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0.26000D+01 | 0.0 |
| 31 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.30000D+01 | 0 0 |
| 32 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0 30000D+01 | 0.0 |
| 33 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 80000D+00 | 0 30000D+01 | 0 0 |
| 34 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 30000D+01 | 0.0 |
| 35 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 16000D+01 | 0 30000D+01 | 0.0 |
| 36 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 34000D+01 | 0.0 |
| 37 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 34000D+01 | 0.0 |
| 38 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 34000D+01 | 0.0 |
| 39 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0.34000D+01 | 0.0 |
| 40 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.16000D+01 | 0 34000D+01 | 0.0 |
| 41 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 38000D+01 | 0.0 |
| 42 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 40000D+00 | 0 38000D+01 | 0.0 |
| 43 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 38000D+01 | 0.0 |
| 44 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 12000D+01 | 0 38000D+01 | 0.0 |
| 45 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 38000D+01 | 0.0 |
| 46 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 420C0D+01 | 0.0 |
| 47 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0.42000D+01 | 0.0 |
| 48 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 80000D+00 | 0 42000D+01 | 0.0 |
| 49 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0 42000D+01 | 0.0 |
| 50 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 42000D+01 | 0.0 |
| 51 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 48000D+01 | 0.0 |
| 52 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 48000D+01 | 0.0 |
| 53 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 80000D+00 | 0.48000D+01 | 0.0 |
| 54 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0.48000D+01 | 0.0 |
| 55 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 16000D+01 | 0 48000D+01 | 0 0 |
| 56 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 54000D+01 | 0.0 |
| 57 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 54000D+01 | 0.0 |
| 58 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0 54000D+01 | 0.0 |
| 59 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 12000D+01 | 0 54000D+01 | 0.0 |
| 60 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 54000D+01 | 0.0 |
| 61 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 60000D+01 | 0.0 |
| 62 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 60000D+01 | 0.0 |
| 63 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 60000D+01 | 0.0 |
| 64 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 12000D+01 | 0 60000D+01 | 0.0 |
| 65 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 60000D+01 | 0 0 |
| 66 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 64000D+01 | 0.0 |
| 67 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 64000D+01 | 0.0 |
| 68 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 80000D+00 | 0 64000D+01 | 0.0 |
| 69 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 12000D+01 | 0 64000D+01 | 0.0 |
| 70 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 64000D+01 | 0 0 |
| 71 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 68000D+01 | 0 0 |
| 72 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.40000D+00 | 0 68000D+01 | 0.0 |
| 73 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 68000D+01 | 0.0 |
| 74 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.12000D+01 | 0 68000D+01 | 0.0 |
| 75 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| 0 800000D+00 | 0.339441D+01 | -0.609879D+00 | 0 0 | 0.139700D+04 | 0.241967D+04 | 0 165000D+04 | 0.0 | 0.0 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 39 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 43 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 44 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 48 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 49 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 53 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 54 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 45000D+02 |
| 1 | OUTER | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.45000D+02 |
| 2 | INNER | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 45000D+02 |
| 2 | OUTER | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.45000D+02 |
| 3 | INNER | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |
| 3 | OUTER | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |
| 4 | INNER | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.45000D+02 |
| 4 | OUTER | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |

SYSTEM ENERGIES(IN-LB)

FRAG. TRANSLATIONAL KINETIC ENERGY = 0.1489128D+04
FRAG. ROTATIONAL KINETIC ENERGY = 0 5194530D+02
WORK INPUT TO STRUCTURE = 0 0
STRUCTURE KINETIC ENERGY = 0.0
STRUCTURE ELASTIC ENERGY = 0 0
STRUCTURE PLASTIC ENERGY = 0 0
ENERGY STORED IN ELASTIC RESTRAINTS= 0.0

***** INCR. NO.= 1 TIME= 0.2000D-05 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0 0 | 0.0 |
| 4 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.0 | 0.0 |
| 5 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.16000D+01 | 0.0 | 0.0 |
| 6 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 40000D+00 | 0.0 |
| 7 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 40000D+00 | 0 40000D+00 | 0.0 |
| 8 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 80000D+00 | 0 40000D+00 | 0.0 |
| 9 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0.40000D+00 | 0.0 |
| 10 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 40000D+00 | 0 0 |
| 11 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0.0 |
| 12 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 80000D+00 | 0.0 |
| 13 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 80000D+00 | 0 80000D+00 | 0.0 |
| 14 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 80000D+00 | 0 0 |
| 15 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 80000D+00 | 0.0 |
| 16 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 14000D+01 | 0.0 |
| 17 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.40000D+00 | 0 14000D+01 | 0 0 |
| 18 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 14000D+01 | 0 0 |
| 19 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 14000D+01 | 0 0 |
| 20 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 14000D+01 | 0.0 |
| 21 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 20000D+01 | 0 0 |
| 22 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 20000D+01 | 0 0 |
| 23 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 20000D+01 | 0.0 |
| 24 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 20000D+01 | 0 0 |
| 25 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.16000D+01 | 0 20000D+01 | 0.0 |
| 26 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 26000D+01 | 0 0 |
| 27 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 40000D+00 | 0 26000D+01 | 0.0 |
| 28 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0.26000D+01 | 0.0 |
| 29 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 26000D+01 | 0 0 |
| 30 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 26000D+01 | 0.0 |
| 31 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 30000D+01 | 0.0 |
| 32 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0 30000D+01 | 0.0 |
| 33 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 30000D+01 | 0.0 |
| 34 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.12000D+01 | 0.30000D+01 | 0.0 |
| 35 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.16000D+01 | 0 30000D+01 | 0.0 |
| 36 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.34000D+01 | 0.0 |
| 37 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.40000D+00 | 0.34000D+01 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 38 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.80000D+00 | 0.34000D+01 | 0.0 |
| 39 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 12000D+01 | 0.34000D+01 | 0.0 |
| 40 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.16000D+01 | 0.34000D+01 | 0.0 |
| 41 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.38000D+01 | 0.0 |
| 42 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 40000D+00 | 0.38000D+01 | 0.0 |
| 43 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 80000D+00 | 0.38000D+01 | 0.0 |
| 44 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.12000D+01 | 0.38000D+01 | 0.0 |
| 45 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.16000D+01 | 0.38000D+01 | 0.0 |
| 46 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.42000D+01 | 0.0 |
| 47 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 42000D+01 | 0.0 |
| 48 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 80000D+00 | 0.42000D+01 | 0.0 |
| 49 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 42000D+01 | 0.0 |
| 50 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 42000D+01 | 0.0 |
| 51 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 48000D+01 | 0.0 |
| 52 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 48000D+01 | 0.0 |
| 53 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 48000D+01 | 0 0 |
| 54 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.48000D+01 | 0.0 |
| 55 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 16000D+01 | 0 48000D+01 | 0 0 |
| 56 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 54000D+01 | 0.0 |
| 57 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0.54000D+01 | 0.0 |
| 58 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 54000D+01 | 0 0 |
| 59 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 12000D+01 | 0 54000D+01 | 0.0 |
| 60 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 16000D+01 | 0 54000D+01 | 0.0 |
| 61 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 60000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0 60000D+01 | 0.0 |
| 63 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 80000D+00 | 0 60000D+01 | 0.0 |
| 64 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 12000D+01 | 0 60000D+01 | 0.0 |
| 65 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 60000D+01 | 0 0 |
| 66 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 64000D+01 | 0.0 |
| 67 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 40000D+00 | 0 64000D+01 | 0.0 |
| 68 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.80000D+00 | 0 64000D+01 | 0.0 |
| 69 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 64000D+01 | 0.0 |
| 70 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 64000D+01 | 0.0 |
| 71 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.68000D+01 | 0.0 |
| 72 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 40000D+00 | 0.68000D+01 | 0.0 |
| 73 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0.68000D+01 | 0.0 |
| 74 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.12000D+01 | 0.68000D+01 | 0.0 |
| 75 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.16000D+01 | 0.68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| 0 800000D+00 | 0.339721D+01 | -0 605039D+00 | 0 0 | 0.139700D+04 | 0.241967D+04 | 0 165000D+04 | 0.0 | 0.0 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 39 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0 4500D+02 |
| 43 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 44 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 48 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 49 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 53 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 4500D+02 | 0.4500D+02 |
| 54 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 45000D+02 |
| 1 | OUTER | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.45000D+02 |
| 2 | INNER | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.45000D+02 |
| 2 | OUTER | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 3 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.45000D+02 |
| 3 | OUTER | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.45000D+02 |
| 4 | INNER | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | OUTER | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |

SYSTEM ENERGIES(IN-LB)

FRAG. TRANSLATIONAL KINETIC ENERGY = 0.1489128D+04
FRAG. ROTATIONAL KINETIC ENERGY = 0.5194530D+02
WORK INPUT TO STRUCTURE = 0.0
STRUCTURE KINETIC ENERGY = 0.0
STRUCTURE ELASTIC ENERGY = 0.0
STRUCTURE PLASTIC ENERGY = 0.0
ENERGY STORED IN ELASTIC RESTRAINTS= 0.0

***** INCR. NO.= 2 TIME= 0.4000D-05 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.80000D+00 | 0.0 | 0.0 |
| 4 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.12000D+01 | 0 0 | 0.0 |
| 5 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 0 | 0.0 |
| 6 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.40000D+00 | 0.0 |
| 7 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0 40000D+00 | 0.0 |
| 8 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.80000D+00 | 0 40000D+00 | 0.0 |
| 9 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 12000D+01 | 0 40000D+00 | 0 0 |
| 10 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 16000D+01 | 0 40000D+00 | 0.0 |
| 11 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 80000D+00 | 0.0 |
| 12 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 40000D+00 | 0 80000D+00 | 0.0 |
| 13 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 80000D+00 | 0 80000D+00 | 0.0 |
| 14 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0 80000D+00 | 0.0 |
| 15 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 16000D+01 | 0 80000D+00 | 0.0 |
| 16 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 14000D+01 | 0.0 |
| 17 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 14000D+01 | 0.0 |
| 18 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 80000D+00 | 0 14000D+01 | 0.0 |
| 19 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0 14000D+01 | 0.0 |
| 20 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0.14000D+01 | 0.0 |
| 21 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 20000D+01 | 0.0 |
| 22 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 40000D+00 | 0.20000D+01 | 0.0 |
| 23 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 20000D+01 | 0.0 |
| 24 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 20000D+01 | 0.0 |
| 25 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 20000D+01 | 0.0 |
| 26 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 26000D+01 | 0 0 |
| 27 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 26000D+01 | 0.0 |
| 28 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 80000D+00 | 0 26000D+01 | 0.0 |
| 29 | 0 0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 26000D+01 | 0.0 |
| 30 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 16000D+01 | 0 26000D+01 | 0.0 |
| 31 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 30000D+01 | 0.0 |
| 32 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 30000D+01 | 0 0 |
| 33 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 30000D+01 | 0 0 |
| 34 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 12000D+01 | 0 30000D+01 | 0.0 |
| 35 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 30000D+01 | 0 0 |
| 36 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 34000D+01 | 0 0 |
| 37 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0 34000D+01 | 0.0 |
| 38 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 34000D+01 | 0 0 |
| 39 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 34000D+01 | 0 0 |
| 40 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.16000D+01 | 0 34000D+01 | 0 0 |
| 41 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.38000D+01 | 0.0 |
| 42 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 40000D+00 | 0.38000D+01 | 0.0 |
| 43 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 80000D+00 | 0 38000D+01 | 0.0 |
| 44 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0 38000D+01 | 0 0 |
| 45 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 16000D+01 | 0 38000D+01 | 0 0 |
| 46 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 42000D+01 | 0.0 |
| 47 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 42000D+01 | 0.0 |
| 48 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 80000D+00 | 0 42000D+01 | 0.0 |
| 49 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0 42000D+01 | 0.0 |
| 50 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.16000D+01 | 0.42000D+01 | 0.0 |
| 51 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.48000D+01 | 0.0 |
| 52 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 40000D+00 | 0.48000D+01 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 53 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 800u0D+00 | 0.48000D+01 | 0.0 |
| 54 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 12000D+01 | 0.48000D+01 | 0.0 |
| 55 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.16000D+01 | 0 48000D+01 | 0.0 |
| 56 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.54000D+01 | 0.0 |
| 57 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.40000D+00 | 0 54000D+01 | 0.0 |
| 58 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 80000D+00 | 0 54000D+01 | 0.0 |
| 59 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0 54000D+01 | 0.0 |
| 60 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 16000D+01 | 0 54000D+01 | 0.0 |
| 61 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 60000D+01 | 0.0 |
| 62 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0 40000D+00 | 0 60000D+01 | 0.0 |
| 63 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0 60000D+01 | 0.0 |
| 64 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 12000D+01 | 0 60000D+01 | 0 0 |
| 65 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 16000D+01 | 0 60000D+01 | 0.0 |
| 66 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 64000D+01 | 0 0 |
| 67 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 40000D+00 | 0 64000D+01 | 0.0 |
| 68 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0 64000D+01 | 0.0 |
| 69 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 12000D+01 | 0 64000D+01 | 0.0 |
| 70 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 64000D+01 | 0.0 |
| 71 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 68000D+01 | 0.0 |
| 72 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.40000D+00 | 0 68000D+01 | 0.0 |
| 73 | 0 0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 80000D+00 | 0 68000D+01 | 0.0 |
| 74 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.12000D+01 | 0.68000D+01 | 0.0 |
| 75 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.16000D+01 | 0.68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| 0.800000D+00 | 0.340000D+01 | -0.600200D+00 | 0 0 | 0 139700D+04 | 0.241967D+04 | 0.165000D+04 | 0.0 | 0.0 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 39 | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 43 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.4500D+02 | 0.4500D+02 |
| 44 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 48 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 49 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 4500D+02 | 0.4500D+02 |
| 53 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |
| 54 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.4500D+02 | 0.4500D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0 45000D+02 |
| 1 | OUTER | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 0 | 0.45000D+02 |
| 2 | INNER | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 2 | OUTER | 0.0 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.45000D+02 |
| 3 | INNER | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.45000D+02 |
| 3 | OUTER | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |
| 4 | INNER | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |
| 4 | OUTER | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.45000D+02 |

SYSTEM ENERGIES(IN-LB)

FRAG TRANSLATIONAL KINETIC ENERGY = 0 148912BD+04
FRAG. ROTATIONAL KINETIC ENERGY = 0 5194530D+02
WORK INPUT TO STRUCTURE = 0.0
STRUCTURE KINETIC ENERGY = 0.0
STRUCTURE ELASTIC ENERGY = 0.0
STRUCTURE PLASTIC ENERGY = 0.0
ENERGY STORED IN ELASTIC RESTRAINTS= 0.0

IMPACT CYCLE= 2

SUBREGIONS PENETRATED: 5 8

POINT OF EFFECTIVE IMPACT(XA,YA,ZA)= 0.8000000D+00 0.3402794D+01 -0.1526557D-15
THIS IS IMPACT NUMBER 1

***** INCR. NO.= 3 TIME= 0.6000D-05 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.0 |
| 2 | 0 0 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0.0 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0 80000D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.0 | 0.0 |
| 5 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.16000D+01 | 0 0 | 0.0 |
| 6 | 0.94752D-08 | 0 19913D-07 | 0 92643D-08 | 0 21756D-06 | -0 52954D-07 | 0 47497D-05 | 0.94752D-08 | 0 40000D+00 | 0 92643D-08 |
| 7 | 0 96341D-09 | 0 18171D-07 | 0.51535D-08 | 0 66400D-07 | -0 22158D-07 | 0.19535D-05 | 0 40000D+00 | 0′ 40000D+00 | 0 51535D-08 |
| 8 | 0.49631D-23 | 0.19127D-07 | 0 13548D-08 | -0 54878D-19 | -0 51714D-07 | -0 18116D-17 | 0 80000D+00 | 0 40000D+00 | 0 13548D-08 |
| 9 | -0 96341D-09 | 0.18171D-07 | 0 51535D-08 | -0 66400D-07 | -0 22158D-07 | -0.19535D-05 | 0 12000D+01 | 0 40000D+00 | 0 51535D-08 |
| 10 | -0 94752D-08 | 0 19913D-07 | 0 92643D-08 | -0 21756D-06 | -0 52954D-07 | -0.47497D-05 | 0.16000D+01 | 0 40000D+00 | 0.92643D-08 |
| 11 | 0 14279D-07 | 0 56708D-07 | -0 38456D-07 | 0 24223D-06 | -0 13260D-05 | 0 22944D-04 | 0 14279D-07 | 0 80000D+00 | -0 38456D-07 |
| 12 | 0 80036D-08 | 0 92721D-07 | -0.58547D-07 | 0 41682D-06 | -0 82053D-06 | 0 15600D-04 | 0.40000D+00 | 0 80000D+00 | -0 58547D-07 |
| 13 | 0 99262D-23 | 0 32213D-07 | -0 11127D-06 | -0 14385D-18 | -0 71064D-06 | -0 74681D-17 | 0 80000D+00 | 0 80000D+00 | -0.11127D-06 |
| 14 | -0 80036D-08 | 0 92721D-07 | -0 58547D-07 | -0 41682D-06 | -0.82053D-06 | -0 15600D-04 | 0.12000D+01 | 0 80000D+00 | -0 58547D-07 |
| 15 | -0 14279D-07 | 0 56708D-07 | -0 38456D-07 | -0 24223D-06 | -0 13260D-05 | -0 22944D-04 | 0 16000D+01 | 0 80000D+00 | -0 38456D-07 |
| 16 | -0 18393D-06 | 0 59656D-06 | 0 12512D-06 | 0 45564D-05 | 0 39544D-05 | 0 72683D-04 | -0 18393D-06 | 0 14000D+01 | 0 12512D-06 |
| 17 | -0 14255D-06 | 0 21974D-06 | 0 49536D-06 | 0 44973D-05 | 0 59962D-05 | 0 63308D-04 | 0 40000D+00 | 0 14000D+01 | 0 49536D-06 |
| 18 | 0 26470D-22 | 0 74318D-06 | 0 14432D-05 | -0.75124D-18 | 0 92807D-05 | -0 24510D-16 | 0 80000D+00 | 0 14000D+01 | 0.14432D-05 |
| 19 | 0 14255D-06 | 0 21974D-06 | 0.49536D-06 | -0 44973D-05 | 0 59962D-05 | -0.63308D-04 | 0.12000D+01 | 0 14000D+01 | 0 49536D-06 |
| 20 | 0 18393D-06 | 0 59656D-06 | 0 12512D-06 | -0 45564D-05 | 0.39544D-05 | -0 72683D-04 | 0 16000D+01 | 0.14000D+01 | 0 12512D-06 |
| 21 | 0 31640D-05 | 0 62356D-05 | 0 26662D-05 | 0 13711D-04 | 0 21682D-04 | 0 85838D-04 | 0 31640D-05 | 0 20000D+01 | 0.26662D-05 |
| 22 | -0 42689D-06 | 0 21997D-05 | -0 30161D-05 | -0 16940D-04 | -0 65843D-04 | 0.41867D-04 | 0 40000D+00 | 0 20000D+01 | -0 30161D-05 |
| 23 | 0.42352D-21 | -0 34230D-05 | -0 14840D-04 | -0.28219D-17 | -0 12957D-03 | -0.11723D-15 | 0 80000D+00 | 0 20000D+01 | -0 14840D-04 |
| 24 | 0 42689D-06 | 0 21997D-05 | -0 30161D-05 | 0 16940D-04 | -0 65843D-04 | -0.41867D-04 | 0 12000D+01 | 0.20000D+01 | -0.30161D-05 |
| 25 | -0 31640D-05 | 0 62356D-05 | 0 26662D-05 | -0 13711D-04 | 0 21682D-04 | -0 85838D-04 | 0 16000D+01 | 0 20000D+01 | 0.26662D-05 |
| 26 | 0 19751D-04 | 0 18194D-04 | -0 16565D-04 | -0.19099D-03 | -0.24342D-03 | 0 13579D-03 | 0 19751D-04 | 0 26000D+01 | -0.16565D-04 |
| 27 | 0 20104D-04 | 0 35848D-04 | -0 45769D-04 | 0 11216D-03 | -0 56973D-04 | 0 36136D-02 | 0 40002D+00 | 0 26000D+01 | -0 45769D-04 |
| 28 | 0 57175D-20 | -0 30321D-05 | -0.47318D-04 | -0 34661D-17 | 0 62331D-03 | -0.26769D-15 | 0 80000D+00 | 0.26000D+01 | -0.47318D-04 |
| 29 | -0.20404D-04 | 0 35848D-04 | -0 45769D-04 | -0.11216D-03 | -0 56973D 04 | -0.36136D-02 | 0 12000D+01 | 0.26000D+01 | -0.45769D-04 |
| 30 | -0 19751D-04 | 0.18194D-04 | -0 16565D-04 | 0 19099D-03 | -0 24342D-03 | -0 13579D-03 | 0 16000D+01 | 0 26000D+01 | -0 16565D-04 |
| 31 | 0 24601D-04 | 0 10648D-04 | -0 97566D-04 | -0 32879D-03 | -0 30127D-03 | 0 52317D-02 | 0 24601D-04 | 0 30000D+01 | -0.97566D-04 |
| 32 | 0 63191D-04 | 0 93528D-04 | 0 25326D-03 | 0 36673D-02 | 0 28345D-02 | 0 27463D-01 | 0.40006D+00 | 0 30001D+01 | 0 25326D-03 |
| 33 | -0 99526D-20 | 0.44415D-03 | 0 17858D-02 | 0 14057D-16 | 0 99969D-02 | -0 11861D-14 | 0 80000D+00 | 0.30004D+01 | 0 17858D-02 |
| 34 | -0 63491D-04 | 0 93528D-04 | 0.25326D-03 | -0 36673D-02 | 0 28345D-02 | -0.27463D-01 | 0 11999D+01 | 0 30001D+01 | 0 25326D-03 |
| 35 | -0 24601D-04 | 0 10648D-04 | -0 97566D-04 | 0 32879D-03 | -0 30127D-03 | -0 52317D-02 | 0 16000D+01 | 0 30000D+01 | -0.97566D-04 |
| 36 | -0.46748D-05 | 0 10144D-04 | -0 10700D-03 | 0 20411D-03 | -0 99241D-05 | 0 21569D-04 | -0 46748D-05 | 0 34000D+01 | -0 10700D-03 |
| 37 | -0 17434D-04 | 0 89706D-04 | 0 12041D-02 | 0 91667D-02 | 0 10296D-03 | 0 84106D-03 | 0 39998D+00 | 0 34001D+01 | 0 12041D-02 |
| 38 | 0 37984D-20 | 0 12590D-02 | 0 72028D-02 | 0 67763D-16 | 0 10507D-02 | -0 29907D-14 | 0 80000D+00 | 0.34013D+01 | 0 72028D-02 |
| 39 | 0 17434D-04 | 0 89706D-04 | 0 12041D-02 | -0 91667D-02 | 0 10296D-03 | -0 84106D-03 | 0.12000D+01 | 0 34001D+01 | 0 12041D-02 |
| 40 | 0 46718D-05 | 0 10144D-04 | -0 10700D-03 | -0 20411D-03 | -0 99241D-05 | -0 21569D-04 | 0 16000D+01 | 0 34000D+01 | -0 10700D-03 |
| 41 | -0 26528D-04 | 0 13732D-04 | -0 10098D-03 | -0 33409D-03 | 0 30038D 03 | -0 53417D-02 | -0 26528D-04 | 0.38000D+01 | -0 10098D-03 |
| 42 | -0 58531D-04 | 0 94139D-04 | 0 27637D-03 | 0 39348D-02 | -0 29333D 02 | -0 28253D-01 | 0 39994D+00 | 0 38001D+01 | 0 27637D-03 |
| 43 | 0 23929D-19 | 0 92132D-04 | 0 13534D-02 | -0 39573D-16 | -0 10850D 01 | 0.11442D-14 | 0 80000D+00 | 0 38001D+01 | 0 19534D-02 |
| 44 | 0 58531D-04 | 0 94139D-04 | 0.27637D-03 | -0 39348D-02 | -0 29333D-02 | 0 28253D-01 | 0 12001D+01 | 0 38001D+01 | 0 27637D-03 |
| 45 | 0 26528D-04 | 0 13732D-04 | -0 10098D-03 | 0 33409D-03 | 0 30038D 03 | 0 53417D-02 | 0 16000D+01 | 0 38000D+01 | -0 10098D-03 |
| 46 | -0 14026D-04 | 0 18309D-04 | -0 17716D-04 | -0 20500D-03 | 0 25573D 03 | -0 12006D-03 | -0 14026D-04 | 0 42000D+01 | -0 17716D-04 |
| 47 | -0.46967D-05 | 0 22945D-04 | -0 46517D-04 | 0 11233D-03 | 0 27307D-04 | -0.39136D-02 | 0 40000D+00 | 0 42000D+01 | -0.48517D-04 |
| 48 | -0 34513D-20 | 0 10260D-03 | -0 50983D-04 | -0 96765D-17 | -0 85785D-03 | 0 24903D-15 | 0 80000D+00 | 0 42001D+01 | -0.50983D-04 |
| 49 | 0 46967D-05 | 0 22945D-04 | -0 46517D-04 | -0 11233D-03 | 0 27307D 04 | 0.39136D-02 | 0 12000D+01 | 0 42000D+01 | -0.48517D-04 |
| 50 | 0.14026D-04 | 0.18309D-04 | -0 17716D-04 | 0 20500D-03 | 0 25573D-03 | 0 12006D-03 | 0 16000D+01 | 0 42000D+01 | -0 17716D-04 |
| 51 | -0 23595D-05 | 0 42261D-05 | 0 29905D-05 | 0 14343D-04 | -0 22166D-04 | -0 60475D-04 | -0 23595D-05 | 0.48000D+01 | 0.29905D-05 |
| 52 | -0 37416D-05 | 0.75080D-05 | -0 40415D-05 | -0 28390D-04 | 0 82972D-04 | 0 55755D-04 | 0 40000D+00 | 0 48000D+01 | -0 40415D-05 |
| 53 | 0 59557D-21 | 0.16292D-04 | -0 19336D-04 | -0 42775D-18 | 0 14496D 03 | 0 70087D-16 | 0 80000D+00 | 0 48000D+01 | -0.19336D-04 |
| 54 | 0 37416D-05 | 0.75080D-05 | -0 40415D-05 | 0 28390D-04 | 0 82972D-04 | -0 55755D-04 | 0.12000D+01 | 0.48000D+01 | -0 40415D-05 |
| 55 | 0 23595D-05 | 0 42261D-05 | 0 29905D-05 | -0.14343D-04 | -0 22166D-04 | 0 60475D-04 | 0 16000D+01 | 0 48000D+01 | 0.29905D-05 |
| 56 | -0 91199D-06 | 0 13593D-05 | 0 17932D-06 | 0 51849D-05 | -0 53658D-05 | -0.71744D-04 | -0 91199D-06 | 0 54000D+01 | 0.17932D-06 |
| 57 | -0 44658D-06 | 0 14016D-05 | 0 55853D-06 | 0 34301D-05 | -0 65969D-05 | -0.45528D-04 | 0 40000D+00 | 0 54000D+01 | 0 55853D-06 |
| 58 | -0 19852D-21 | 0 81180D-06 | 0 14659D-05 | -0 88752D-18 | -0 11450D-04 | 0.35853D-16 | 0 80000D+00 | 0.54000D+01 | 0.14859D-05 |
| 59 | 0 44658D-06 | 0.14016D-05 | 0 55853D-06 | -0 34301D-05 | -0 65969D-05 | 0 45528D-04 | 0 12000D+01 | 0.54000D+01 | 0.55853D-06 |
| 60 | 0.91199D-06 | 0.13599D-05 | 0 17932D-06 | -0.51849D-05 | -0 53858D-05 | 0.71744D-04 | 0.16000D+01 | 0.54000D+01 | 0.17932D-06 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 61 | -0 97107D-07 | 0.28811D-06 | -0 47744D-07 | 0 19653D-06 | 0 14862D-05 | -0.21553D-04 | -0.97107D-07 | 0.60000D+01 | -0.47744D-07 |
| 62 | -0 21180D-07 | 0 18578D-06 | -0 71318D-07 | 0.22843D-06 | 0.99607D-06 | -0.11304D-04 | 0.40000D+00 | 0.60000D+01 | -0.71318D-07 |
| 63 | 0.33501D-22 | 0 24343D-06 | -0 14502D-06 | -0.18677D-18 | 0.67754D-06 | 0.10526D-16 | 0.80000D+00 | 0.60000D+01 | -0.14502D-06 |
| 64 | 0 21180D-07 | 0 18578D-06 | -0 71318D-07 | -0 22843D-06 | 0 99607D-06 | 0.11304D-04 | 0.12000D+01 | 0.60000D+01 | -0.71318D-07 |
| 65 | 0 97107D-07 | 0 28811D-06 | -0 47744D-07 | -0 19653D-06 | 0 14862D-05 | 0.21553D-04 | 0.16000D+01 | 0.60000D+01 | -0.47744D-07 |
| 66 | -0 18817D-07 | 0 66375D-07 | 0 10832D-07 | 0 20639D-06 | 0 33206D-07 | -0.42535D-05 | -0.18817D-07 | 0.64000D+01 | 0.10832D-07 |
| 67 | -0.61808D-08 | 0 53457D-07 | 0 63938D-08 | 0 30544D-07 | 0 13509D-07 | -0.13561D-05 | 0.40000D+00 | 0.64000D+01 | 0.63938D-08 |
| 68 | 0.70827D-23 | 0 50815D-07 | -0.49228D-08 | -0.57315D-19 | 0 46282D-07 | 0 17750D-17 | 0.80000D+00 | 0.64000D+01 | -0.49228D-08 |
| 69 | 0.61808D-08 | 0.53457D-07 | 0 63938D-08 | -0.30544D-07 | 0.13509D-07 | 0.13561D-05 | 0.12000D+01 | 0.64000D+01 | 0.63938D-08 |
| 70 | 0 18817D-07 | 0 66375D-07 | 0 10832D-07 | -0.20639D-06 | 0 33206D-07 | 0.42535D-05 | 0.16000D+01 | 0.64000D+01 | 0.10832D-07 |
| 71 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.68000D+01 | 0.0 |
| 72 | 0 0 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0.40000D+00 | 0.68000D+01 | 0.0 |
| 73 | 0.0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.80000D+00 | 0.68000D+01 | 0.0 |
| 74 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.12000D+01 | 0.68000D+01 | 0.0 |
| 75 | 0.0 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.16000D+01 | 0.68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| 0.800000D+00 | 0.340273D+01 | -0 595705D+00 | 0.0 | 0 136252D+04 | 0.224726D+04 | 0.182237D+04 | 0.0 | 0.0 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | -0.1787D-01 | 0.1796D-01 | -0.1523D-01 | 0 1435D-01 | -0 5985D-15 | 0 5967D-15 | 0.0 | 0.1796D-01 | 0.0 | 0.4740D-11 |
| 39 | 0 8889D-02 | -0 8792D-02 | -0.2079D-02 | 0 2082D-02 | -0 1578D-02 | -0.1241D-02 | 0.8945D-02 | 0.2117D-02 | -0.4093D+01 | -0.8674D+02 |
| 43 | -0 4175D-02 | 0 4468D-02 | 0.5175D-02 | -0 7948D-02 | 0 2290D-15 | -0.2284D-15 | 0.5175D-02 | 0.4468D-02 | 0.9000D+02 | -0.5271D-12 |
| 44 | 0.2394D-02 | -0 2312D-02 | 0.1070D-02 | -0 1228D-02 | 0 5548D-02 | -0 5753D-02 | 0.4584D-02 | 0.1157D-02 | 0 3829D+02 | -0.5033D+02 |
| 48 | 0.7716D-04 | -0.5366D-04 | 0.8461D-03 | -0 9638D-03 | 0 4937D-16 | -0 4940D-16 | 0.8461D-03 | 0.0 | 0.9000D+02 | 0.0 |
| 49 | 0.1942D-04 | 0 1566D-04 | -0 1982D-03 | -0 5466D-05 | 0 6093D-03 | -0 9562D-03 | 0.2341D-03 | 0.4833D-03 | 0.3517D+02 | -0.4437D+02 |
| 53 | 0.5252D-04 | -0 3381D-04 | -0 2349D-03 | 0 6528D-04 | 0 1401D-16 | -0 1402D-16 | 0.5252D-04 | 0.6528D-04 | 0.1396D-11 | -0.9000D+02 |
| 54 | -0.8958D-05 | 0 1486D-04 | -0 4553D-04 | 0.9629D-05 | -0 2977D-04 | -0 7471D-05 | 0.0 | 0.1680D-04 | 0.0 | -0.2751D+02 |



STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | -0 18074D-02 | -0 36266D-02 | -0 89513D-02 | -0 72187D-02 | 0.17571D-02 | 0 18502D-02 | -0.39256D+02 |
| 1 | OUTER | 0.21953D-02 | 0.83495D-03 | 0.11468D-01 | 0.72233D-02 | -0.42280D-02 | 0.72896D-02 | 0 41618D+02 |
| 2 | INNER | -0 18074D-02 | -0 36266D-02 | 0.89513D-02 | 0 17571D-02 | -0.72187D-02 | 0.18502D-02 | 0.39256D+02 |
| 2 | OUTER | 0 21953D-02 | 0 83495D-03 | -0 11468D-01 | -0 42280D-02 | 0.72233D-02 | 0.72896D-02 | -0.41618D+02 |
| 3 | INNER | -0 12130D-03 | 0 18511D-02 | -0.25985D-02 | -0.12130D-03 | 0.18494D-02 | 0.24960D-02 | -0.63600D+02 |
| 3 | OUTER | 0.28199D-03 | -0.19955D-02 | 0.29187D-02 | 0 28195D-03 | -0.19975D-02 | 0.99429D-03 | 0.26017D+02 |
| 4 | INNER | -0.12130D-03 | 0 18511D-02 | 0.25985D-02 | -0 12130D-03 | 0.18494D-02 | 0.24960D-02 | 0.63600D+02 |
| 4 | OUTER | 0 28199D-03 | -0 19955D-02 | -0 29187D-02 | 0.28195D-03 | -0.19975D-02 | 0.99429D-03 | -0.26017D+02 |

SYSTEM ENERGIES(IN-LB)

FRAG TRANSLATIONAL KINETIC ENERGY = 0 1317489D+04
FRAG ROTATIONAL KINETIC ENERGY = 0 6336536D+02
WORK INPUT TO STRUCTURE = 0 1602192D+03
STRUCTURE KINETIC ENERGY = 0 6523067D+02
STRUCTURE ELASTIC ENERGY = 0.1507899D+02
STRUCTURE PLASTIC ENERGY = 0.7990952D+02
ENERGY STORED IN ELASTIC RESTRAINTS= 0.3324755D-04

***** INCR. NO.=  140  TIME=  0.2800D-03 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0.0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.80000D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.12000D+01 | 0.0 | 0.0 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.16000D+01 | 0.0 | 0.0 |
| 6 | 0 99196D-03 | 0 24400D-02 | 0.99401D-02 | 0.49670D-02 | 0 48266D-01 | -0.32008D-01 | 0.99196D-03 | 0.40244D+00 | 0.99401D-02 |
| 7 | 0 46903D-03 | 0 28891D-02 | 0.10568D-01 | -0.12682D-02 | 0 47366D-01 | 0.92273D-03 | 0.40047D+00 | 0.40289D+00 | 0.10568D-01 |
| 8 | 0 89356D-04 | 0.46029D-02 | 0.99245D-02 | -0.38251D-04 | 0 42851D-01 | -0.86186D-01 | 0.80009D+00 | 0.40460D+00 | 0.99245D-02 |
| 9 | -0.56806D-03 | 0.27553D-02 | 0.10436D-01 | -0.14594D-03 | 0 46013D-01 | -0.12171D-01 | 0.11994D+01 | 0.40276D+00 | 0.10436D-01 |
| 10 | -0.10819D-02 | 0.23835D-02 | 0.94952D-02 | -0.56482D-02 | 0.46630D-01 | 0.27413D-01 | 0.15989D+01 | 0.40238D+00 | 0.94952D-02 |
| 11 | 0.25261D-03 | 0.26791D-02 | 0.28321D-01 | 0.22139D-02 | 0.43855D-01 | -0.70363D-02 | 0.25261D-03 | 0.80268D+00 | 0.28321D-01 |
| 12 | 0 46802D-01 | 0 31205D-02 | 0 29308D-01 | 0 25126D-02 | 0 41381D-01 | -0.63338D-03 | 0.40005D+00 | 0.80312D+00 | 0.29308D-01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 13 | -0.64037D-04 | 0.38945D-02 | 0.29786D-01 | -0.37840D-02 | 0.48266D-01 | -0.10363D+00 | 0.79994D+00 | 0.80389D+00 | 0.29786D-01 |
| 14 | -0.12221D-03 | 0.31523D-02 | 0 28502D-01 | -0.19518D-02 | 0 44930D-01 | -0 14778D-01 | 0 11999D+01 | 0.80315D+00 | 0.28502D-01 |
| 15 | -0 33801D-03 | 0 25894D-02 | 0 27552D-01 | -0 23488D-02 | 0 43781D-01 | 0.27290D-02 | 0.15997D+01 | 0.80259D+00 | 0.27552D-01 |
| 16 | 0 17980D-03 | 0 30637D-02 | 0 51833D-01 | 0 44222D-02 | 0 36364D-01 | -0.16044D-02 | 0.17980D-03 | 0 14031D+01 | 0.51833D-01 |
| 17 | 0 64511D-04 | 0.30468D-02 | 0 53751D-01 | 0 44449D-02 | 0.39214D-01 | 0.85365D-02 | 0 40006D+00 | 0 14030D+01 | 0.53751D-01 |
| 18 | 0 77478D-04 | 0 33495D-02 | 0 54779D-01 | -0 63504D-03 | 0 47665D-01 | -0.12763D+00 | 0 80008D+00 | 0.14033D+01 | 0.54779D-01 |
| 19 | 0 25879D-04 | 0 30279D-02 | 0 53630D-01 | -0.42469D-02 | 0.39875D-01 | -0 13496D-01 | 0.12000D+01 | 0 14030D+01 | 0.53630D-01 |
| 20 | -0 79804D-04 | 0.29218D-02 | 0.51869D-01 | -0 39302D-02 | 0 38145D-01 | 0 14448D-02 | 0.15993D+01 | 0 14029D+01 | 0.51869D-01 |
| 21 | 0 59588D-03 | 0 32050D-02 | 0.71213D-01 | 0 11536D-01 | 0.31894D-01 | -0 30163D-03 | 0 59588D-03 | 0 20032D+01 | 0.71213D-01 |
| 22 | 0 47843D-03 | 0 30222D-02 | 0 75866D-01 | 0 11337D-01 | 0 35848D-01 | 0 11173D-01 | 0.40048D+00 | 0 20030D+01 | 0.75866D-01 |
| 23 | 0.13487D-03 | 0.31389D-02 | 0 78876D-01 | -0 67013D-03 | 0 45332D-01 | -0.13068D+00 | 0 80013D+00 | 0.20031D+01 | 0.78876D-01 |
| 24 | -0 22389D-03 | 0 29576D-02 | 0 76338D-01 | -0 96331D-02 | 0.36665D-01 | -0 18474D-01 | 0 11998D+01 | 0 20030D+01 | 0 76338D-01 |
| 25 | -0 32976D-03 | 0 30225D-02 | 0 72255D-01 | -0 10236D-01 | 0 33173D-01 | -0 17539D-02 | 0 15997D+01 | 0 20030D+01 | 0.72255D-01 |
| 26 | 0 77599D-03 | 0 28136D-02 | 0 89014D-01 | 0 21819D-01 | 0 32925D 01 | 0 41101D-02 | 0 77599D-03 | 0 26028D+01 | 0.89014D-01 |
| 27 | 0 72531D-03 | 0 31157D-02 | 0 97296D-01 | 0 19304D-01 | 0 38622D 01 | 0.13858D-01 | 0 40073D+00 | 0.26031D+01 | 0 97296D-01 |
| 28 | 0 24961D-03 | 0 40500D-02 | 0 10268D+00 | 0 43313D-02 | 0 46896D-01 | -0.11939D+00 | 0 80025D+00 | 0.26040D+01 | 0.10268D+00 |
| 29 | -0 37638D-03 | 0 29431D-02 | 0 98575D-01 | -0 19335D-01 | 0 39719D-01 | -0 33544D-01 | 0 11996D+01 | 0.26029D+01 | 0 98575D-01 |
| 30 | -0 42173D-03 | 0 26500D 02 | 0 90180D-01 | -0 21056D-01 | 0 32795D-01 | -0 80318D-02 | 0 15996D+01 | 0 26026D+01 | 0 90480D-01 |
| 31 | 0 11305D-03 | 0 21916D-02 | 0.10291D+00 | 0 25606D-01 | 0 37231D 01 | -0.14363D-02 | 0 11305D-03 | 0.30022D+01 | 0 10294D+00 |
| 32 | 0 20664D-03 | 0 27126D-02 | 0.11370D+00 | 0 29493D-01 | 0 40771D-01 | 0 18374D-01 | 0 40021D+00 | 0 30027D+01 | 0 11370D+00 |
| 33 | 0 92122D-04 | 0 55840D-02 | 0 12366D+00 | 0 57339D-02 | 0 50325D-01 | -0 70995D-01 | 0 80009D+00 | 0 30056D+01 | 0 12366D+00 |
| 34 | 0 94928D-04 | 0 25584D-02 | 0 11451D+00 | -0 28708D-01 | 0.39588D-01 | -0.31057D-01 | 0 12001D+01 | 0 30026D+01 | 0.11451D+00 |
| 35 | 0 20836D-03 | 0 20152D-02 | 0.10399D+00 | -0 24741D-01 | 0 35098D-01 | -0.17568D-02 | 0 16002D+01 | 0.30020D+01 | 0 10399D+00 |
| 36 | -0 88932D-03 | 0 12373D-02 | 0 11666D+00 | 0 23265D-01 | 0 28165D-01 | -0 98305D-02 | -0 88932D-03 | 0.31012D+01 | 0.11686D+00 |
| 37 | -0 64267D-03 | 0 16382D-02 | 0.12847D+00 | 0 34930D-01 | 0 29862D-01 | 0 97245D-02 | 0 39936D+00 | 0 34016D+01 | 0.12847D+00 |
| 38 | 0 17079D-03 | 0 35595D-02 | 0 14051D+00 | -0 61408D-07 | 0 32186D-01 | 0 42548D-02 | 0 80017D+00 | 0.34036D+01 | 0.14051D+00 |
| 39 | 0 95695D-03 | 0 15453D-02 | 0 12836D+00 | -0.34992D-01 | 0.27951D 01 | -0 18519D-01 | 0 12010D+01 | 0.34015D+01 | 0.12836D+00 |
| 40 | 0 12160D-02 | 0 10331D-02 | 0.11668D+00 | -0 23070D-01 | 0 23858D-01 | 0 49471D-02 | 0 16012D+01 | 0 34010D+01 | 0.11668D+00 |
| 41 | -0 85138D-03 | -0 10775D-03 | 0 12054D+00 | 0 25948D-01 | -0 76157D-02 | 0.12060D-01 | -0 85138D-03 | 0 37999D+01 | 0.12054D+00 |
| 42 | -0 64987D-03 | -0 30391D-03 | 0 13263D+00 | 0 34613D-01 | -0 10014D-01 | -0.20471D-01 | 0.39935D+00 | 0 37997D+01 | 0.13263D+00 |
| 43 | 0 16590D-03 | -0 34919D-02 | 0 14344D+00 | -0 87462D-02 | -0 17388D-01 | -0 80059D-01 | 0 80017D+00 | 0 37965D+01 | 0.14344D+00 |
| 44 | 0 82773D-03 | -0 58601D-03 | 0.13095D+00 | -0 36018D-01 | -0 13076D-01 | 0 15091D-01 | 0 12008D+01 | 0 37994D+01 | 0.13095D+00 |
| 45 | 0 98572D-03 | -0 21653D-03 | 0 11829D+00 | -0 28169D-01 | -0 10894D-01 | -0 12162D-01 | 0.16010D+01 | 0 37998D+01 | 0.11829D+00 |
| 46 | -0 39103D-04 | -0 66278D-03 | 0 11639D+00 | 0 23321D-01 | -0 12072D-01 | -0 12482D-01 | -0 39103D-04 | 0.41993D+01 | 0.11639D+00 |
| 47 | 0 79633D-04 | -0 11735D-02 | 0 12604D+00 | 0 25506D-01 | -0 22013D 01 | -0.40168D-01 | 0 40008D+00 | 0.41988D+01 | 0.12604D+00 |
| 48 | 0 73362D-04 | -0.32488D-02 | 0.13220D+00 | -0 72098D-02 | -0.32528D 01 | -0.13654D+00 | 0 80007D+00 | 0.41968D+01 | 0.13220D+00 |
| 49 | -0 56315D-04 | -0 13173D-02 | 0.12405D+00 | -0 25893D-01 | -0 19193D-01 | 0.26987D-01 | 0 11999D+01 | 0 41987D+01 | 0.12405D+00 |
| 50 | 0 34697D-04 | -0 71851D-03 | 0 11409D+00 | -0 24498D-01 | -0.10665D-01 | 0 11047D-01 | 0 16000D+01 | 0 41993D+01 | 0 11409D+00 |
| 51 | 0 23015D-03 | -0 13801D-02 | 0 10753D+00 | 0 39210D-02 | -0 26407D 01 | -0 85511D-02 | 0 23015D-03 | 0 47986D+01 | 0.10753D+00 |
| 52 | 0 29263D-03 | -0 16111D-02 | 0 11003D+00 | 0 77195D-02 | -0 33394D-01 | -0.24254D-01 | 0 40029D+00 | 0.47984D+01 | 0 11003D+00 |
| 53 | 0.11038D-04 | -0 21064D-02 | 0 11251D+00 | 0 16155D-02 | -0 44457D-01 | -0 16006D+00 | 0 80001D+00 | 0 47979D+01 | 0.11251D+00 |
| 54 | -0 23905D-03 | -0 15526D-02 | 0.10970D+00 | -0.98937D-02 | -0 32169D-01 | 0 24161D-01 | 0 11998D+01 | 0.47984D+01 | 0.10970D+00 |
| 55 | -0 19464D-03 | -0 13998D-02 | 0 10658D+00 | -0 53361D-02 | -0 23726D-01 | 0 91710D-02 | 0,15998D+01 | 0 47986D+01 | 0.10658D+00 |
| 56 | 0 12991D-03 | -0 17241D-02 | 0 84843D-01 | -0 46787D-02 | -0 54147D-01 | 0 89120D-02 | 0.12991D-03 | 0.53983D+01 | 0.84848D-01 |
| 57 | 0 46674D-04 | -0 21183D-02 | 0 84147D-01 | 0 40799D-04 | -0 54287D-01 | -0 12250D-01 | 0 40005D+00 | 0 53979D+01 | 0.84147D-01 |
| 58 | 0 34266D-04 | -0 28138D-02 | 0 84485D-01 | 0 16687D-02 | -0 63805D-01 | -0 16006D+00 | 0 80003D+00 | 0 53972D+01 | 0.84485D-01 |
| 59 | 0 88249D-04 | -0 20008D-02 | 0 84825D-01 | 0.65936D-03 | -0 53411D-01 | 0.98318D-02 | 0 12001D+01 | 0 53980D+01 | 0.84825D-01 |
| 60 | 0 75374D-05 | -0 16630D-02 | 0 85782D-01 | 0 52926D-02 | -0 51338D-01 | -0 73828D-02 | 0 16000D+01 | 0 53983D+01 | 0.85782D-01 |
| 61 | 0 53580D-03 | -0 21328D-02 | 0 45790D-01 | -0 66178D-03 | -0 74389D-01 | 0.13603D-01 | 0 53580D-03 | 0 59979D+01 | 0 45790D-01 |
| 62 | 0 25468D-03 | -0 29596D-02 | 0.45825D-01 | 0 53322D-03 | -0 72729D-01 | -0 11969D-01 | 0 40025D+00 | 0 59970D+01 | 0.45825D-01 |
| 63 | 0 14991D-03 | -0.43250D-02 | 0 47201D-01 | 0 55865D-02 | -0 77008D-01 | -0 13376D+00 | 0 80015D+00 | 0 59957D+01 | 0 47201D-01 |
| 64 | 0 64211D-05 | -0 28189D-02 | 0 47245D-01 | -0 36087D-03 | -0 72982D-01 | -0 62363D-02 | 0 12000D+01 | 0.59972D+01 | 0.47245D-01 |
| 65 | -0 27532D-03 | -0 20173D-02 | 0 47504D-01 | 0 17674D-02 | -0 75620D-01 | -0 19804D-01 | 0 15997D+01 | 0 59980D+01 | 0.47504D-01 |
| 66 | 0 13501D-02 | -0 26553D-02 | 0.15546D-01 | 0 66908D-02 | -0 74475D-01 | 0 40837D-01 | 0 13501D-02 | 0 63973D+01 | 0.15546D-01 |
| 67 | 0 77363D-03 | -0 30733D-02 | 0 16399D-01 | -0 43493D-03 | -0 72122D 01 | -0 94422D-02 | 0 40077D+00 | 0 63969D+01 | 0.16399D-01 |
| 68 | -0 92938D-04 | -0 57059D-02 | 0 15661D-01 | 0 12028D-03 | -0 68498D-01 | -0 10592D+00 | 0 79991D+00 | 0 63943D+01 | 0.15661D-01 |
| 69 | -0 61820D-03 | -0 31794D-02 | 0 16704D-01 | 0 25726D-02 | -0 74396D-01 | -0 46922D-02 | 0 11994D+01 | 0 63968D+01 | 0.16704D-01 |
| 70 | -0 11747D-02 | -0.25377D-02 | 0 16362D-01 | -0 56024D-02 | -0 77524D-01 | -0 48024D-01 | 0 15988D+01 | 0 63975D+01 | 0.16382D-01 |
| 71 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0.0 | 0.68000D+01 | 0.0 |
| 72 | 0 0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0,40000D+00 | 0 68000D+01 | 0.0 |
| 73 | 0.0 | 0 0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.80000D+00 | 0.68000D+01 | 0.0 |
| 74 | 0 0 | 0 0 | 0.0 | 0 0 | 0.0 | 0 0 | 0.12000D+01 | 0 68000D+01 | 0.0 |
| 75 | 0.0 | 0 0 | 0 0 | 0 0 | 0 0 | 0.0 | 0 16000D+01 | 0 68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.800143D+00 | 0.373791D+01 | -0.457382D+00 | -0.736983D+00 | 0.121677D+04 | -0.105726D+04 | 0.238109D+04 | 0.105022D+00 | 0.105182D+01 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | -0 2588D-01 | 0.2989D-01 | -0 2000D-01 | -0 1643D-02 | 0.8270D-03 | 0 8749D-03 | 0 0 | 0 2989D-01 | 0.0 | -0.7947D+00 |
| 39 | 0 8590D-02 | -0 4747D-02 | -0 1148D-01 | 0.4401D-02 | -0 6924D-02 | 0 4837D-03 | 0 9171D-02 | 0.4408D-02 | -0.9519D+01 | 0.8849D+02 |
| 43 | -0 2413D-01 | 0.2792D-01 | -0 1874D-01 | 0 2036D-02 | -0.1633D-01 | 0 1569D-01 | 0 0 | 0 3011D-01 | 0.0 | 0.1561D+02 |
| 44 | 0.6217D-02 | -0 2861D-02 | -0 9500D-02 | 0 2525D-02 | 0 6279D-02 | 0 2429D-03 | 0 6821D-02 | 0 2527D-02 | 0.1089D+02 | 0.8871D+02 |
| 48 | -0.1411D-01 | 0 1383D-01 | 0 4036D-02 | -0 5219D-03 | -0 2742D-01 | 0 2720D-01 | 0 1145D-01 | 0 2203D-01 | -0.6180D+02 | 0 3109D+02 |
| 49 | 0 1206D-02 | -0 6264D-03 | -0 1060D-02 | -0 7858D-03 | 0 7800D-02 | -0 2995D-02 | 0 4134D-02 | 0 7935D-03 | 0 3690D+02 | -0.4348D+02 |
| 53 | -0 6198D-02 | 0 4872D-02 | -0 3854D-03 | 0 3087D-02 | -0 3204D-01 | 0 3198D-01 | 0.1299D-01 | 0 1999D-01 | -0.5014D+02 | 0.4340D+02 |
| 54 | 0.9766D-03 | -0.1393D-02 | -0.3109D-02 | 0 3005D-02 | 0 6154D-02 | -0 3510D-02 | 0 2627D-02 | 0 3620D-02 | 0.2821D+02 | -0.7070D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG (DIR 1) | ELONG (DIR 2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | -0 64382D-02 | -0 21699D-01 | -0 31511D-02 | -0 15768D-01 | -0 12572D-01 | 0 0 | 0 0 |
| 1 | OUTER | 0 11781D-01 | -0 68949D-03 | 0 64753D-03 | 0 58525D-02 | 0 52085D-02 | 0 11790D-01 | 0 14862D+01 |
| 2 | INNER | -0 67066D-02 | -0 24259D-01 | 0 59900D-03 | -0 15300D-01 | -0 15909D-01 | 0 0 | 0 0 |
| 2 | OUTER | 0 11807D-01 | 0 13396D-02 | 0.82501D-03 | 0.69618D-02 | 0 61421D-02 | 0 11824D-01 | 0.22532D+01 |
| 3 | INNER | -0 76950D-02 | -0 33406D-02 | -0 18255D-01 | -0.77248D-02 | -0.33462D-02 | 0.38659D-02 | -0.51708D+02 |
| 3 | OUTER | 0 10405D-01 | 0 25247D-02 | 0 52819D-02 | 0.10351D-01 | 0.25215D-02 | 0.11208D-01 | 0.16917D+02 |
| 4 | INNER | -0 54403D-02 | -0.38710D-02 | 0 10052D-01 | -0 51552D-02 | -0 38785D-02 | 0 43115D-03 | 0.49437D+02 |
| 4 | OUTER | 0 77580D-02 | 0.31527D-02 | 0 98022D-03 | 0.77281D-02 | 0.31478D-02 | 0.78095D-02 | 0.60080D+01 |

SYSTEM ENERGIES(IN-LB)

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    FRAG. TRANSLATIONAL KINETIC ENERGY =  0 4956483D+03
    FRAG. ROTATIONAL KINETIC ENERGY    =  0.1081762D+03
    WORK INPUT TO STRUCTURE            =  0 9372492D+03
    STRUCTURE KINETIC ENERGY           =  0 9763470D+02
    STRUCTURE ELASTIC ENERGY           =  0 7251976D+02
    STRUCTURE PLASTIC ENERGY           =  0 6649751D+03
    ENERGY STORED IN ELASTIC RESTRAINTS=  0.1021197D+03

        IMPACT CYCLE=        140

        SUBREGIONS PENETRATED:   1   4

        POINT OF EFFECTIVE IMPACT(XA,YA,ZA)=  0.8004215D+00  0.3736101D+01  0.1400903D+00
        THIS IS IMPACT NUMBER     120

        IMPACT CYCLE=        143

        SUBREGIONS PENETRATED    1   4

        POINT OF EFFECTIVE IMPACT(XA,YA,ZA)=  0.8004963D+00  0.3743301D+01  0.1336815D+00
        THIS IS IMPACT NUMBER     121

        IMPACT CYCLE=        146  ELEMENT=   31  SUBREGION =  2
        GLOBAL IMPACT LOCATION (XN,YN,ZN)=  0.8193398D+00  0.3750430D+01  0.1272757D+00
        PENETRATION DISTANCE(IN.)=  0.2084767D-04
        THIS IS IMPACT NUMBER     122

        IMPACT CYCLE=        149

        SUBREGIONS PENETRATED.   1   4

        POINT OF EFFECTIVE IMPACT(XA,YA,ZA)=  0.8004189D+00  0.3757495D+01  0.1207581D+00
        THIS IS IMPACT NUMBER     123
```

***** INCR. NO.=  150  TIME=  0.3000D-03 SEC.

| NODE | U | V | W | PSIX | PSIY | TWIST | X-POS. | Y-POS. | Z-POS. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.40000D+00 | 0.0 | 0.0 |
| 3 | 0 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.80000D+00 | 0.0 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0 0 | 0 0 | 0.0 | 0.12000D+01 | 0.0 | 0.0 |
| 5 | 0 0 | 0 0 | 0.0 | 0.0 | 0 0 | 0.0 | 0.16000D+01 | 0.0 | 0.0 |
| 6 | 0.93203D-03 | 0.23300D-02 | 0.99185D-02 | 0.30337D-02 | 0.47319D-01 | -0.36416D-01 | 0.93203D-03 | 0.40233D+00 | 0.99185D-02 |
| 7 | 0 44385D-03 | 0.28118D-02 | 0 99480D-02 | -0.22490D-02 | 0 44517D-01 | -0.24245D-02 | 0.40014D+00 | 0.40281D+00 | 0.99480D-02 |
| 8 | 0 93689D-04 | 0 44886D-02 | 0.91159D-02 | 0.73653D-04 | 0 39360D-01 | -0.82803D-01 | 0 80009D+00 | 0.40449D+00 | 0.91159D-02 |
| 9 | -0 53439D-03 | 0 26747D-02 | 0 98466D-02 | 0 96593D-03 | 0 43263D-01 | -0.71419D-02 | 0.11995D+01 | 0.40267D+00 | 0.98466D-02 |
| 10 | -0 10110D-02 | 0 22608D-02 | 0 95270D-02 | -0 36793D-02 | 0 45939D-01 | 0.32480D-01 | 0.15990D+01 | 0 40226D+00 | 0.95270D-02 |
| 11 | 0 19409D-03 | 0 25056D-02 | 0 27474D-01 | -0 16926D-02 | 0 40411D-01 | -0.90191D-02 | 0.19409D-03 | 0 80251D+00 | 0 27474D-01 |
| 12 | 0 23054D-04 | 0 30031D-02 | 0 27127D-01 | -0.11893D-03 | 0 39758D-01 | -0.21976D-02 | 0.40002D+00 | 0 80300D+00 | 0.27127D-01 |
| 13 | -0 52787D-04 | 0 37848D-02 | 0.27025D-01 | -0 34218D-02 | 0 42555D-01 | -0.95194D-01 | 0.79995D+00 | 0.80378D+00 | 0.27025D-01 |
| 14 | -0 74987D-04 | 0 30211D-02 | 0 26444D-01 | 0 79995D-03 | 0 40394D-01 | -0 10799D-01 | 0.11999D+01 | 0 80302D+00 | 0.26444D-01 |
| 15 | -0 25549D-03 | 0 23924D-02 | 0 26878D-01 | 0 16678D-02 | 0.10680D-01 | 0 56336D-02 | 0.15997D+01 | 0.80239D+00 | 0 26878D-01 |
| 16 | 0 11767D-03 | 0 28567D-02 | 0 47873D-01 | -0 71733D-03 | 0 28778D 01 | -0 25421D-03 | 0.11767D-03 | 0.14029D+01 | 0.47873D-01 |
| 17 | 0 42455D-04 | 0 29001D-02 | 0.48082D-01 | 0 11912D-02 | 0 32040D-01 | 0 10373D-01 | 0 40004D+00 | 0.14029D+01 | 0 48082D-01 |
| 18 | 0 10105D-03 | 0 32159D-02 | 0 48422D-01 | -0 26058D-03 | 0 39555D-01 | -0.11611D+00 | 0 80010D+00 | 0.14032D+01 | 0 48422D-01 |
| 19 | 0 94309D-04 | 0 28716D-02 | 0 48145D-01 | -0 59261D-03 | 0 32710D-01 | -0.13563D-01 | 0.12001D+01 | 0.14029D+01 | 0.48145D-01 |
| 20 | 0 26484D-04 | 0 20898D-02 | 0 48242D-01 | 0 15897D-02 | 0 30718D-01 | 0 74521D-03 | 0 16000D+01 | 0 14027D+01 | 0 48242D-01 |
| 21 | 0 53086D-03 | 0 30321D-02 | 0 61280D-01 | 0.90973D-02 | 0 20316D-01 | 0 51245D-02 | 0 53086D-03 | 0 20030D+01 | 0 61280D-01 |
| 22 | 0 45560D-03 | 0 23802D-02 | 0.65107D-01 | 0.97148D-02 | 0.26355D-01 | 0 16539D-01 | 0.40046D+00 | 0 20029D+01 | 0.65107D-01 |
| 23 | 0 16823D-03 | 0 29639D-02 | 0 67782D-01 | -0.42763D-03 | 0 35467D-01 | -0.11756D+00 | 0 80017D+00 | 0.20030D+01 | 0.67782D-01 |
| 24 | -0 13518D-03 | 0 28177D-02 | 0 65740D-01 | -0 76166D-02 | 0 27011D-01 | -0.22372D-01 | 0.11999D+01 | 0 20028D+01 | 0.65740D-01 |
| 25 | -0 20203D-03 | 0 28505D-02 | 0.62652D-01 | -0 73332D-02 | 0 21432D-01 | -0 68323D-02 | 0 15998D+01 | 0 20029D+01 | 0.62652D-01 |
| 26 | 0 76300D-03 | 0 27454D-02 | 0.71586D-01 | 0 24097D-01 | 0.20667D-01 | 0 11314D-01 | 0.76300D-03 | 0.26027D+01 | 0.71586D-01 |
| 27 | 0 71546D-03 | 0 30255D-02 | 0 80653D-01 | 0 20989D-01 | 0.28779D-01 | 0.19996D-01 | 0 40072D+00 | 0 26030D+01 | 0.80653D-01 |
| 28 | 0 27851D-03 | 0 38577D-02 | 0 86474D-01 | 0 41684D-02 | 0 37469D-01 | -0.10767D+00 | 0.80028D+00 | 0 26039D+01 | 0 86474D-01 |
| 29 | -0 30690D-03 | 0 28773D-02 | 0 81974D-01 | -0 20622D-01 | 0 29649D-01 | -0 37189D-01 | 0.11997D+01 | 0 26029D+01 | 0 81974D-01 |
| 30 | -0 35267D-03 | 0 26177D-02 | 0 73241D-01 | -0 22967D-01 | 0 20365D-01 | -0.14684D-01 | 0.15996D+01 | 0.26026D+01 | 0.73241D-01 |
| 31 | 0 19580D-03 | 0 22425D-02 | 0 81068D-01 | 0 30224D-01 | 0 27513D-01 | 0 46610D-02 | 0 19580D-03 | 0.30022D+01 | 0.81068D-01 |
| 32 | 0 24537D-03 | 0 27051D-02 | 0 93362D-01 | 0.32408D-01 | 0.32861D-01 | 0.22711D-01 | 0.40025D+00 | 0.30027D+01 | 0.93362D-01 |
| 33 | 0 10972D-03 | 0 54660D-02 | 0 10393D+00 | 0 55760D-02 | 0 43208D-01 | -0 65467D-01 | 0 80011D+00 | 0 30055D+01 | 0.10393D+00 |
| 34 | 0 89155D-04 | 0 25780D-02 | 0 94206D-01 | -0 31512D-01 | 0 31543D-01 | -0.33141D-01 | 0 12001D+01 | 0 30026D+01 | 0.94206D-01 |
| 35 | 0.15639D-03 | 0 21139D-02 | 0 82244D-01 | -0 29103D-01 | 0 25215D-01 | -0 76747D-02 | 0 16002D+01 | 0.30021D+01 | 0.82244D-01 |
| 36 | -0 70007D-03 | 0.14001D-02 | 0 91962D-01 | 0 30376D-01 | 0 23823D-01 | -0 38757D-02 | -0.70007D-03 | 0.34014D+01 | 0.91962D-01 |
| 37 | -0 55901D-03 | 0.17371D-02 | 0 10576D+00 | 0 38932D-01 | 0 26659D-01 | 0 12896D-01 | 0.39944D+00 | 0.34017D+01 | 0.10576D+00 |
| 38 | 0 16688D-03 | 0.35638D-02 | 0 11867D+00 | -0 10582D-04 | 0 26665D-01 | 0.45187D-02 | 0 80017D+00 | 0.34036D+01 | 0.11867D+00 |
| 39 | 0 85953D-03 | 0.16704D-02 | 0 10566D+00 | -0 38879D-01 | 0 24609D-01 | -0.20289D-01 | 0 12009D+01 | 0 34017D+01 | 0.10566D+00 |
| 40 | 0 10151D-02 | 0.12559D-02 | 0 91852D-01 | -0 30072D-01 | 0.19405D-01 | -0.72400D-03 | 0 16010D+01 | 0 34013D+01 | 0.91852D-01 |
| 41 | -0.67696D-03 | 0.12295D-03 | 0 95966D-01 | 0 32897D-01 | -0 22583D-02 | 0.95011D-02 | -0.67696D-03 | 0 38001D+01 | 0 95966D-01 |
| 42 | -0 58053D-03 | -0 76708D-04 | 0 11019D+00 | 0.38289D-01 | -0 52433D-02 | -0 20436D-01 | 0.39942D+00 | 0 37999D+01 | 0.11019D+00 |
| 43 | 0 13879D-03 | -0 31895D-02 | 0 12165D+00 | -0 84848D-02 | -0 13319D-01 | -0 71592D-01 | 0 80014D+00 | 0 37968D+01 | 0.12185D+00 |
| 44 | 0 69710D-03 | -0 33749D-03 | 0 10852D+00 | -0 39659D-01 | -0 65443D-02 | 0 16227D-01 | 0 12007D+01 | 0 37997D+01 | 0.10852D+00 |
| 45 | 0 74881D-03 | 0 39873D-04 | 0.93737D-01 | -0 35011D-01 | -0.58576D-02 | -0 94532D-02 | 0 16007D+01 | 0 38000D+01 | 0 93737D-01 |
| 46 | -0 19267D-05 | -0 42510D-03 | 0 95486D-01 | 0 28255D-01 | 0 65264D-03 | -0 19984D-01 | -0 19267D-05 | 0 41996D+01 | 0 95486D-01 |
| 47 | 0 77928D-04 | -0 88932D-03 | 0 10669D+00 | 0 28478D-01 | -0 11692D-01 | -0 44032D-01 | 0 40008D+00 | 0 41991D+01 | 0.10669D+00 |
| 48 | 0 29364D-04 | -0 27965D-02 | 0 11361D+00 | -0 70276D-02 | -0 22452D 01 | -0.12254D+00 | 0 80003D+00 | 0 41972D+01 | 0.11361D+00 |
| 49 | -0.14931D-03 | -0 10102D-02 | 0 10464D+00 | -0 29278D-01 | -0 94036D-02 | 0 33758D-01 | 0 11999D+01 | 0 41990D+01 | 0 10464D+00 |
| 50 | -0 10209D-03 | -0 44977D-03 | 0 92975D-01 | -0 29770D-01 | 0 14320D-02 | 0 18692D-01 | 0 15999D+01 | 0 41996D+01 | 0 92975D-01 |
| 51 | 0 42062D-04 | -0 11120D-02 | 0 96165D-01 | 0 24965D-02 | -0 91667D-02 | -0 18101D-01 | 0 42062D-04 | 0 47989D+01 | 0 96165D-01 |
| 52 | 0 14820D-03 | -0 12717D-02 | 0 98067D-01 | 0 67526D-02 | -0 19964D 01 | -0 31816D-01 | 0 40015D+00 | 0 47987D+01 | 0 98067D-01 |
| 53 | -0 40706D-04 | -0.15962D-02 | 0 10008D+00 | 0 83953D-03 | -0 31487D-01 | -0 14323D+00 | 0 79996D+00 | 0 47984D+01 | 0.10008D+00 |
| 54 | -0.19374D-03 | -0 12226D-02 | 0 97402D-01 | -0 92515D-02 | -0 19137D-01 | 0 33117D-01 | 0 11938D+01 | 0 47988D+01 | 0.97402D-01 |
| 55 | -0 97240D-04 | -0 11517D-02 | 0 94572D-01 | -0 46922D-02 | -0 71319D-02 | 0.18772D-01 | 0 15999D+01 | 0 47988D+01 | 0 94572D-01 |
| 56 | -0 95131D-04 | -0 12860D-02 | 0 83134D-01 | -0 13682D-01 | -0 42425D-01 | 0.32901D-02 | -0 95131D-04 | 0 53987D+01 | 0 83134D-01 |
| 57 | -0 13676D-03 | -0 18153D-02 | 0 79460D-01 | -0 61814D-02 | -0 44582D-01 | -0.16011D-01 | 0 39986D+00 | 0 53982D+01 | 0 79460D-01 |
| 58 | 0 58865D-05 | -0 25541D-02 | 0 78106D-01 | 0 20503D-03 | -0 54137D-01 | -0 15152D+00 | 0 80001D+00 | 0 53974D+01 | 0 78106D-01 |
| 59 | 0 22811D-03 | -0 17534D-02 | 0 79591D-01 | 0 58179D-02 | -0 43993D-01 | 0.14674D-01 | 0 12002D+01 | 0 53982D+01 | 0 79591D-01 |
| 60 | 0 19488D-03 | -0 13228D-02 | 0 83134D-01 | 0 13227D-01 | -0 39751D 01 | -0.10684D-02 | 0 16002D+01 | 0 53987D+01 | 0 83134D-01 |
| 61 | 0 41008D-03 | -0 16383D-02 | 0 47531D-01 | -0 84142D-02 | -0 73034D-01 | 0 19187D-01 | 0 41008D-03 | 0 59984D+01 | 0.47531D-01 |
| 62 | 0 16092D-03 | -0 27670D-02 | 0 44905D-01 | -0 48460D-02 | -0.69046D-01 | -0 59663D-02 | 0 40016D+00 | 0 59972D+01 | 0.44905D-01 |
| 63 | 0 15469D-03 | -0.43551D-02 | 0 44938D-01 | 0 51133D-02 | -0 71429D-01 | -0.13143D+00 | 0 80015D+00 | 0 59956D+01 | 0.44938D-01 |
| 64 | 0 88169D-04 | -0.26419D-02 | 0.45996D-01 | 0 43647D-02 | -0 69071D-01 | -0 10141D-01 | 0 12001D+01 | 0 59974D+01 | 0.45996D-01 |
| 65 | -0.17014D-03 | -0 15689D-02 | 0 48662D-01 | 0 89343D-02 | -0 73752D-01 | -0.24856D-01 | 0 15998D+01 | 0.59984D+01 | 0.48662D-01 |
| 66 | 0.12752D-02 | -0 23381D-02 | 0 16882D-01 | 0 24671D-02 | -0.77292D-01 | 0.50831D-01 | 0 12752D-02 | 0.63977D+01 | 0.16882D-01 |
| 67 | 0.75786D-03 | -0.29417D-02 | 0.16345D-01 | -0.30153D-02 | -0.71265D-01 | -0.10615D-02 | 0.40076D+00 | 0.63971D+01 | 0.16345D-01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 68 | -0.10223D-03 | -0.56997D-02 | 0.15070D-01 | 0.58109D-05 | -0 65642D-01 | -0.10663D+00 | 0.79990D+00 | 0.63943D+01 | 0.15070D-01 |
| 69 | -0.61369D-03 | -0.30584D-02 | 0.16515D-01 | 0.47010D-02 | -0.73064D-01 | -0.11233D-01 | 0.11994D+01 | 0.63969D+01 | 0.16515D-01 |
| 70 | -0.11145D-02 | -0 22437D-02 | 0.17411D-01 | -0.18308D-02 | -0 79391D-01 | -0.56187D-01 | 0.15989D+01 | 0.63978D+01 | 0.17411D-01 |
| 71 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.68000D+01 | 0.0 |
| 72 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.40000D+00 | 0.68000D+01 | 0.0 |
| 73 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.80000D+00 | 0.68000D+01 | 0.0 |
| 74 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.12000D+01 | 0.68000D+01 | 0.0 |
| 75 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.16000D+01 | 0.68000D+01 | 0.0 |

FRAGMENT GLOBAL LOCATION AND VELOCITY COMPONENTS

| X-LOC | Y-LOC | Z-LOC | VEL-X | VEL-Y | VEL-Z | OMEGA-X | OMEGA-Y | OMEGA-Z |
|---|---|---|---|---|---|---|---|---|
| 0.800126D+00 | 0.376221D+01 | -0.478863D+00 | -0.109028D+01 | 0.121354D+04 | -0.108176D+04 | 0.239815D+04 | -0.100262D+01 | 0.915274D+00 |

STRAIN COMPONENTS, PRINCIPAL (TENSILE) STRAIN AND DIRECTION AT USER-SPECIFIED NODES

| | EPS-X STRAIN | | EPS-Y STRAIN | | SHEAR STRAIN | | PRINC. STRAIN(T) | | DIRECTION(DEG.) | |
|---|---|---|---|---|---|---|---|---|---|---|
| NODE | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER | INNER | OUTER |
| 38 | -0.2735D-01 | 0.3090D-01 | -0.1792D-01 | -0.2876D-02 | 0.8563D-03 | -0.9511D-03 | 0.0 | 0.3091D-01 | 0.0 | -0.8064D+00 |
| 39 | 0 7825D-02 | -0.4189D-02 | -0.9437D-02 | 0.2759D-02 | -0.7140D-02 | 0.9759D-03 | 0.8535D-02 | 0.2793D-02 | -0.1123D+02 | 0.8600D+02 |
| 43 | -0.2572D-01 | 0.2900D-01 | -0.1643D-01 | 0.7198D-03 | -0.1470D-01 | 0.1393D-01 | 0.0 | 0.3062D-01 | 0.0 | 0.1312D+02 |
| 44 | 0.5460D-02 | -0.2353D-02 | -0.7445D-02 | 0.8261D-03 | 0.6359D-02 | -0 1321D-03 | 0.6201D-02 | 0.8274D-03 | 0.1312D+02 | -0.8881D+02 |
| 48 | -0.1562D-01 | 0.1511D-01 | 0.4081D-02 | -0.5885D-03 | -0.2470D-01 | 0.2432D-01 | 0.1003D-01 | 0.2173D-01 | -0.6429D+02 | 0.2858D+02 |
| 49 | 0.9222D-03 | -0.3887D-03 | -0.2326D-03 | -0.1710D-02 | 0.8865D-02 | -0.4638D-02 | 0.4815D-02 | 0.1362D-02 | 0.4129D+02 | -0.3705D+02 |
| 53 | -0.5334D-02 | 0.4480D-02 | -0.1050D-02 | 0.2446D-02 | -0.2863D-01 | 0.2866D-01 | 0.1128D-01 | 0.1783D-01 | -0.4925D+02 | 0.4297D+02 |
| 54 | 0.1065D-02 | -0.1121D-02 | -0.3486D-02 | 0.2613D-02 | 0.7671D-02 | -0.5576D-02 | 0.3249D-02 | 0.4102D-02 | 0.2966D+02 | -0.6190D+02 |

STRAIN COMPONENTS, PRINCIPAL(TENSILE) STRAIN AND DIRECTION, AND ELONGATIONS AT SPECIFIED ADDITIONAL POINTS

| POINT NO. | SURFACE | EPS-X STRAIN | EPS-Y STRAIN | SHEAR STRAIN | ELONG.(DIR.1) | ELONG.(DIR.2) | PRINC. STRN(T) | DIRECTION(DEG.) |
|---|---|---|---|---|---|---|---|---|
| 1 | INNER | -0.76503D-02 | -0.19381D-01 | -0.29144D-02 | -0.15087D-01 | -0.12132D-01 | 0.0 | 0.0 |
| 1 | OUTER | 0.12709D-01 | -0.19287D-02 | 0.36435D-03 | 0.55571D-02 | 0.51947D-02 | 0.12712D-01 | 0.71290D+00 |
| 2 | INNER | -0.77716D-02 | -0.21754D-01 | 0.52492D-03 | -0.13607D-01 | -0.15140D-01 | 0.0 | 0.0 |
| 2 | OUTER | 0.12555D-01 | -0.10363D-03 | 0.84862D-03 | 0.60279D-02 | 0.57845D-02 | 0.12569D-01 | 0.19177D+01 |
| 3 | INNER | -0 87068D-02 | -0.19412D-02 | -0.17577D-01 | -0.87450D-02 | -0.19430D-02 | 0.40929D-02 | -0.55526D+02 |
| 3 | OUTER | 0.11179D-01 | 0.12706D-02 | 0.53272D-02 | 0.11118D-01 | 0.12698D-02 | 0.11850D-01 | 0.14132D+02 |
| 4 | INNER | -0.65011D-02 | -0.23416D-02 | 0.99634D-02 | -0.65224D-02 | -0.23443D-02 | 0.97710D-03 | 0.56330D+02 |
| 4 | OUTER | 0.85670D-02 | 0.18683D-02 | 0.21520D-03 | 0.85306D-02 | 0.18666D-02 | 0.85687D-02 | 0.92004D+00 |

SYSTEM ENERGIES(IN-LB)

FRAG. TRANSLATIONAL KINETIC ENERGY = 0.5041488D+03
FRAG. ROTATIONAL KINETIC ENERGY = 0.1097311D+03
WORK INPUT TO STRUCTURE = 0 9271939D+03
STRUCTURE KINETIC ENERGY = 0.1662348D+03
STRUCTURE ELASTIC ENERGY = 0.7717805D+02
STRUCTURE PLASTIC ENERGY = 0.6094116D+03
ENERGY STORED IN ELASTIC RESTRAINTS= 0.7436948D+02

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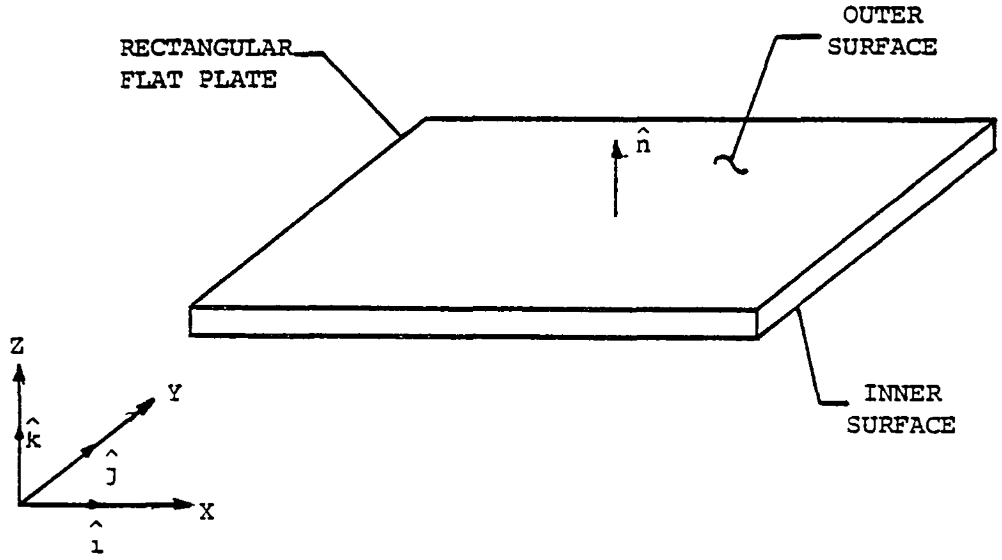
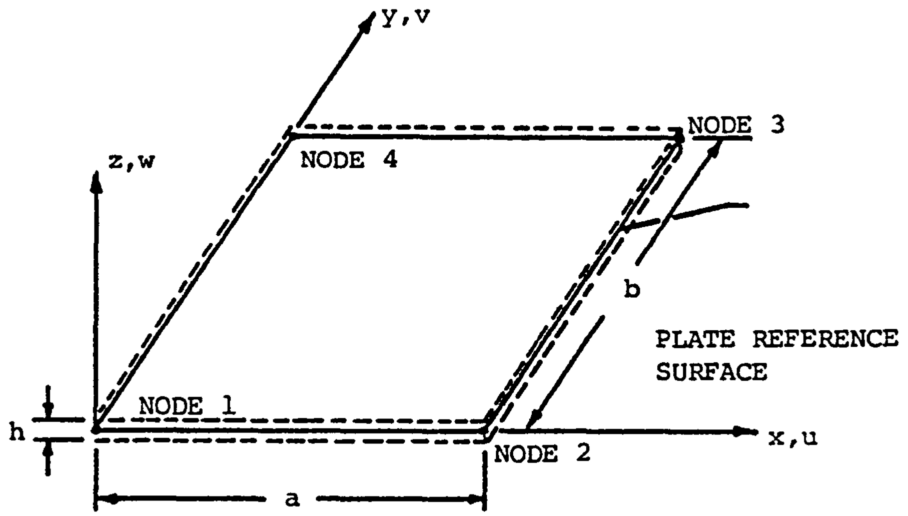


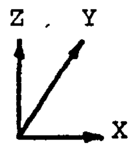

X,Y,Z -- GLOBAL RECTANGULAR CARTESIAN COORDINATE SYSTEM

$\hat{i}, \hat{j}, \hat{k}$ -- UNIT VECTORS IN THE GLOBAL COORDINATE DIRECTIONS

$\hat{n}$ -- UNIT-OUTWARD NORMAL OF THE PLATE

FIG. 1 GLOBAL COORDINATES AND UNIT-OUTWARD-NORMAL DIRECTION FOR A FLAT PLATE




X,Y,Z - Global Rectangular Cartesian Coordinate System

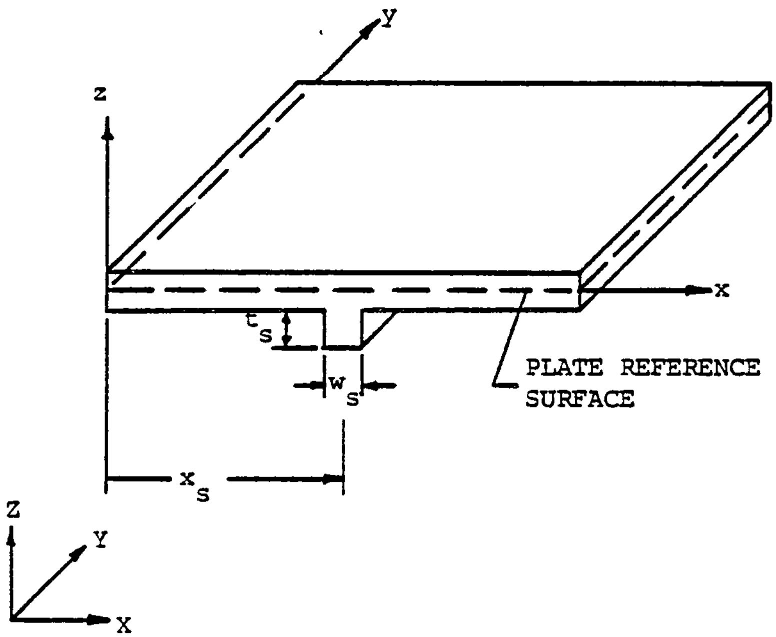
x,y,z - Local (element) Coordinate System

h - Total Element Thickness

a,b - Element Spanwise Dimension in the x and y Directions

$$\left.\begin{array}{l} u,\ v,\ w \\ w_{,x} = \dfrac{\partial w}{\partial x} \\ w_{,y} = \dfrac{\partial w}{\partial y} \\ w_{,xy} = \dfrac{\partial^2 w}{\partial x \partial y} \end{array}\right\} \begin{array}{l}\text{Generalized Nodal} \\ \text{Displacements for} \\ \text{u,v,w: LLC Element}\end{array}$$

FIG. 2 GEOMETRY AND NOMENCLATURE FOR A UNIFORM-THICKNESS RECTANGULAR PLATE ELEMENT




$w_s$ - Width of Stiffener

$t_s$ - Thickness of Stiffener

$x_s$ - Local x-Location of Stiffener

FIG. 3 GEOMETRY AND NOMENCLATURE FOR A STIFFENED RECTANGULAR PLATE ELEMENT

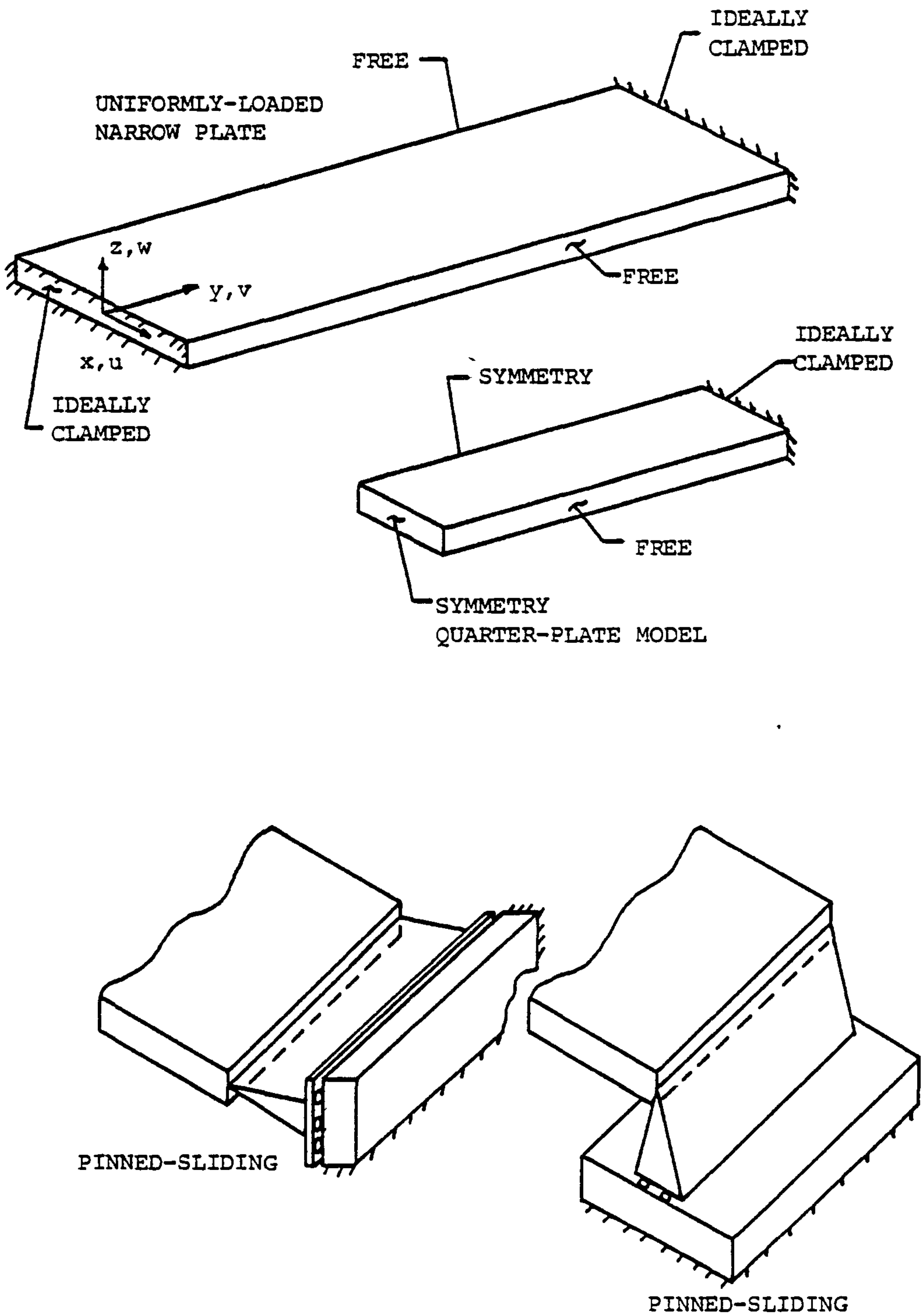
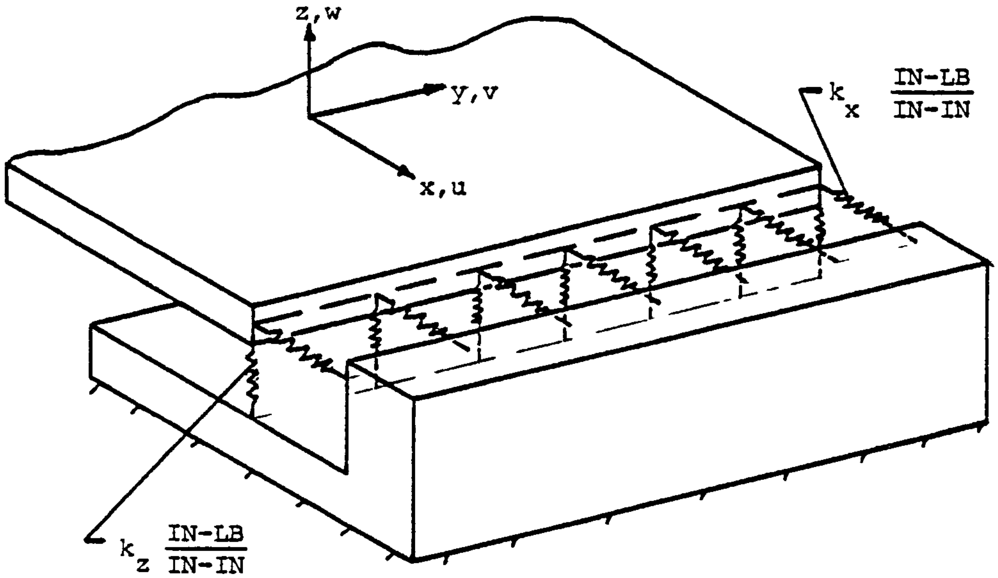
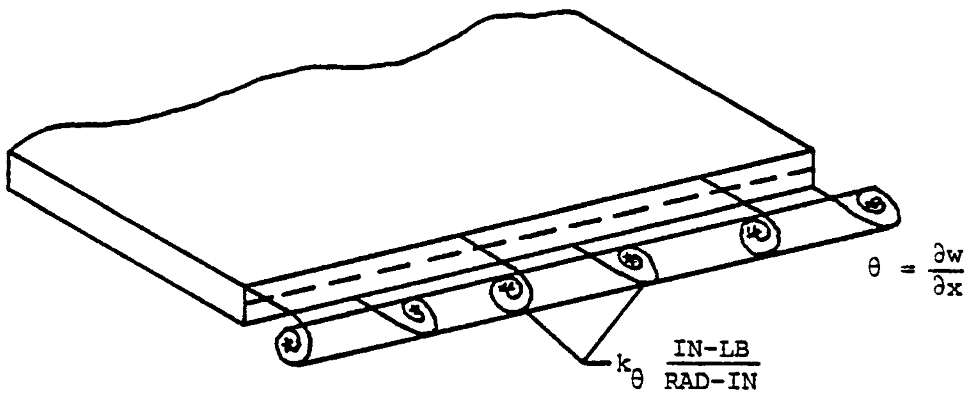



FIG. 4 SCHEMATICS OF PLATE-SIDE BOUNDARY AND ELASTIC-SPRING SUPPORT CONDITIONS

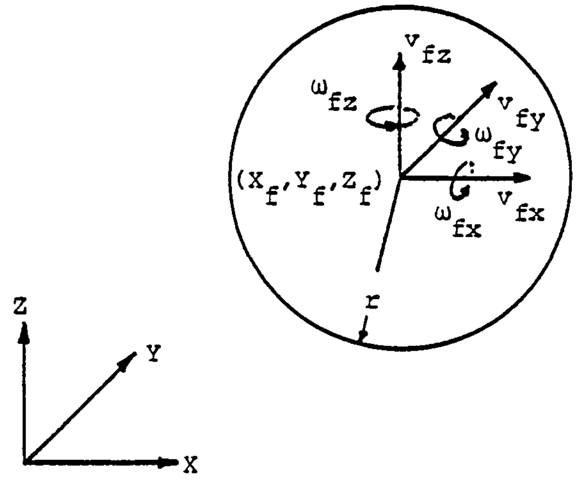



DISTRIBUTED LINE TRANSLATIONAL SPRINGS




DISTRIBUTED LINE ROTATIONAL SPRINGS

FIG. 4 CONCLUDED




$X, Y, Z$ - Global Coordinate System.

$X_f, Y_f, Z_f$ - Global Coordinates which Locate the Center of Gravity (c.g.) of the Fragment.

$r$ - Radius of the Spherical Fragment.

$v_{fx}, v_{fy}, v_{fz}$ - Translational Velocity Components of the Fragment in the X,Y, and Z Directions, Respectively.

$\omega_{fx}, \omega_{fy}, \omega_{fz}$ - Rotational Velocity Components of the Fragment in the X,Y, and Z Directions, Respectively.

FIG. 5 GEOMETRY AND NOMENCLATURE FOR AN IDEALIZED NON-DEFORMABLE SPHERICAL FRAGMENT

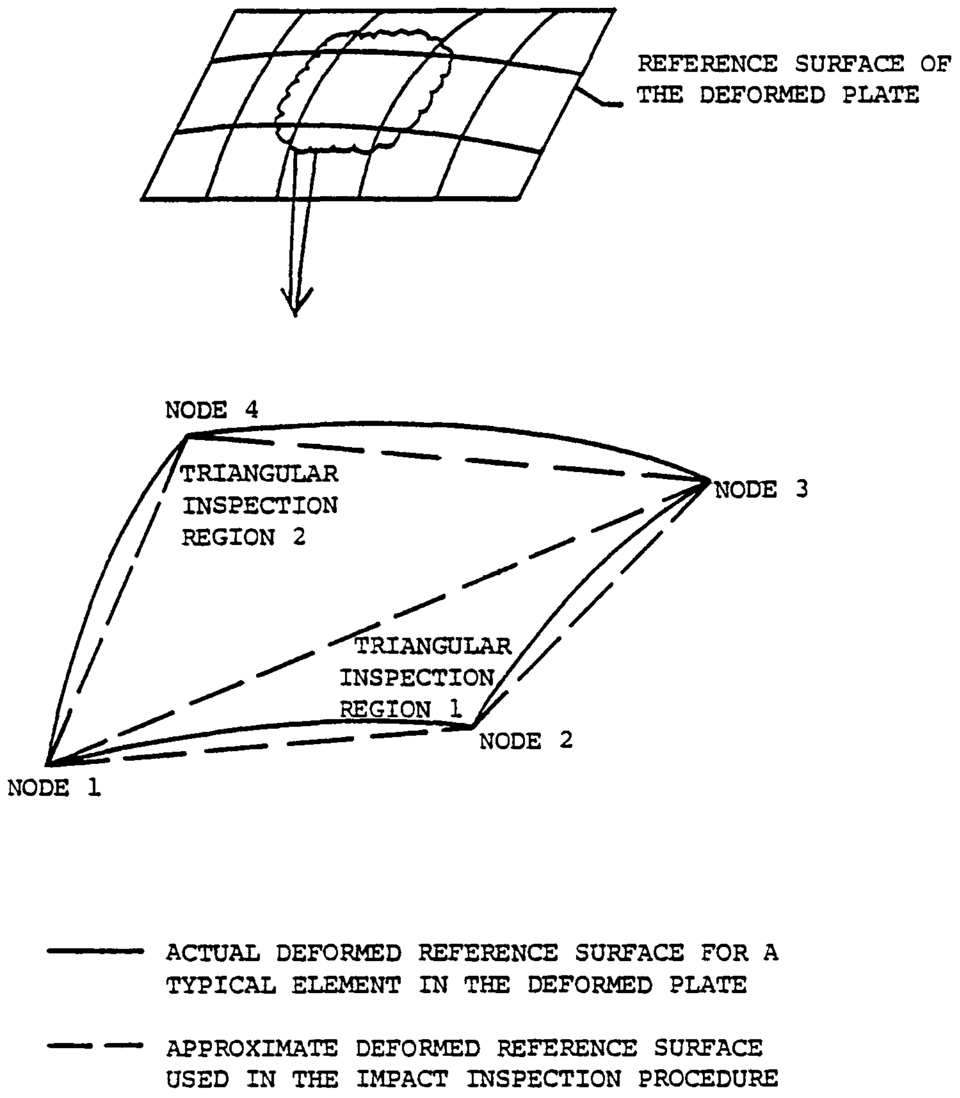



FIG. 6 IDEALIZATION OF DEFORMED PLATE FOR IMPACT INSPECTION PURPOSES

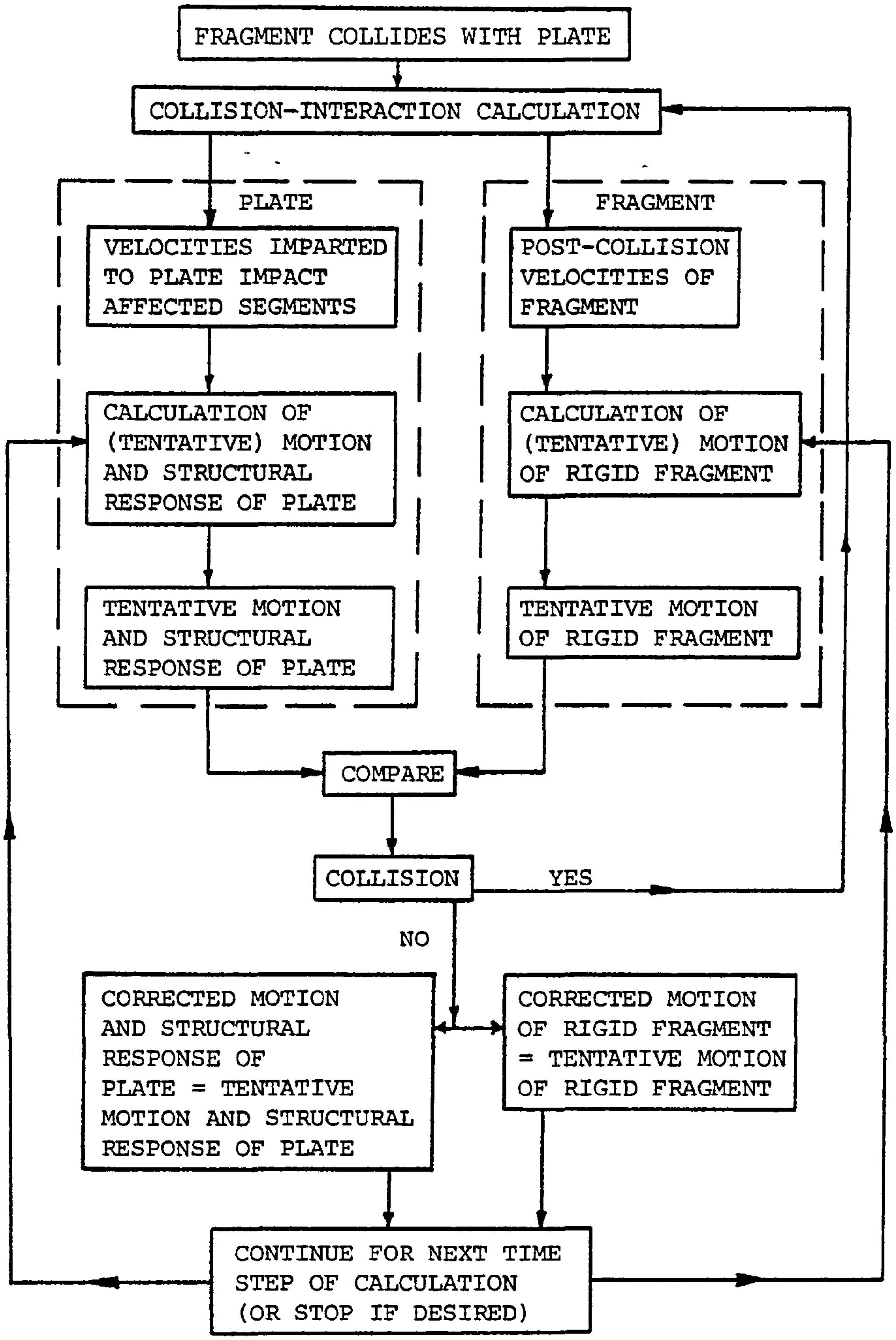



FIG. 7 INFORMATION FLOW SCHEMATIC FOR PREDICTING PLATE AND FRAGMENT MOTIONS IN THE COLLISION-IMPARTED VELOCITY METHOD

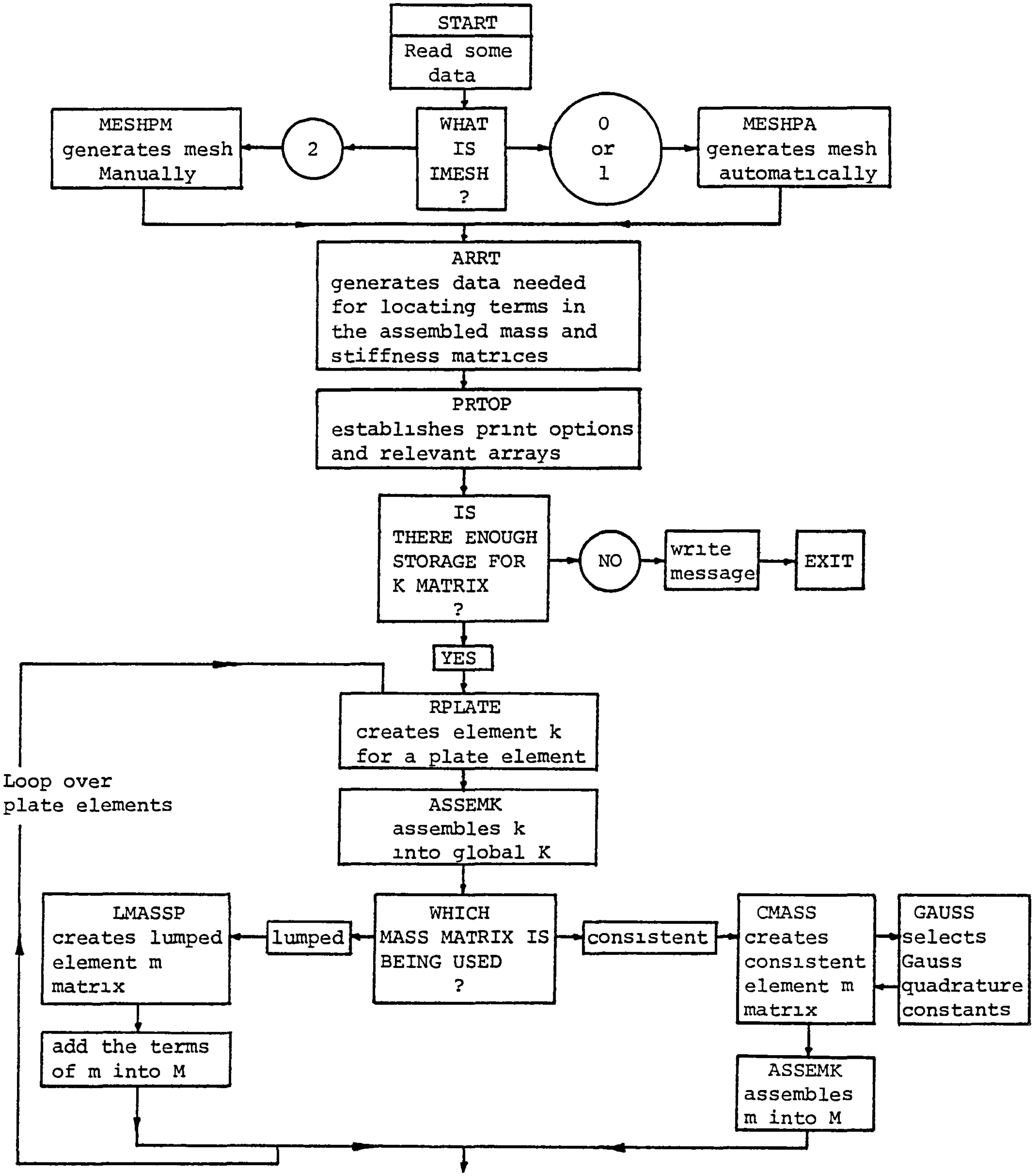



FIG. 8 FLOW CHART OF MAINP FOR THE PLATE 1 PROGRAM

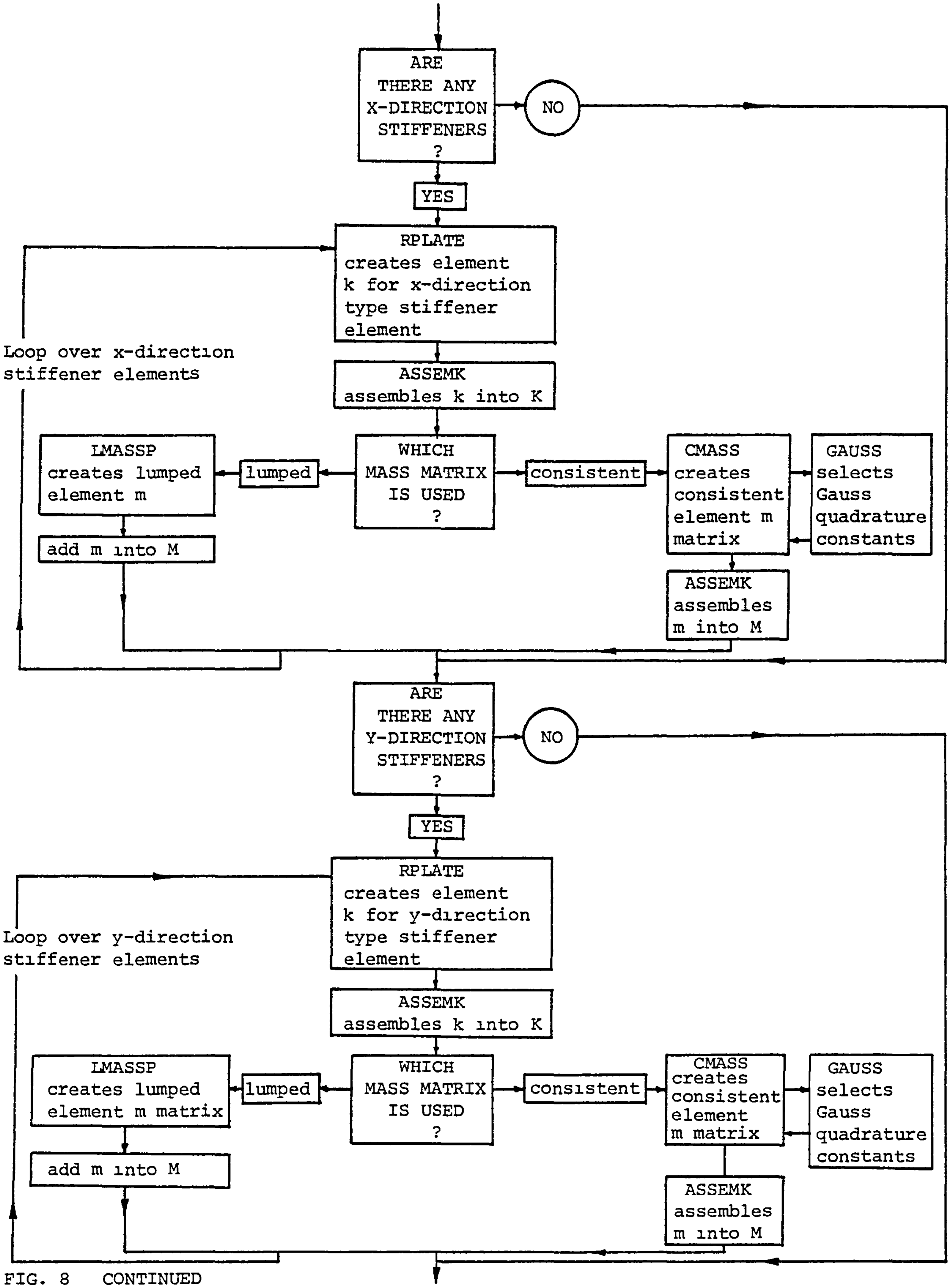



FIG. 8 CONTINUED

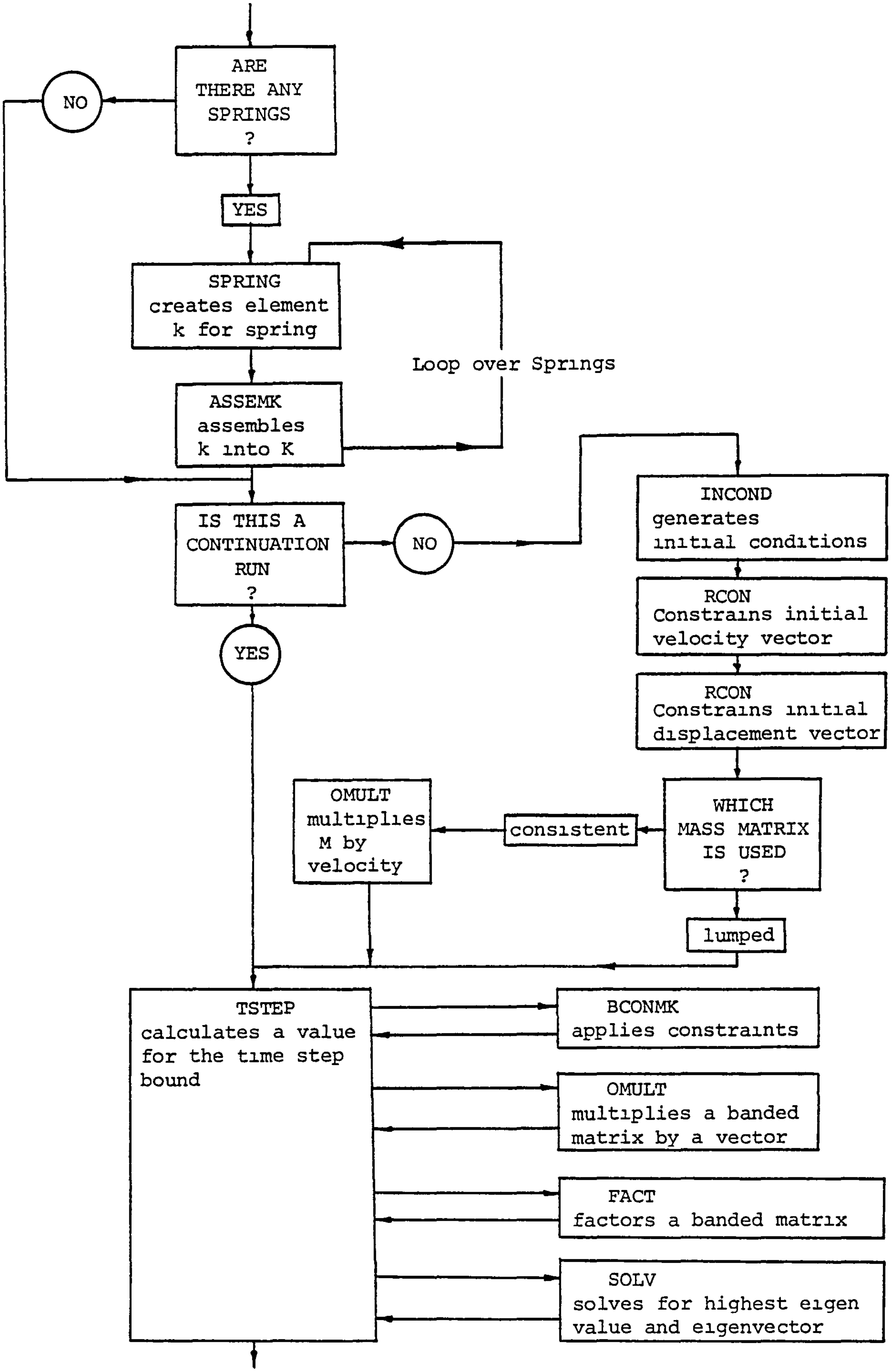



FIG. 8 CONTINUED

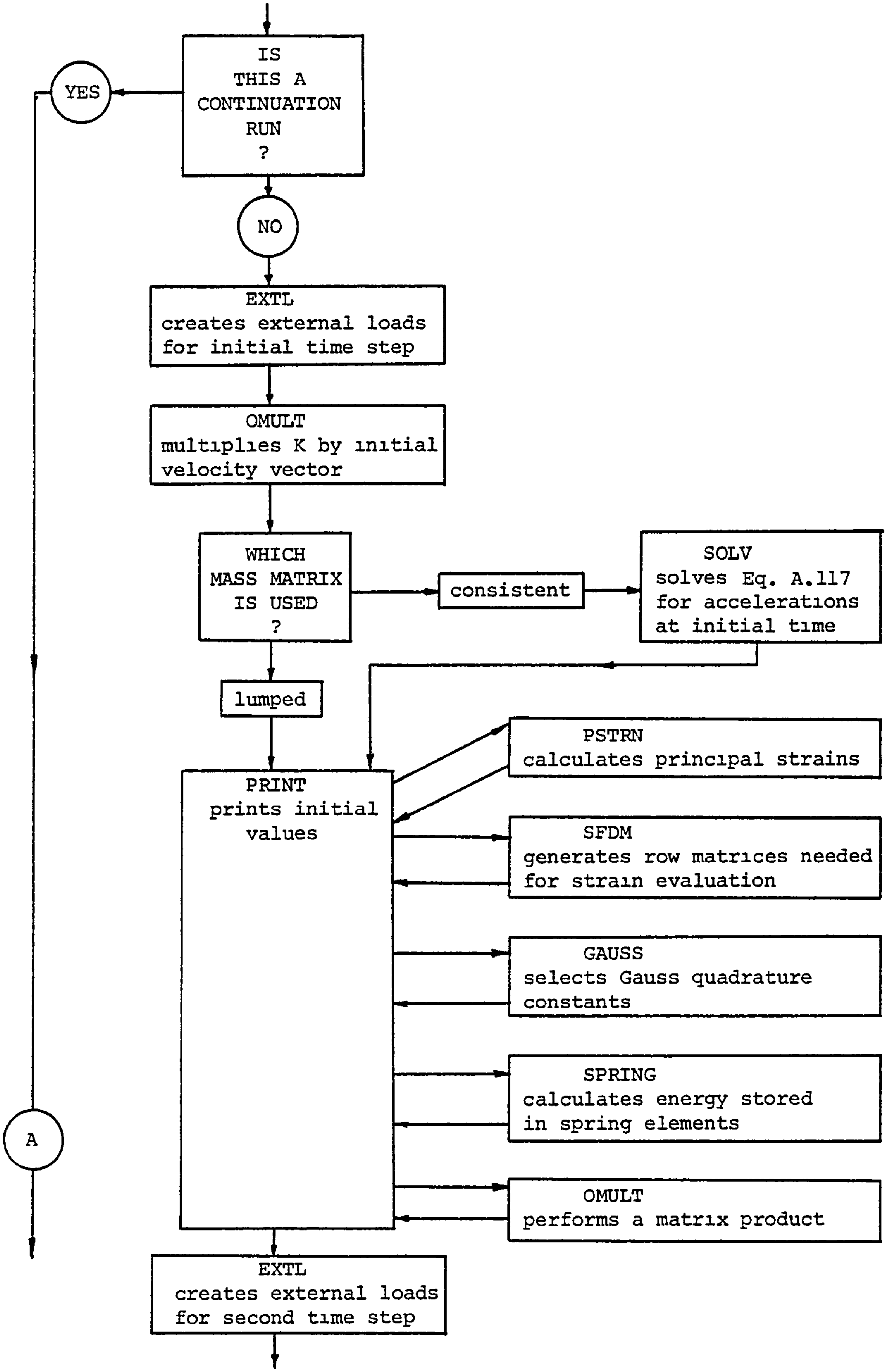



FIG. 8 CONTINUED

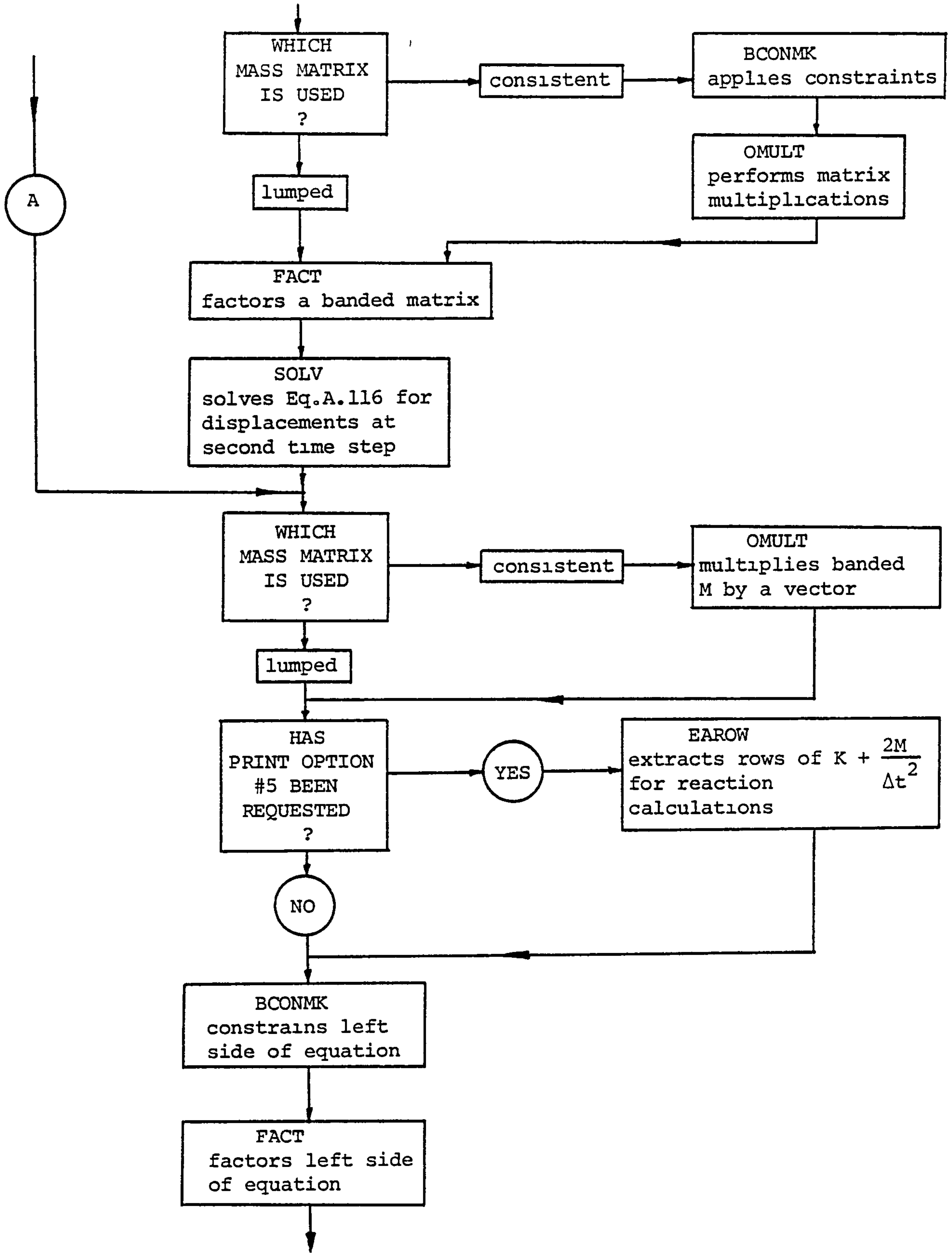



FIG. 8 CONTINUED

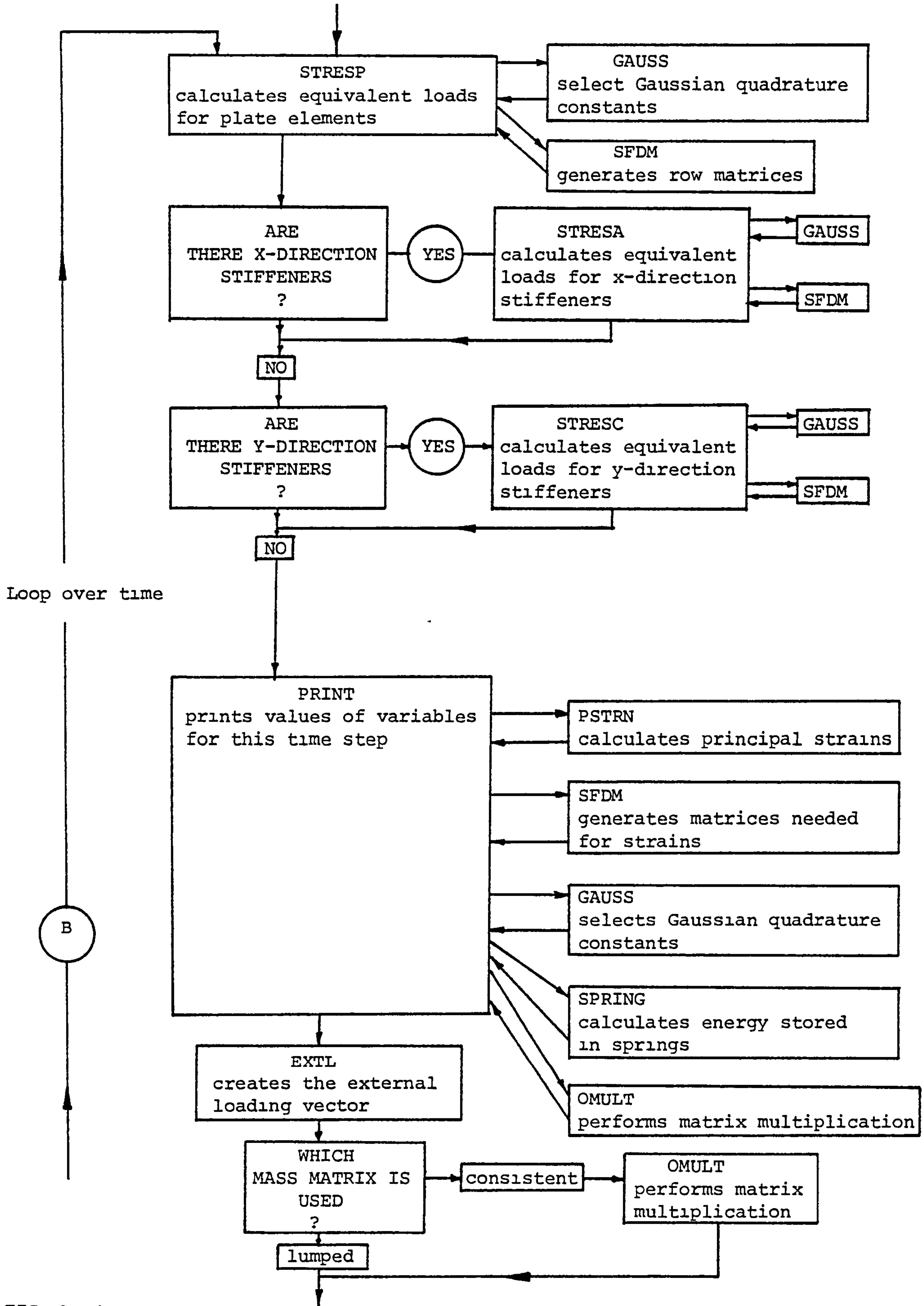



FIG. 8 (CONTINUED)

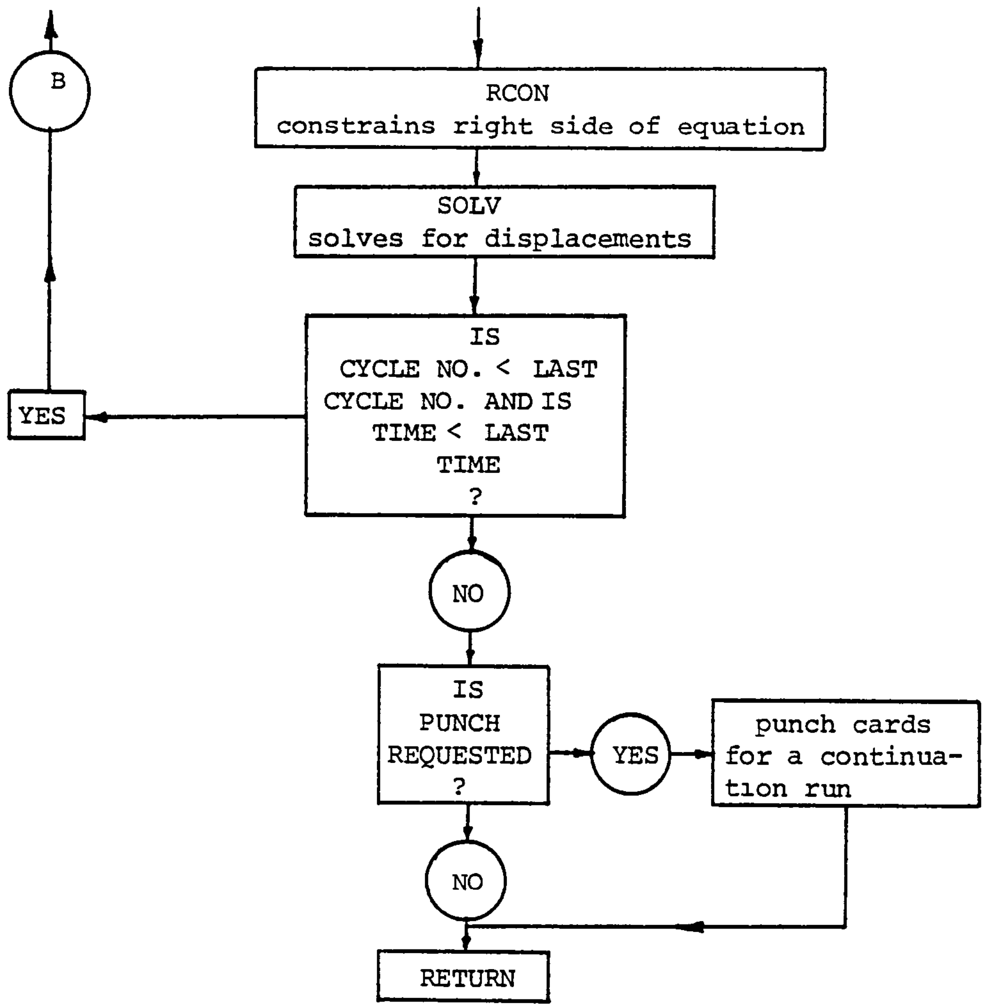



FIG. 8 CONCLUDED

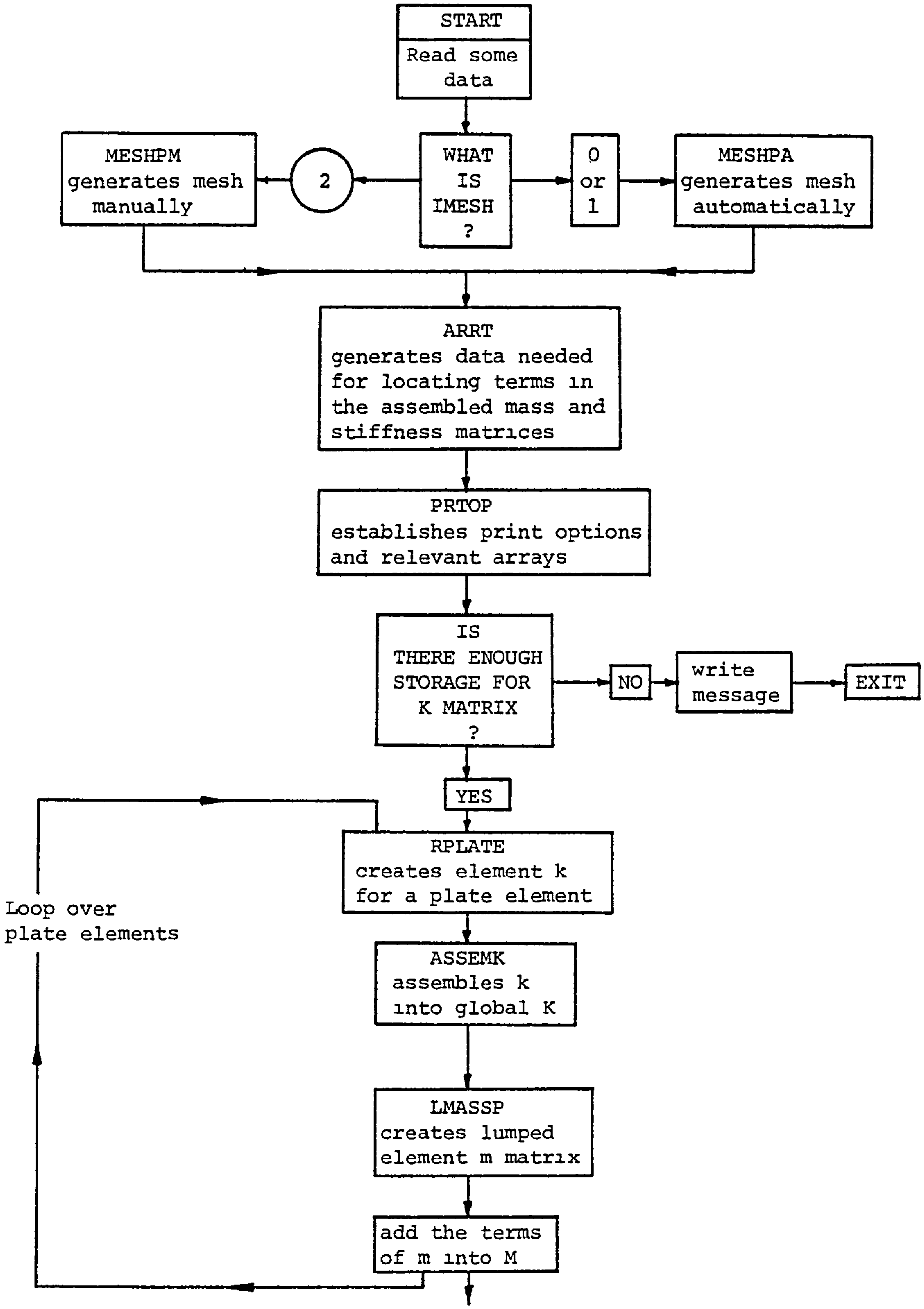



FIG. 9 FLOW CHART OF IMAINP FOR THE CIVM-PLATE 1 PROGRAM

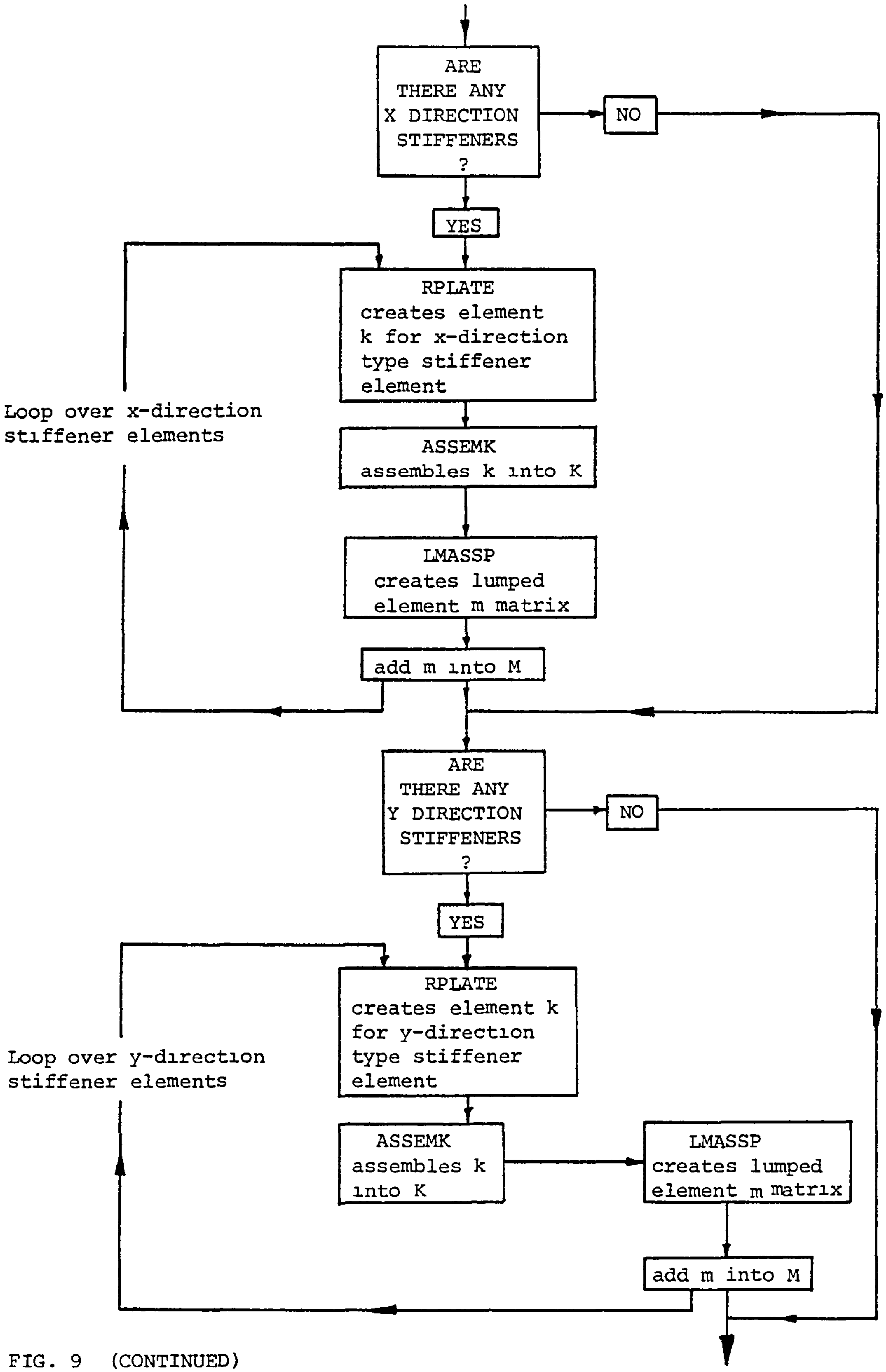



FIG. 9 (CONTINUED)

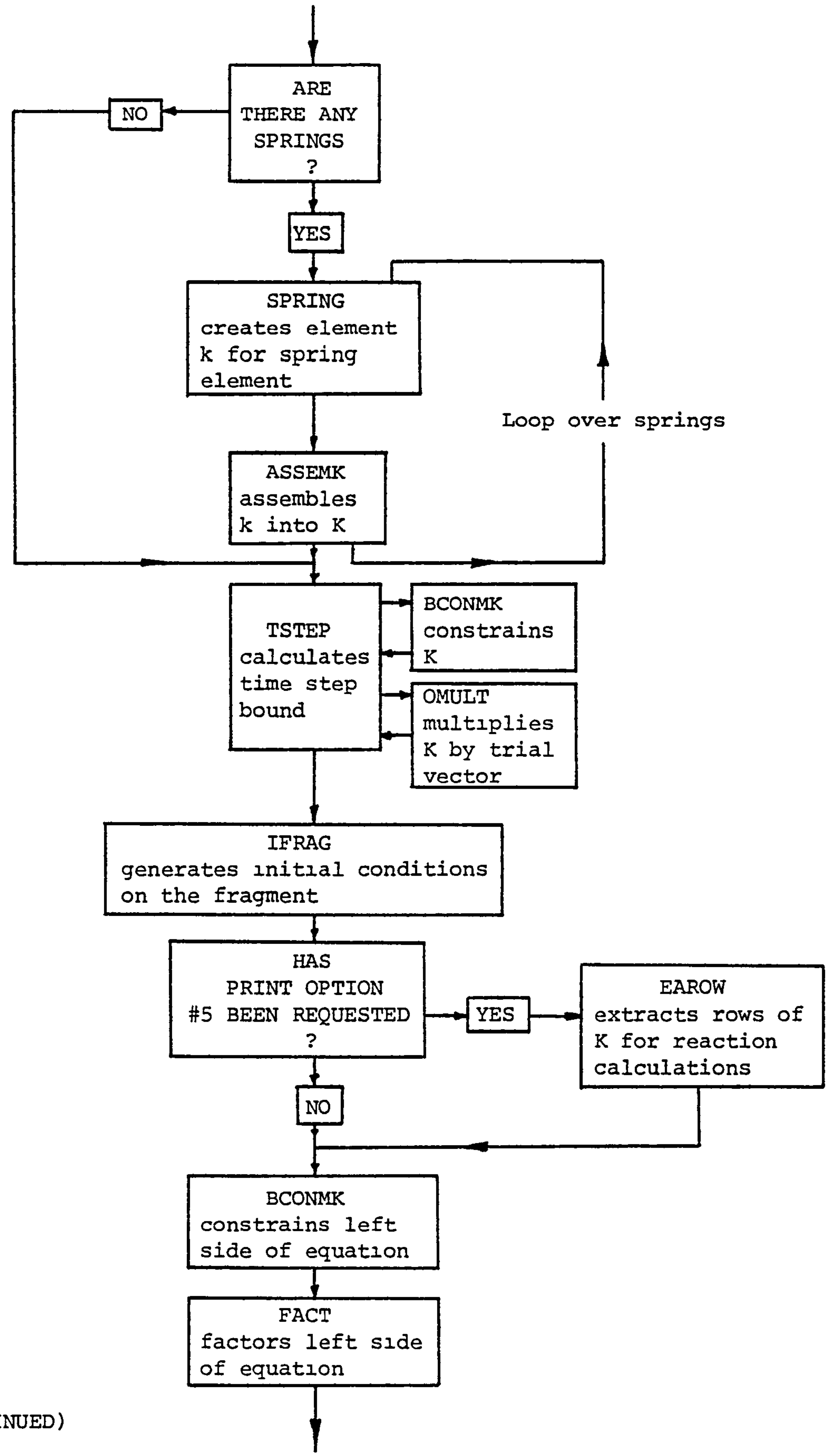



FIG. 9 (CONTINUED)

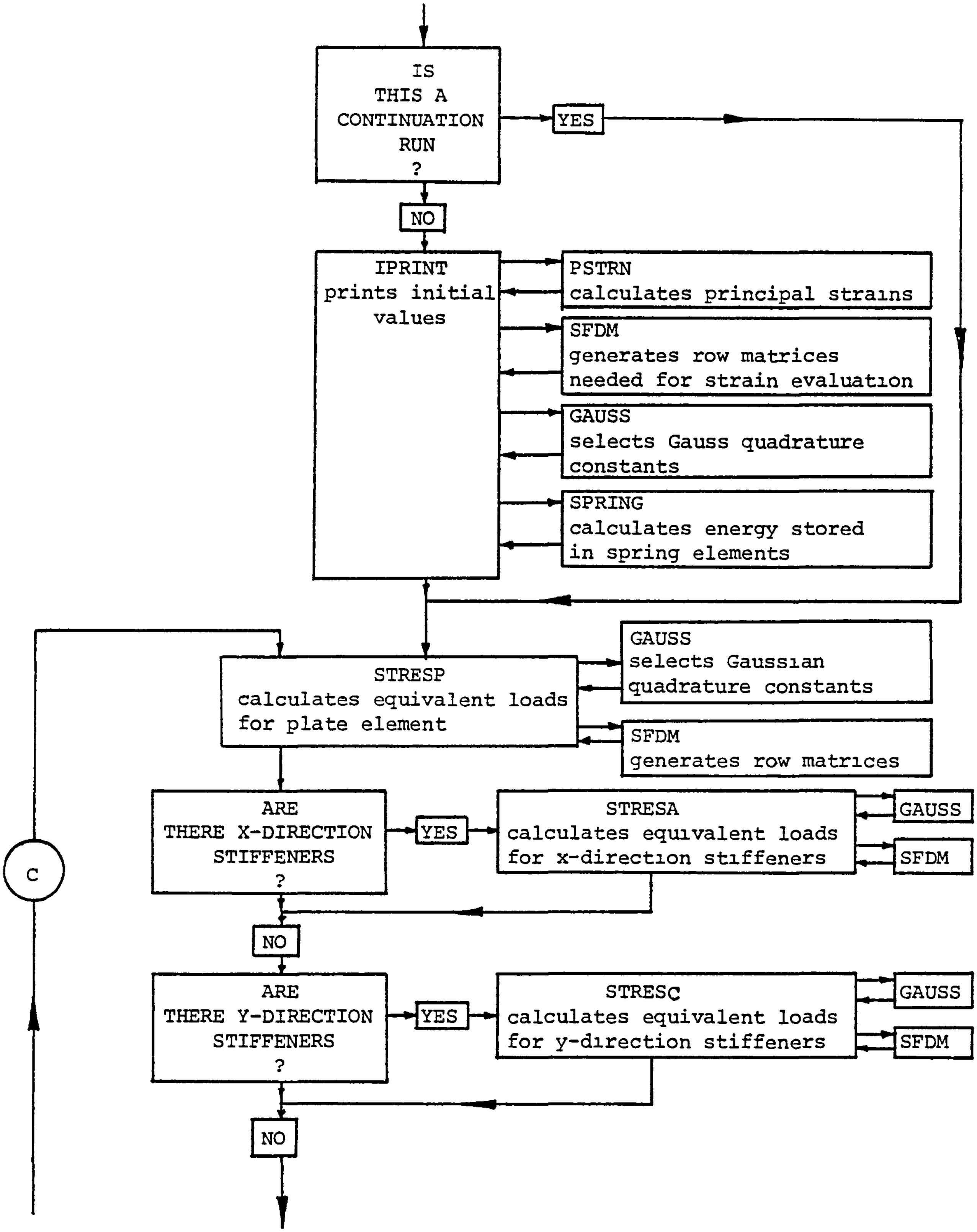



FIG. 9 (CONTINUED)

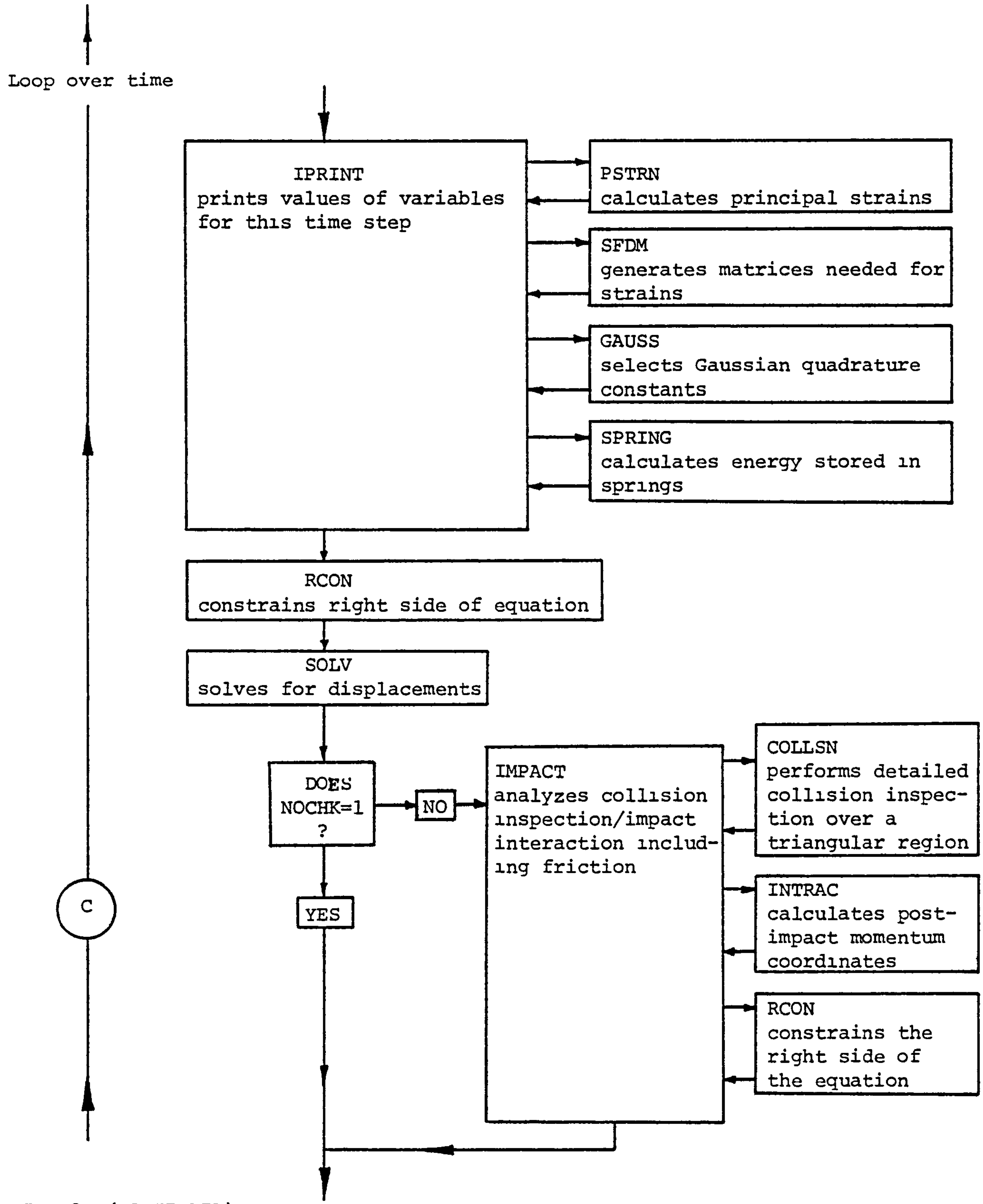



FIG. 9 (CONTINUED)

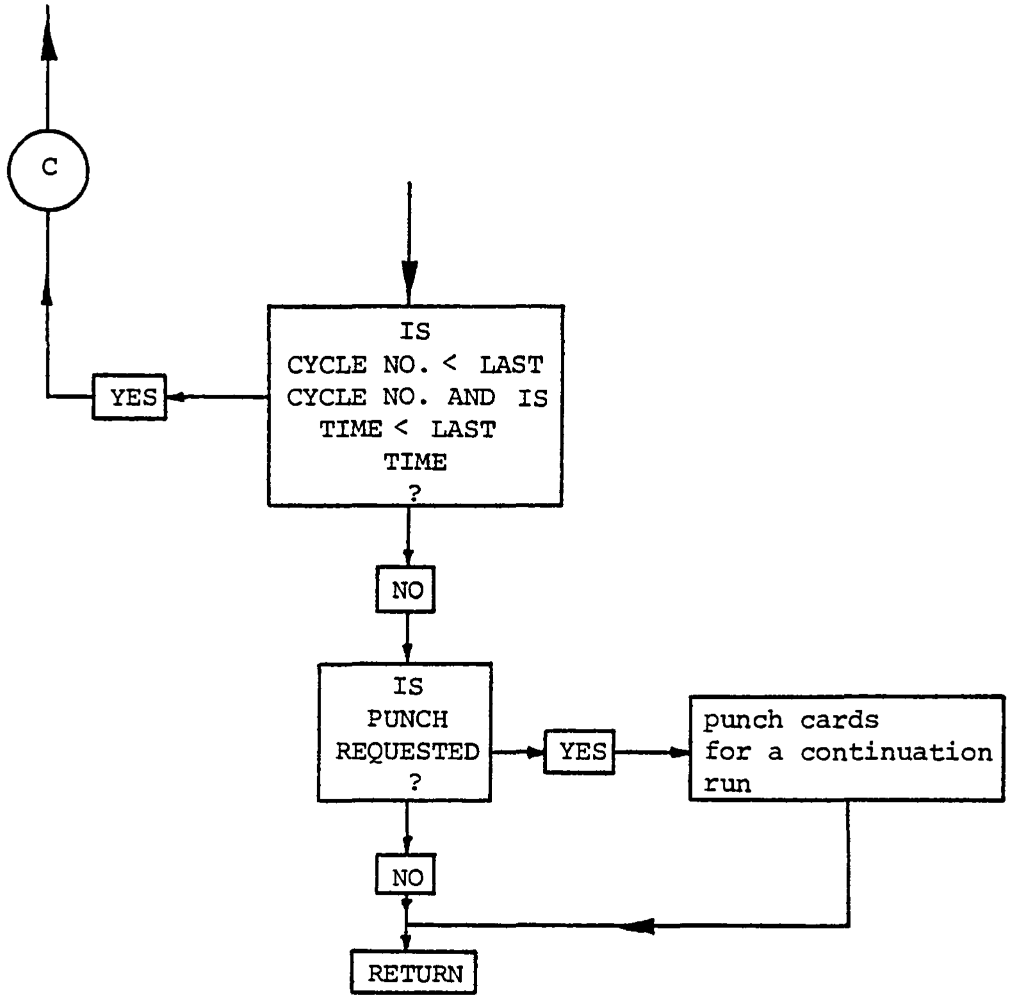



FIG. 9 (CONCLUDED)

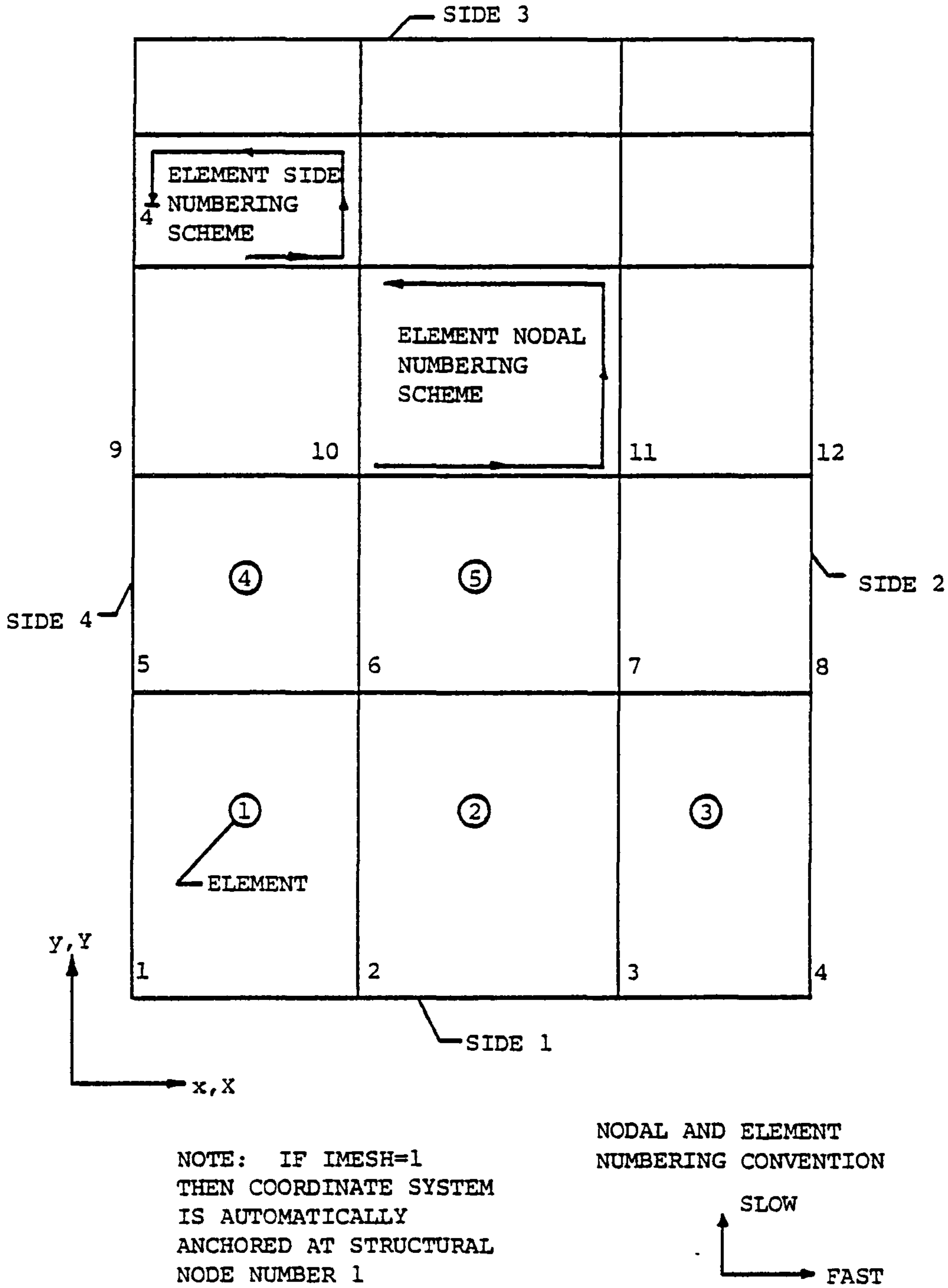



FIG. 10 NUMBERING CONVENTION ADOPTED IN THE AUTOMATED MESH GENERATION PROCEDURE

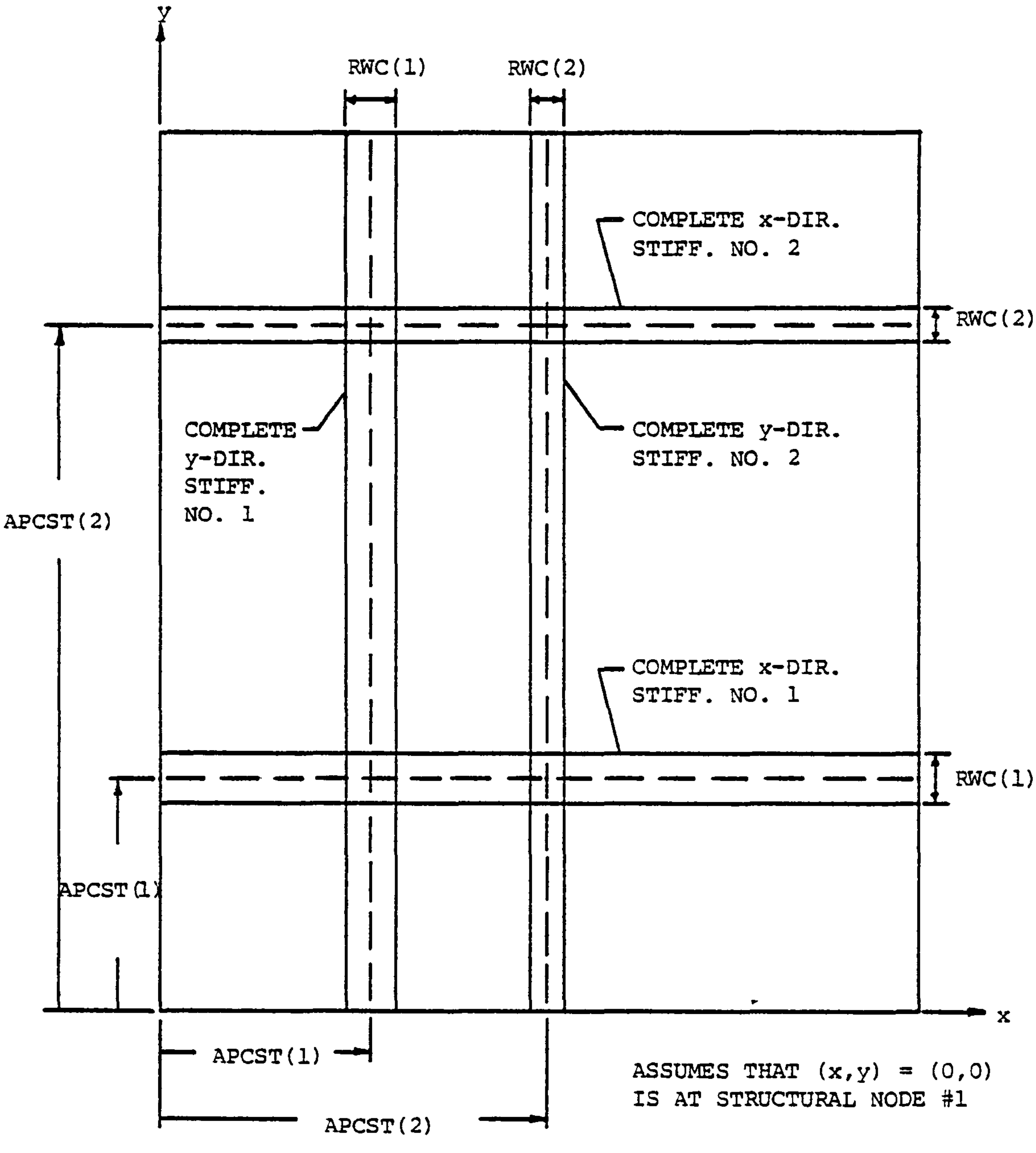
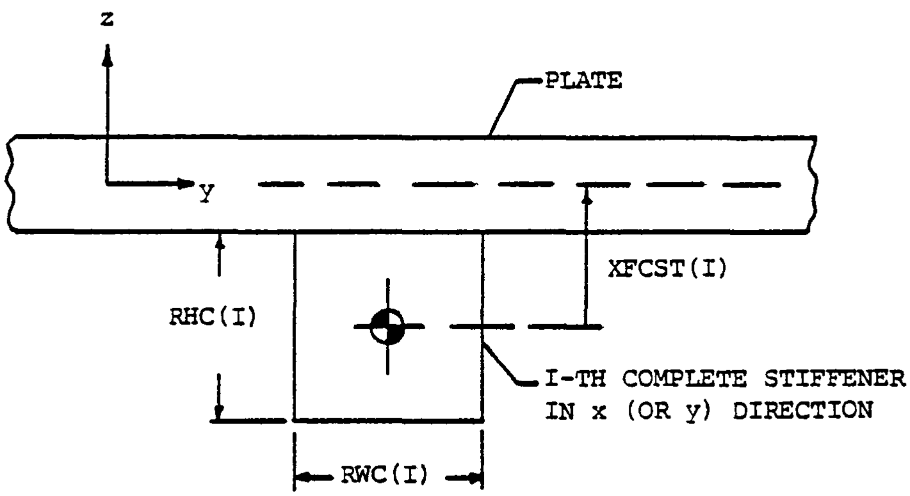



FIG. 11 ILLUSTRATIVE COMPLETE X- AND Y-DIRECTION STIFFENERS

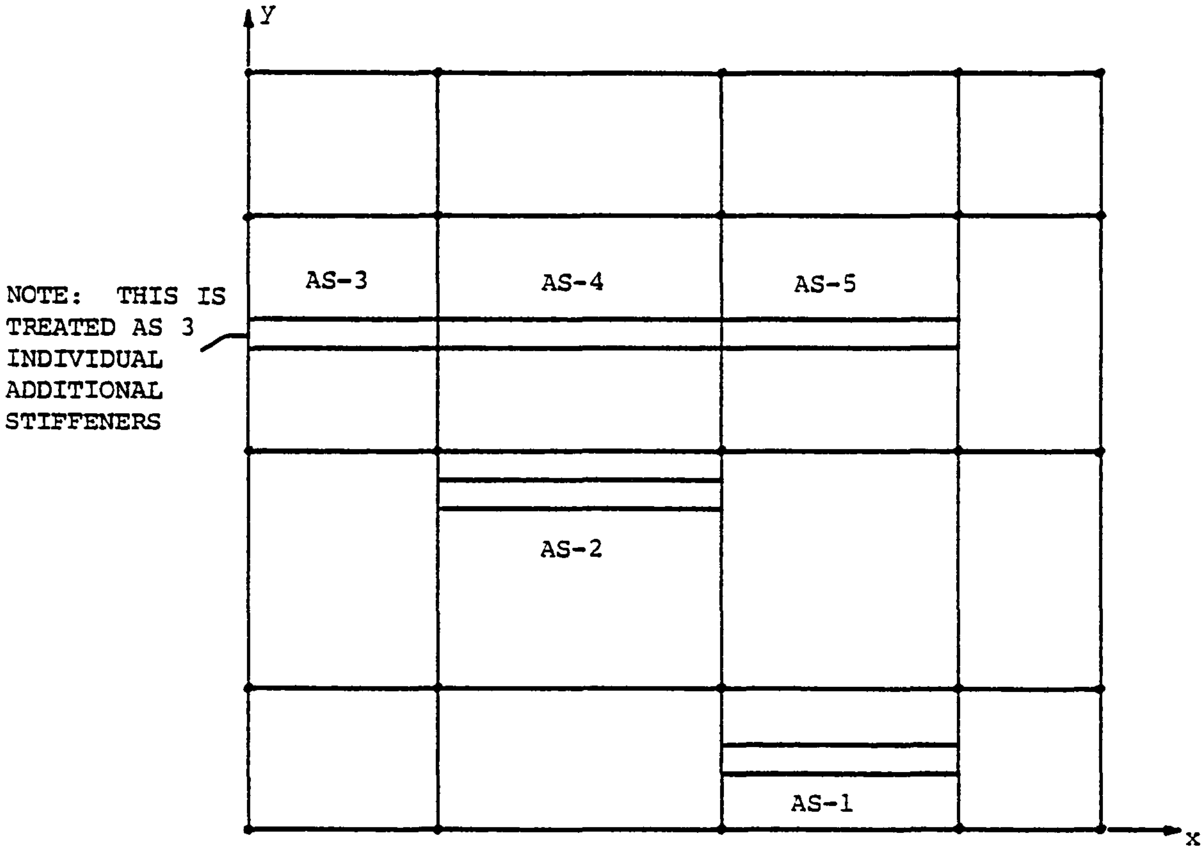



Note:

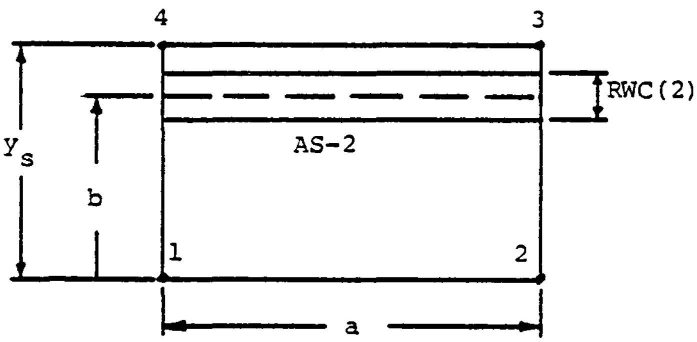
1. Stiffener has a rectangular cross-section which is constant along the stiffener.

2. If Stiffener were placed on "top" of plate, XFCST(I) would be a negative number.

FIG. 12 CROSS-SECTIONAL GEOMETRIC PROPERTIES OF STIFFENERS

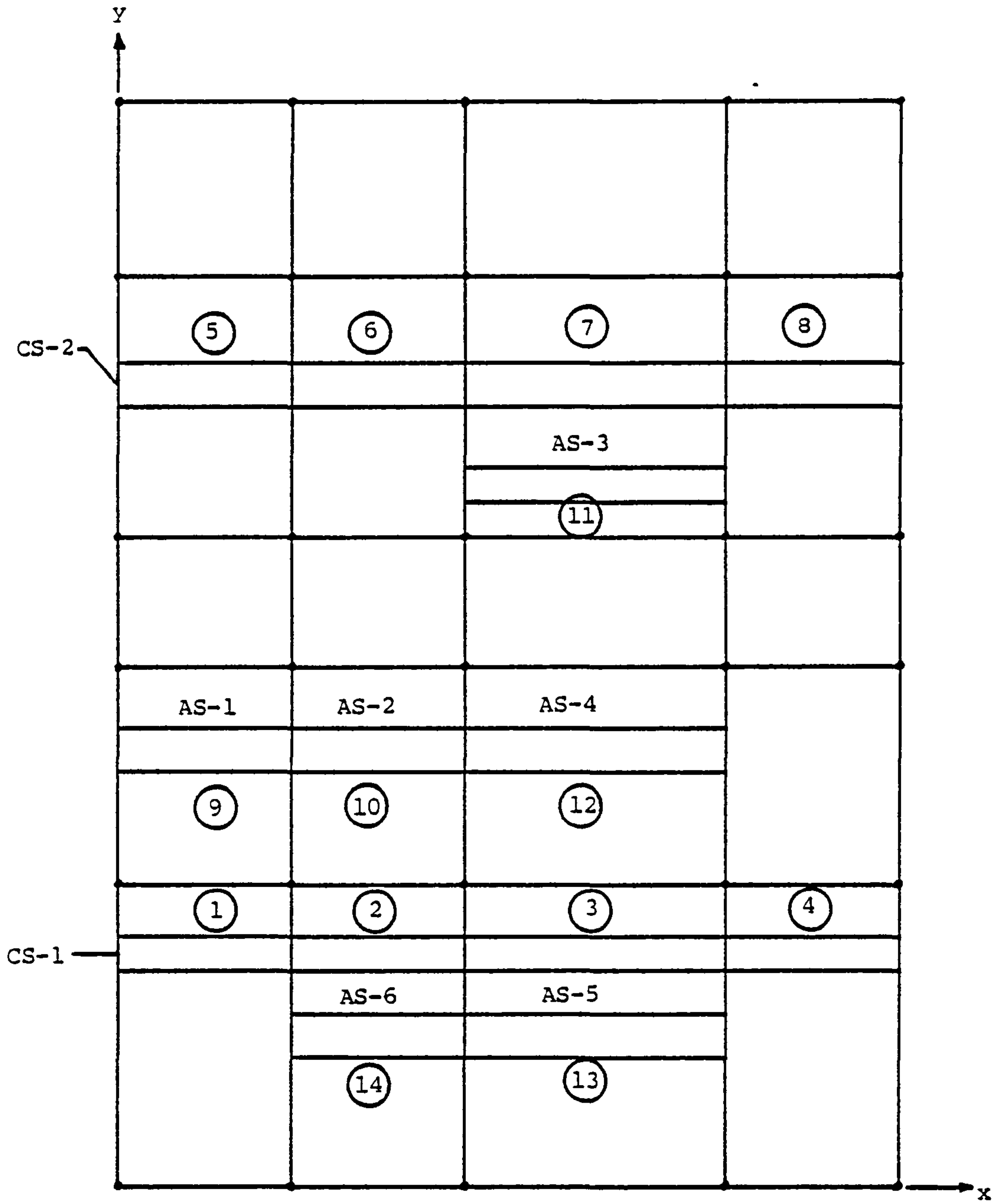



AS-n refers to Additional Stiffener Number n.




POSITION OF ADDITIONAL STIFFENER IS GIVEN IN TERMS OF THE NORMALIZED POSITION RELATIVE TO ELEMENT NODE 1: $\text{POS} = \frac{y_s}{b}$

FIG. 13 ILLUSTRATIVE X-DIRECTION ADDITIONAL STIFFENERS



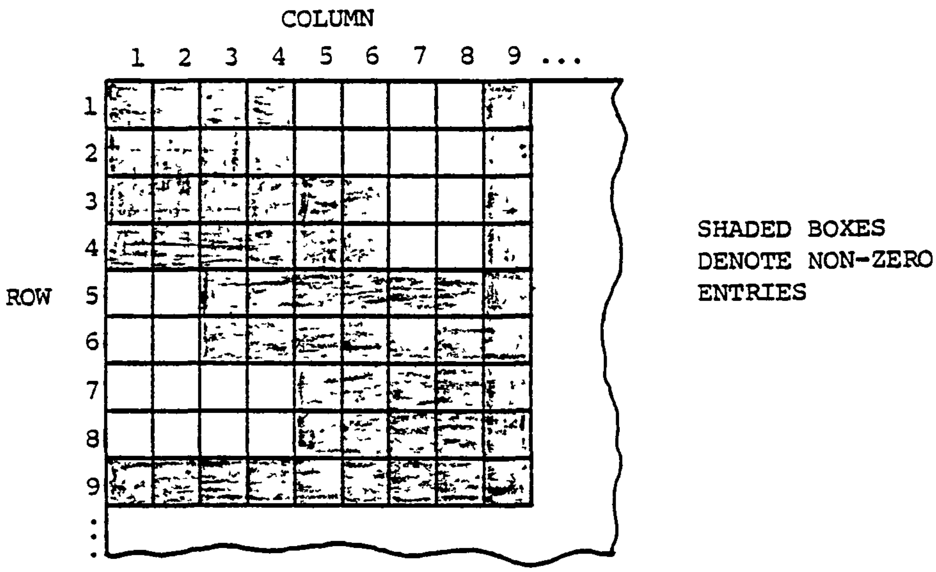

NOTATION

CS-n : n-th stiffener defined as complete stiffener.

AS-n : n-th stiffener defined as an additional stiffener.

n : Stiffener number assigned internally to each stiffener on each element.

FIG. 14 CONVENTION USED FOR INTERNAL RENUMBERING OF STIFFENERS




a. Portion of a Hypothetical Assembled Stiffness Matrix

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $K_{11}$ | $K_{21}$ | $K_{22}$ | $K_{31}$ | $K_{32}$ | $K_{33}$ | $K_{41}$ | $K_{42}$ | $K_{43}$ | $K_{44}$ | $K_{53}$ |

$K_{51}$, $K_{52}$ NOT STORED

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $K_{54}$ | $K_{55}$ | $K_{63}$ | $K_{64}$ | $K_{65}$ | $K_{66}$ | $K_{75}$ | $K_{76}$ | $K_{77}$ | $K_{85}$ | $K_{86}$ |

$K_{61}$, $K_{62}$ NOT STORED

$K_{71}$-$K_{74}$, $K_{81}$-$K_{84}$ NOT STORED

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $K_{87}$ | $K_{88}$ | $K_{91}$ | $K_{92}$ | $K_{93}$ | $K_{94}$ | $K_{95}$ | $K_{96}$ | $K_{97}$ | $K_{98}$ | $K_{99}$ | ... |

b. Storage of Assembled Stiffness Matrix (Lower Triangle) In Vector Form

FIG. 15 STORAGE OF THE ASSEMBLED STIFFNESS MATRIX

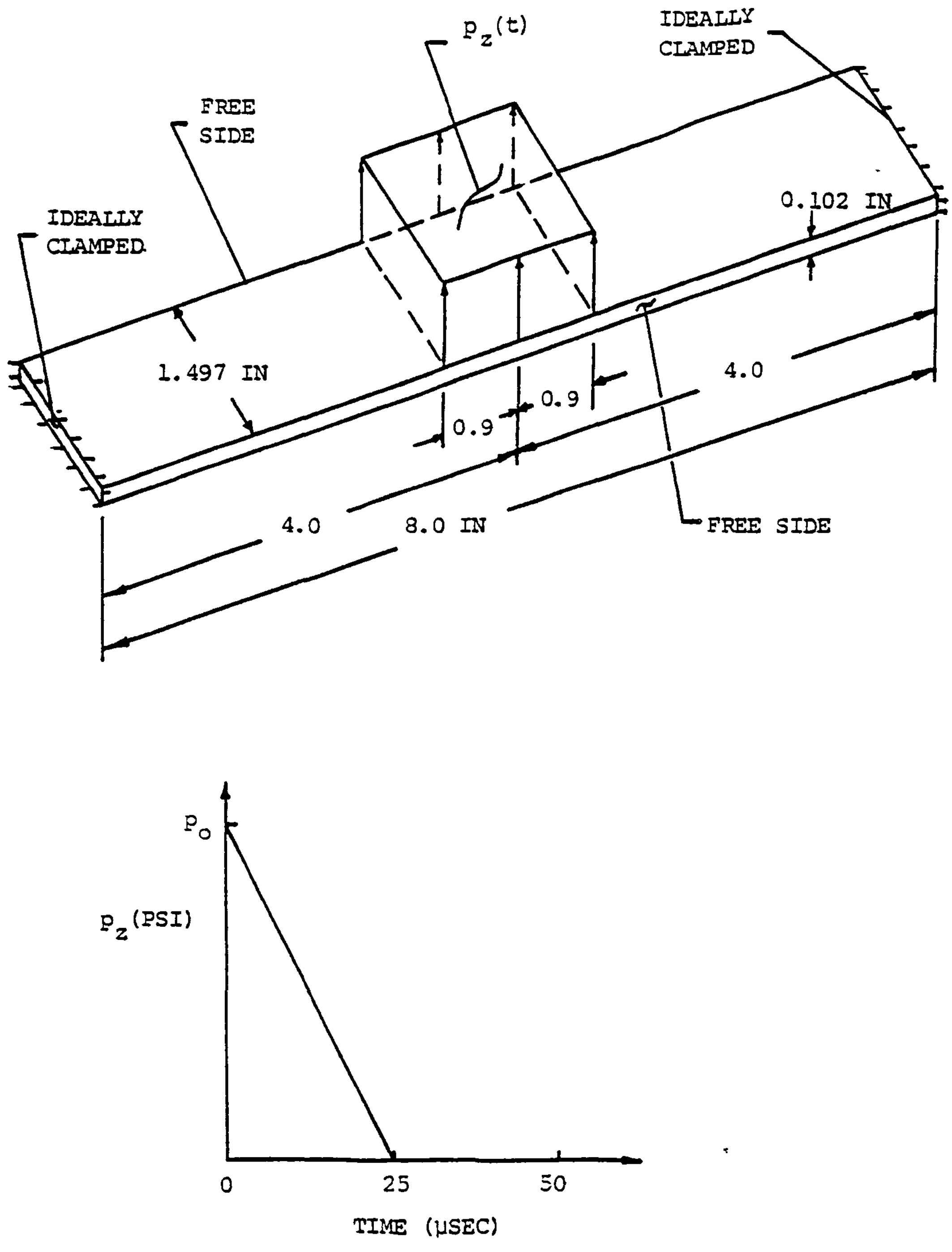
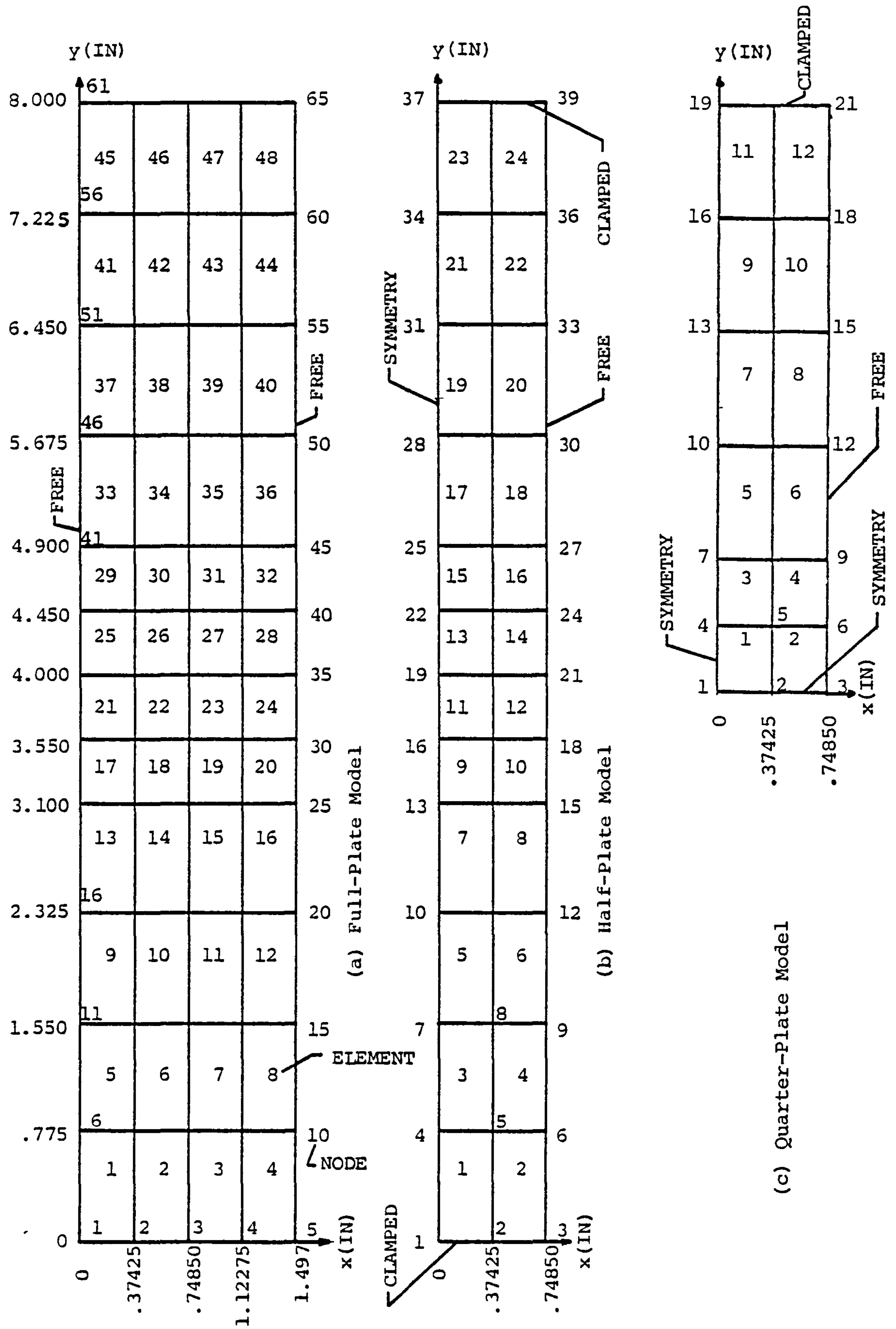



FIG. 16 SCHEMATIC OF A TRANSIENTLY-LOADED THIN NARROW PLATE

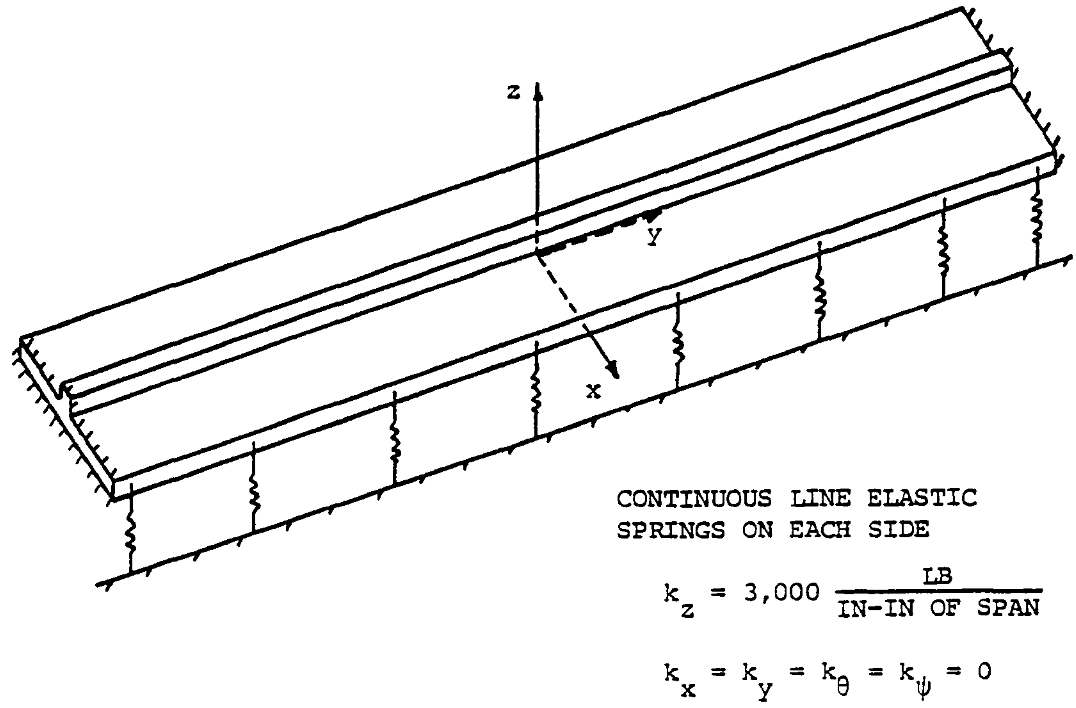


(a) Full-Plate Model

(b) Half-Plate Model

(c) Quarter-Plate Model

FIG. 17 FINITE-ELEMENT MODELS OF THE TRANSIENTLY-LOADED THIN NARROW PLATE



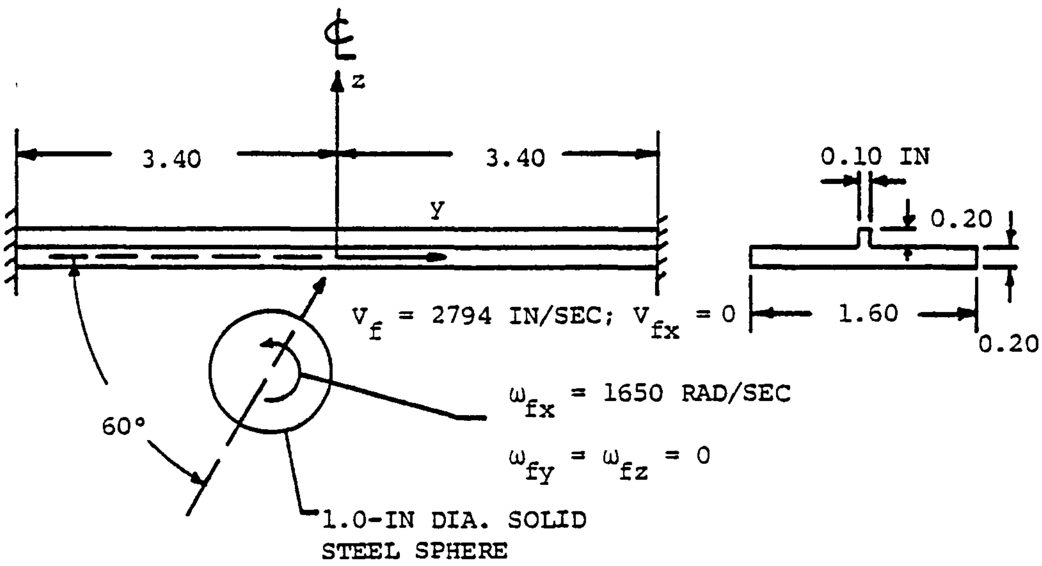




FIG. 18 SCHEMATIC OF A GEOMETRICALLY-STIFFENED SPRING-RESTRAINED THIN FLAT PLATE SUBJECTED TO STEEL-SPHERE IMPACT

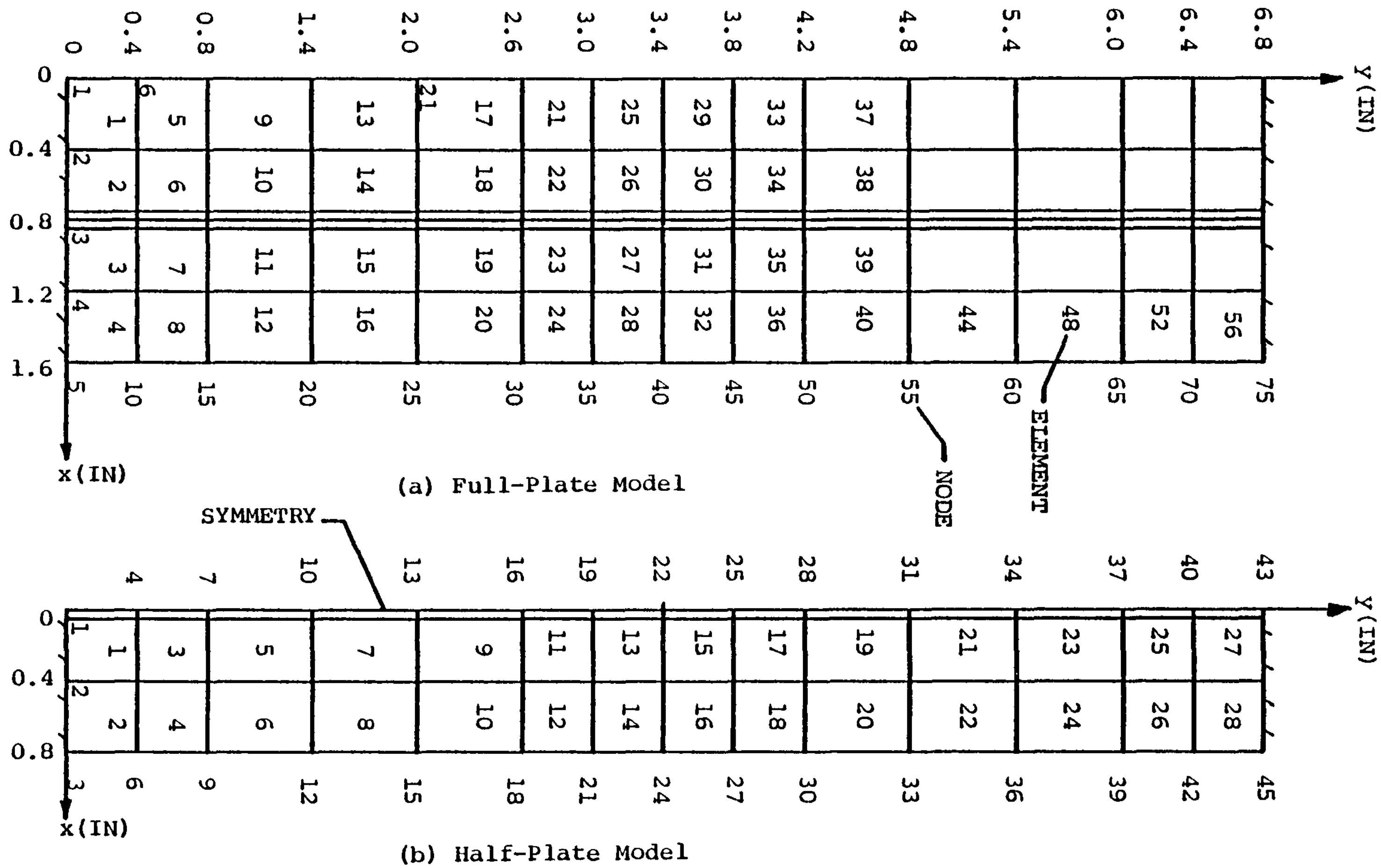



FIG. 19 FINITE-ELEMENT MODELS OF THE GEOMETRICALLY-STIFFENED SPRING-RESTRAINED THIN FLAT PLATE SUBJECTED TO STEEL-SPHERE IMPACT

## APPENDIX A

## GOVERNING EQUATIONS ON WHICH THE PLATE PROGRAM IS BASED

### A.1 Formulation for Constant-Thickness Unstiffened Rectangular Flat Plate Elements

#### A.1.1 Interpolation Functions and Strain-Displacement Relations

The PLATE program accommodates both unstiffened and longeron integrally-stiffened thin plate structures utilizing a constant-thickness plate element of rectangular planform. The present subsection deals with the unstiffened element; as discussed in Subsection A.2, a stiffened plate element is formulated by combining a "stiffener" element with the unstiffened plate element to which it is attached.

The geometry and nomenclature of a typical rectangular thin plate element are shown in Fig. 2. The element has constant thickness, h, and spanwise dimensions a and b in the x and y directions, respectively, with the origin of the element xyz coordinate system located at element node number 1.

Since the Kirchhoff hypothesis[+] is employed, the displacement field $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ may be approximated by the plate middle surface displacements u, v, and w and rotations $\theta$ and $\psi$ as follows:

$$\begin{aligned} \tilde{u}(x,y,z) &= u(x,y) - z\,\theta(x,y) \\ \tilde{v}(x,y,z) &= v(x,y) - z\,\Psi(x,y) \\ \tilde{w}(x,y,z) &= w(x,y) \end{aligned} \tag{A.1}$$

where

$$\begin{aligned} \theta(x,y) &= \frac{\partial w(x,y)}{\partial x} \\ \psi(x,y) &= \frac{\partial w(x,y)}{\partial y} \end{aligned} \tag{A.2}$$

The midsurface displacements u, v, and w are approximated within each element

[+]The Kirchhoff hypothesis is appropriate since attention herein is restricted to thin plates. It should be noted, however, that recently less restrictive plate bending elements have developed, based on Mindlin theory [24]; such elements employ independent interpolations for the transverse displacement w and for the rotations. These elements often suffer from excessive stiffness in the thin-plate limit; this "defect" has been remedied by the use of selective reduced numerical integration [25-27].

by assuming a bilinear interpolation for the inplane displacements u and v, and a bicubic Hermetian interpolation for the transverse displacement, w. The interpolations written in terms of element x,y,z coordinates are

$$u(x,y) = \alpha_1 + x\alpha_2 + y\alpha_3 + xy\alpha_4 \tag{A.3a}$$

$$v(x,y) = \alpha_5 + x\alpha_6 + y\alpha_7 + xy\alpha_8 \tag{A.3b}$$

$$\begin{aligned} w(x,y) = {} & \alpha_9 + x\alpha_{10} + y\alpha_{11} + xy\alpha_{12} + x^2\alpha_{13} + y^2\alpha_{14} \\ & + x^2y\alpha_{15} + xy^2\alpha_{16} + x^2y^2\alpha_{17} + x^3\alpha_{18} + y^3\alpha_{19} \\ & + x^3y\alpha_{20} + xy^3\alpha_{21} + x^3y^2\alpha_{22} + x^2y^3\alpha_{23} + x^3y^3\alpha_{24} \end{aligned} \tag{A.3c}$$

where $\alpha_1$, $\alpha_2$ ... $\alpha_{24}$ are unknown parameters which will be related to the generalized nodal displacements; the rotations $\theta$ and $\psi$ are evaluated by substituting Eq. A.3c into Eqs. A.2.

In order to obtain a set of generalized nodal displacements which can be related to the 24 $\alpha_i$'s, the generalized nodal displacements chosen are the parameters u, v, w, $\theta$, $\psi$, and $\chi = (\partial^2 w/\partial x \partial y)$ at each of the four corner nodes of the element.+ The nodal displacement vector, $\underset{\sim}{q}$, for the element is thus

$$\begin{aligned} \{q\}^T = \lfloor & u_1\ v_1\ w_1\ \theta_1\ \psi_1\ \chi_1 \quad u_2\ v_2\ w_2\ \theta_2\ \psi_2\ \chi_2 \\ & u_3\ v_3\ w_3\ \cdots\cdots \quad \chi_4 \rfloor \end{aligned} \tag{A.4}$$

By evaluating Eqs. A.3 and $\theta(x,y)$, $\psi(x,y)$, and $\chi(x,y)$ (obtained by differentiating Eqs. A.3) at the element nodes, a unique (invertable) relation between $\underset{\sim}{q}$ and $\underset{\sim}{\alpha}$ is obtained:

$$\underset{\sim}{q} = \underset{\sim}{G}\ \underset{\sim}{\alpha} \tag{A.5}$$

The 24 $\alpha_i$'s can be related to the 24 $q_i$'s by inversion of Eq. A.5 so that

$$\underset{\sim}{\alpha} = \underset{\sim}{G}^{-1}\ \underset{\sim}{q} \tag{A.6}$$

and the displacement interpolation (Eq. A.3) can, in principle, be written in terms of nodal generalized displacements, $\underset{\sim}{q}$; details of the calculation of

+These six generalized displacements are convenient to use because of the ways in which these quantities appear in the strain-displacement relations (Eqs. A.7 and A.8) for the Kirchhoff hypothesis.

$\underset{\sim}{G}^{-1}$ will be discussed later.

The distribution of strains $\tilde{\varepsilon}_{xx}$, $\tilde{\varepsilon}_{yy}$, and $\tilde{\varepsilon}_{xy}$ may be represented by mid-surface inplane strain components $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\varepsilon_{xy}$ and curvatures $\kappa_{xx}$, $\kappa_{yy}$, and $\kappa_{xy}$, and are written under the Kirchhoff assumption,+ as

$$\begin{aligned} \tilde{\epsilon}_{xx} &= \epsilon_{xx} - z\mathcal{K}_{xx} \\ \tilde{\epsilon}_{yy} &= \epsilon_{yy} - z\mathcal{K}_{yy} \\ \tilde{\epsilon}_{xy} &= \epsilon_{xy} - z\mathcal{K}_{xy} \end{aligned} \tag{A.7}$$

For the present element, the nonlinear strain-displacement relations of Sanders [12] for moderately small rotations are used, and are given by++

$$\begin{aligned} \epsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left[\frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\right]^2 ; \quad \mathcal{K}_{xx} = \frac{\partial^2 w}{\partial x^2} \equiv \frac{\partial \theta}{\partial x} \\ \epsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}\left[\frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\right]^2 ; \quad \mathcal{K}_{yy} = \frac{\partial^2 w}{\partial y^2} \equiv \frac{\partial \psi}{\partial y} \\ \epsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) ; \quad \mathcal{K}_{xy} = 2\frac{\partial^2 w}{\partial x\,\partial y} \equiv 2\chi \end{aligned} \tag{A.8}$$

When Eq. A.6 is substituted into the interpolation functions of Eqs. A.3, and the result substituted into Eqs. A.8, then Eqs. A.8 can be written in the form:

$$\begin{aligned} \epsilon_{xx} &= \underset{\sim}{D}_1^T \underset{\sim}{q} + \frac{1}{2}\underset{\sim}{q}^T\left(\underset{\sim}{D}_7 \underset{\sim}{D}_7^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right)\underset{\sim}{q} \\ \epsilon_{yy} &= \underset{\sim}{D}_2^T \underset{\sim}{q} + \frac{1}{2}\underset{\sim}{q}^T\left(\underset{\sim}{D}_8 \underset{\sim}{D}_8^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right)\underset{\sim}{q} \\ \epsilon_{xy} &= \underset{\sim}{D}_3^T \underset{\sim}{q} + \frac{1}{2}\underset{\sim}{q}^T\left(\underset{\sim}{D}_7 \underset{\sim}{D}_8^T + \underset{\sim}{D}_8 \underset{\sim}{D}_7^T\right)\underset{\sim}{q} \\ \mathcal{K}_{xx} &= \underset{\sim}{D}_4^T \underset{\sim}{q} \\ \mathcal{K}_{yy} &= \underset{\sim}{D}_5^T \underset{\sim}{q} \\ \mathcal{K}_{xy} &= \underset{\sim}{D}_6^T \underset{\sim}{q} \end{aligned} \tag{A.9}$$

+The Kirchhoff assumption reduces the number of "displacement variables" but introduces higher order derivatives in the formulation of plate and shell elements.

++Note that $\varepsilon_{xy}$ is the engineering strain and $\kappa_{xy}$ is the engineering twist curvature at the reference surface; these quantities are equal to twice their tensorial counterparts.

where $\underset{\sim}{D}_1, \underset{\sim}{D}_2, \ldots \underset{\sim}{D}_9$ are defined from the relations

$$\frac{\partial u}{\partial x} = \underset{\sim}{D}_1^T \underset{\sim}{q} \qquad 2\frac{\partial^2 w}{\partial x \partial y} = \underset{\sim}{D}_6^T \underset{\sim}{q}$$

$$\frac{\partial v}{\partial y} = \underset{\sim}{D}_2^T \underset{\sim}{q} \qquad \frac{\partial w}{\partial x} = \underset{\sim}{D}_7^T \underset{\sim}{q}$$

$$\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \underset{\sim}{D}_3^T \underset{\sim}{q} \qquad \frac{\partial w}{\partial y} = \underset{\sim}{D}_8^T \underset{\sim}{q}$$

$$\frac{\partial^2 w}{\partial x^2} = \underset{\sim}{D}_4^T \underset{\sim}{q} \qquad \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \underset{\sim}{D}_9^T \underset{\sim}{q}$$

$$\frac{\partial^2 w}{\partial y^2} = \underset{\sim}{D}_5^T \underset{\sim}{q} \tag{A.10}$$

The nonlinear geometric behavior of the plate material will be expressed in terms of the increment in strain from time $t_{m-1}$ to time $t_m$. Assuming that the displacements $\underset{\sim}{q}_m$ and the increment in nodal displacements, $\Delta \underset{\sim}{q}_m$ are known, the strain increments can be calculated by

$$(\Delta \tilde{\epsilon}_{xx})_m = (\Delta \epsilon_{xx})_m - z(\Delta K_{xx})_m$$

$$(\Delta \tilde{\epsilon}_{yy})_m = (\Delta \epsilon_{yy})_m - z(\Delta K_{yy})_m$$

$$(\Delta \tilde{\epsilon}_{xy})_m = (\Delta \epsilon_{xy})_m - z(\Delta K_{xy})_m \tag{A.11}$$

where

$$(\Delta \epsilon_{xx})_m = \underset{\sim}{D}_1^T \Delta \underset{\sim}{q}_m + \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_7 \underset{\sim}{D}_7^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right) \Delta \underset{\sim}{q}_m - \frac{1}{2} \Delta \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_7 \underset{\sim}{D}_7^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right) \Delta \underset{\sim}{q}_m$$

$$(\Delta \epsilon_{yy})_m = \underset{\sim}{D}_2^T \Delta \underset{\sim}{q}_m + \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_8 \underset{\sim}{D}_8^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right) \Delta \underset{\sim}{q}_m - \frac{1}{2} \Delta \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_8 \underset{\sim}{D}_8^T + \underset{\sim}{D}_9 \underset{\sim}{D}_9^T\right) \Delta \underset{\sim}{q}_m$$

$$(\Delta \epsilon_{xy})_m = \underset{\sim}{D}_3^T \Delta \underset{\sim}{q}_m + \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_7 \underset{\sim}{D}_8^T + \underset{\sim}{D}_8 \underset{\sim}{D}_7^T\right) \Delta \underset{\sim}{q}_m - \frac{1}{2} \Delta \underset{\sim}{q}_m^T \left(\underset{\sim}{D}_7 \underset{\sim}{D}_8^T + \underset{\sim}{D}_8 \underset{\sim}{D}_7^T\right) \Delta \underset{\sim}{q}_m \tag{A.12}$$

$$(\Delta \mathcal{K}_{xx})_m = \underset{\sim}{D}_4^T \, \Delta \underset{\sim}{q}_m \qquad (\Delta \mathcal{K}_{xy})_m = \underset{\sim}{D}_6^T \, \Delta \underset{\sim}{q}_m$$

$$(\Delta \mathcal{K}_{yy})_m = \underset{\sim}{D}_5^T \, \Delta \underset{\sim}{q}_m$$

and

$$\Delta \underset{\sim}{q}_m = \underset{\sim}{q}_m - \underset{\sim}{q}_{m-1} \tag{A.13}$$

A.1.2 Normalized Coordinates and Nodal Degrees of Freedom

In operations related to the element matrix formulation, it is generally convenient to deal with normalized coordinates $(\xi,\eta)$ where

$$\xi = \frac{x}{a}$$

$$\eta = \frac{y}{b} \tag{A.14}$$

Normalized nodal coordinates $(\bar{u}, \bar{v}, \bar{w}, \bar{\theta}, \bar{\psi}, \bar{\chi})$ are then defined by the relations

$$\bar{u} = u \qquad \bar{\theta} = a\,\theta$$

$$\bar{v} = v \qquad \bar{\Psi} = b\,\Psi$$

$$\bar{w} = w \qquad \bar{\chi} = a\,b\,\chi \tag{A.15}$$

and normalized new parameters $\bar{\alpha}_i$ are defined by

$$\begin{array}{lll}
\bar{\alpha}_1 = \alpha_1 & \bar{\alpha}_9 = \alpha_9 & \bar{\alpha}_{17} = a^2 b^2 \alpha_{17} \\
\bar{\alpha}_2 = a\,\alpha_2 & \bar{\alpha}_{10} = a\,\alpha_{10} & \bar{\alpha}_{18} = a^3 \alpha_{18} \\
\bar{\alpha}_3 = b\,\alpha_3 & \bar{\alpha}_{11} = b\,\alpha_{11} & \bar{\alpha}_{19} = b^3 \alpha_{19} \\
\bar{\alpha}_4 = a b\,\alpha_4 & \bar{\alpha}_{12} = a b\,\alpha_{12} & \bar{\alpha}_{20} = a^3 b\,\alpha_{20} \\
\bar{\alpha}_5 = \alpha_5 & \bar{\alpha}_{13} = a^2 \alpha_{13} & \bar{\alpha}_{21} = a b^3 \alpha_{21} \\
\bar{\alpha}_6 = a\,\alpha_6 & \bar{\alpha}_{14} = b^2 \alpha_{14} & \bar{\alpha}_{22} = a^3 b^2 \alpha_{22} \\
\bar{\alpha}_7 = b\,\alpha_7 & \bar{\alpha}_{15} = a^2 b\,\alpha_{15} & \bar{\alpha}_{23} = a^2 b^3 \alpha_{23} \\
\bar{\alpha}_8 = a b\,\alpha_8 & \bar{\alpha}_{16} = a b^2 \alpha_{16} & \bar{\alpha}_{24} = a^3 b^3 \alpha_{24}
\end{array} \tag{A.16}$$

Substituting Eqs. A.14 through A.16 into Eqs. A.3 yields the following interpolations:

$$\bar{u} = \bar{\alpha}_1 + \xi\bar{\alpha}_2 + \eta\bar{\alpha}_3 + \xi\eta\bar{\alpha}_4 \tag{A.17a}$$

$$\bar{v} = \bar{\alpha}_5 + \xi\bar{\alpha}_6 + \eta\bar{\alpha}_7 + \xi\eta\bar{\alpha}_8 \tag{A.17b}$$

$$\begin{aligned}\bar{w} = {} & \bar{\alpha}_9 + \xi\bar{\alpha}_{10} + \eta\bar{\alpha}_{11} + \xi\eta\bar{\alpha}_{12} + \xi^2\bar{\alpha}_{13} + \eta^2\bar{\alpha}_{14} \\ & + \xi^2\eta\bar{\alpha}_{15} + \xi\eta^2\bar{\alpha}_{16} + \xi^2\eta^2\bar{\alpha}_{17} + \xi^3\bar{\alpha}_{18} + \eta^3\bar{\alpha}_{19} \\ & + \xi^3\eta\bar{\alpha}_{20} + \xi\eta^3\bar{\alpha}_{21} + \xi^3\eta^2\bar{\alpha}_{22} + \xi^2\eta^3\bar{\alpha}_{23} + \xi^3\eta^3\bar{\alpha}_{24}\end{aligned} \tag{A.17c}$$

$$\begin{aligned}\bar{\theta} = {} & \bar{\alpha}_{10} + \eta\bar{\alpha}_{12} + 2\xi\bar{\alpha}_{13} + 2\xi\eta\bar{\alpha}_{15} + \eta^2\bar{\alpha}_{16} + 2\xi\eta^2\bar{\alpha}_{17} \\ & + 3\xi^2\bar{\alpha}_{18} + 3\xi^2\eta\bar{\alpha}_{20} + \eta^3\bar{\alpha}_{21} + 3\xi^2\eta^2\bar{\alpha}_{22} \\ & + 2\xi\eta^3\bar{\alpha}_{23} + 3\xi^2\eta^3\bar{\alpha}_{24}\end{aligned} \tag{A.17d}$$

$$\begin{aligned}\bar{\psi} = {} & \bar{\alpha}_{11} + \xi\bar{\alpha}_{12} + 2\eta\bar{\alpha}_{14} + \xi^2\bar{\alpha}_{15} + 2\xi\eta\bar{\alpha}_{16} + 2\xi^2\eta\bar{\alpha}_{17} \\ & + 3\eta^2\bar{\alpha}_{19} + \xi^3\bar{\alpha}_{20} + 3\xi\eta^2\bar{\alpha}_{21} + 2\xi^3\eta\bar{\alpha}_{22} + 3\xi^2\eta^2\bar{\alpha}_{23} + 3\xi^3\eta^2\bar{\alpha}_{24}\end{aligned} \tag{A.17e}$$

$$\begin{aligned}\bar{\chi} = {} & \bar{\alpha}_{12} + 2\xi\bar{\alpha}_{15} + 2\eta\bar{\alpha}_{16} + 4\xi\eta\bar{\alpha}_{17} + 3\xi^2\bar{\alpha}_{20} + 3\eta^2\bar{\alpha}_{21} \\ & + 6\xi^2\eta\bar{\alpha}_{22} + 6\xi\eta^2\bar{\alpha}_{23} + 9\xi^2\eta^2\bar{\alpha}_{24}\end{aligned} \tag{A.17f}$$

Note that $\bar{u}$ and $\bar{v}$ are bilinear whereas $\bar{w}$ is expressed as a bicubic interpolation function of $\xi$ and $\eta$. When Eqs. A.17 are evaluated at the nodal coordinates, the normalized nodal degrees of freedom should result; these conditions define the matrix $\underset{\sim}{\bar{G}}$ (see Eqs. A.5) relating $\underset{\sim}{\bar{q}}$ and $\underset{\sim}{\bar{\alpha}}$. Because of the simple rectangular geometry of the element and the use of normalized quantities, the $\bar{\alpha}_i$'s can be obtained in terms of the $\bar{q}$'s in closed form. The resulting

expressions are

$$\bar{\alpha}_1 = \bar{u}_1 \qquad \bar{\alpha}_7 = -\bar{v}_1 + \bar{v}_4$$

$$\bar{\alpha}_2 = -\bar{u}_1 + \bar{u}_2 \qquad \bar{\alpha}_8 = \bar{v}_1 - \bar{v}_2 + \bar{v}_3 - \bar{v}_4$$

$$\bar{\alpha}_3 = -\bar{u}_1 + \bar{u}_4 \qquad \bar{\alpha}_9 = \bar{w}_1$$

$$\bar{\alpha}_4 = \bar{u}_1 - \bar{u}_2 + \bar{u}_3 - \bar{u}_4 \qquad \bar{\alpha}_{10} = \bar{\theta}_1$$

$$\bar{\alpha}_5 = \bar{v}_1 \qquad \bar{\alpha}_{11} = \bar{\psi}_1$$

$$\bar{\alpha}_6 = -\bar{v}_1 + \bar{v}_2 \qquad \bar{\alpha}_{12} = \bar{\chi}_1$$

$$\begin{aligned}
\bar{\alpha}_{13} &= 3(-\bar{w}_1 + \bar{w}_2) - (2\bar{\theta}_1 + \bar{\theta}_2) \\
\bar{\alpha}_{14} &= 3(-\bar{w}_1 + \bar{w}_4) - (2\bar{\psi}_1 + \bar{\psi}_4) \\
\bar{\alpha}_{15} &= 3(-\bar{\psi}_1 + \bar{\psi}_2) - (2\bar{\chi}_1 + \bar{\chi}_2) \\
\bar{\alpha}_{16} &= 3(-\bar{\theta}_1 + \bar{\theta}_4) - (2\bar{\chi}_1 + \bar{\chi}_4) \\
\bar{\alpha}_{17} &= 9(\bar{w}_1 - \bar{w}_2 + \bar{w}_3 - \bar{w}_4) + 3(2\bar{\theta}_1 + \bar{\theta}_2 - \bar{\theta}_3 - 2\bar{\theta}_4) \\
&\quad + 3(2\bar{\psi}_1 - 2\bar{\psi}_2 - \bar{\psi}_3 - \bar{\psi}_4) + (4\bar{\chi}_1 + 2\bar{\chi}_2 + \bar{\chi}_3 + 2\bar{\chi}_4) \\
\bar{\alpha}_{18} &= 2(\bar{w}_1 - \bar{w}_2) + (\bar{\theta}_1 + \bar{\theta}_2) \\
\bar{\alpha}_{19} &= 2(\bar{w}_1 - \bar{w}_4) + (\bar{\psi}_1 + \bar{\psi}_4) \\
\bar{\alpha}_{20} &= 2(\bar{\psi}_1 - \bar{\psi}_2) + (\bar{\chi}_1 + \bar{\chi}_2) \\
\bar{\alpha}_{21} &= 2(\bar{\theta}_1 - \bar{\theta}_4) + (\bar{\chi}_1 + \bar{\chi}_4) \\
\bar{\alpha}_{22} &= 6(-\bar{w}_1 + \bar{w}_2 - \bar{w}_3 + \bar{w}_4) + 3(-\bar{\theta}_1 - \bar{\theta}_2 + \bar{\theta}_3 + \bar{\theta}_4) \\
&\quad + 2(-2\bar{\psi}_1 + 2\bar{\psi}_2 + \bar{\psi}_3 - \bar{\psi}_4) + (-2\bar{\chi}_1 - 2\bar{\chi}_2 - \bar{\chi}_3 - \bar{\chi}_4) \\
\bar{\alpha}_{23} &= 6(-\bar{w}_1 + \bar{w}_2 - \bar{w}_3 + \bar{w}_4) + 2(-2\bar{\theta}_1 - \bar{\theta}_2 + \bar{\theta}_3 + 2\bar{\theta}_4) \\
&\quad + 3(-\bar{\psi}_1 + \bar{\psi}_2 + \bar{\psi}_3 - \bar{\psi}_4) + 2(-2\bar{\chi}_1 - \bar{\chi}_2 - \bar{\chi}_3 - 2\bar{\chi}_4) \\
\bar{\alpha}_{24} &= 4(\bar{w}_1 - \bar{w}_2 + \bar{w}_3 - \bar{w}_4) + 2(\bar{\theta}_1 + \bar{\theta}_2 - \bar{\theta}_3 - \bar{\theta}_4) \\
&\quad + 2(\bar{\psi}_1 - \bar{\psi}_2 - \bar{\psi}_3 + \bar{\psi}_4) + (\bar{\chi}_1 + \bar{\chi}_2 + \bar{\chi}_3 + \bar{\chi}_4)
\end{aligned} \tag{A.18}$$

If Eqs. A.18 are written in matrix form, the $\underset{\sim}{\bar{G}}^{-1}$ matrix is obtained:

$$\underset{\sim}{\bar{\alpha}} = \underset{\sim}{\bar{G}}^{-1} \underset{\sim}{\bar{q}} \tag{A.19}$$

It is useful to note that $\underset{\sim}{\bar{G}}^{-1}$ is independent of element geometry and may be formed once and used for all elements.

The internal energy of the plate element is defined in terms of strain components, or, more specifically, midsurface strains and curvatures. It will, therefore, be necessary to relate these strain components and curvatures to the derivatives of the normalized displacements in the normalized coordinates. It can easily be seen that

$$
\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{1}{a}\frac{\partial \bar{u}}{\partial \xi} & \qquad \frac{\partial u}{\partial y} &= \frac{1}{b}\frac{\partial \bar{u}}{\partial \eta} \\
\frac{\partial v}{\partial x} &= \frac{1}{a}\frac{\partial \bar{v}}{\partial \xi} & \qquad \frac{\partial v}{\partial y} &= \frac{1}{b}\frac{\partial \bar{v}}{\partial \eta} \\
\frac{\partial \theta}{\partial x} &= \frac{1}{a^2}\frac{\partial \bar{\theta}}{\partial \xi} & \qquad \frac{\partial \psi}{\partial y} &= \frac{1}{b^2}\frac{\partial \bar{\psi}}{\partial \eta} \\
2\chi &= \frac{1}{ab}(2\bar{\chi})
\end{aligned}
\tag{A.20}
$$

from which the following matrix relation may be written

$$
\begin{Bmatrix} \dfrac{\partial u}{\partial x} \\[2mm] \dfrac{\partial v}{\partial y} \\[2mm] \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\[2mm] \dfrac{\partial \theta}{\partial x} \\[2mm] \dfrac{\partial \psi}{\partial y} \\[2mm] 2\chi \end{Bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{b} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{b} & \frac{1}{a} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{a^2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{b^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{ab}
\end{bmatrix}}_{\underset{\sim}{A}}
\begin{Bmatrix} \dfrac{\partial \bar{u}}{\partial \xi} \\[2mm] \dfrac{\partial \bar{v}}{\partial \eta} \\[2mm] \dfrac{\partial \bar{u}}{\partial \eta} \\[2mm] \dfrac{\partial \bar{v}}{\partial \xi} \\[2mm] \dfrac{\partial \bar{\theta}}{\partial \xi} \\[2mm] \dfrac{\partial \bar{\psi}}{\partial \eta} \\[2mm] 2\bar{\chi} \end{Bmatrix}
\tag{A.21}
$$

A.1.3 Definition and Calculation of Element Property Matrices

In the previous subsections, the characteristic displacement behavior of the element and the associated strain components were defined. These interpolations/relations are now used in conjunction with the concepts of work and energy to derive the various plate element property matrices.

A.1.3.1 Element Mass Matrix

The consistent mass matrix $\underset{\sim}{m}_c$ for a rectangular plate element is derived from the expression for the kinetic energy, $(T)_n$, for a typical (nth) element which is given by

$$T_n = \frac{1}{2}\int_{V_n} \rho\left(\dot{\tilde{u}}^2 + \dot{\tilde{v}}^2 + \dot{\tilde{w}}^2\right) dV$$

$$= \frac{1}{2}\int_0^b \int_0^a \int_{-\frac{h}{2}}^{+\frac{h}{2}} \rho\left[(\dot{u} - z\dot{\theta})^2 + (\dot{v} - z\dot{\psi})^2 + \dot{w}^2\right] dx\, dy\, dz \tag{A.22}$$

with $\rho$ taken as the mass per unit volume of the plate material. Introducing normalized coordinates (Eqs. A.14) and normalized displacements (Eqs. A.15), and integrating Eq. A.22 through the thickness of the element yields

$$T_n = \frac{1}{2} a b \int_0^1 \int_0^1 \lfloor \dot{\bar{u}}\ \dot{\bar{v}}\ \dot{\bar{w}}\ \dot{\bar{\theta}}\ \dot{\bar{\psi}} \rfloor [\tilde{m}] \begin{Bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{w}} \\ \dot{\bar{\theta}} \\ \dot{\bar{\psi}} \end{Bmatrix} d\xi\, d\eta \tag{A.23}$$

where $\underset{\sim}{\tilde{m}}$ is the diagonal matrix

$$[\tilde{m}] = \begin{bmatrix} \rho h & & & & \\ & \rho h & & 0 & \\ & & \rho h & & \\ & 0 & & \frac{\rho h^3}{12 a^2} & \\ & & & & \frac{\rho h^3}{12 b^2} \end{bmatrix} \tag{A.24}$$

It is reasonable to assume that the spatial variation of the velocity field is the same as that of the displacement field. From Eqs. A.17a through A.17e, the velocity field is written as

$$\begin{Bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{w}} \\ \dot{\bar{\theta}} \\ \dot{\bar{\psi}} \end{Bmatrix} = \left[ N(\xi,\eta) \right] \begin{Bmatrix} \dot{\bar{\alpha}}_1 \\ \dot{\bar{\alpha}}_2 \\ \vdots \\ \cdot \\ \dot{\bar{\alpha}}_{24} \end{Bmatrix} \tag{A.25}$$

Substitution of Eq. A.19 (valid also for velocities) into Eq. A.25 and the result into Eq. A.23 yields,

$$T_n = \frac{1}{2} \dot{\bar{\underset{\sim}{q}}}^T a\, b \left(\underset{\sim}{\bar{G}}^{-1}\right)^T \int_0^1 \int_0^1 \underset{\sim}{N}^T \underset{\sim}{\tilde{m}} \underset{\sim}{N}\, d\xi\, d\eta\, \underset{\sim}{\bar{G}}^{-1} \dot{\bar{\underset{\sim}{q}}} = \frac{1}{2} \dot{\bar{\underset{\sim}{q}}}^T \underset{\sim}{\bar{m}}\, \dot{\bar{\underset{\sim}{q}}} \tag{A.26}$$

The $\underset{\sim}{\bar{m}}$ matrix is the normalized consistent element mass matrix for the element corresponding to normalized nodal (velocity) degrees of freedom, $\dot{\bar{\underset{\sim}{q}}}$. By using Eq. A.15, the normalized $\underset{\sim}{\bar{q}}$ vector can be related to $\underset{\sim}{\dot{q}}$ by

$$\dot{\bar{\underset{\sim}{q}}} = \underset{\sim}{T}\, \underset{\sim}{\dot{q}} \tag{A.27}$$

so that by substituting Eq. A.27 into Eq. A.26, the consistent element mass matrix $\underset{\sim}{m}_c$ is defined

$$\underset{\sim}{m}_c = \underset{\sim}{T}^T \underset{\sim}{\bar{m}}\, \underset{\sim}{T} \tag{A.28}$$

where

$$\underset{\sim}{\bar{m}} = a\, b \left(\underset{\sim}{\bar{G}}^{-1}\right)^T \int_0^1 \int_0^1 \underset{\sim}{N}^T \underset{\sim}{\tilde{m}} \underset{\sim}{N}\, d\xi\, d\eta\, \underset{\sim}{\bar{G}}^{-1} \tag{A.29}$$

The integral in Eq. A.29 is evaluated numerically. It should be noted that the matrix multiplication indicated in Eq. A.28 is accomplished simply by scaling the rows and columns of the $\underset{\sim}{\bar{m}}$ matrix since $\underset{\sim}{T}$ is a diagonal matrix.

A diagonal lumped mass matrix can also be formed for the rectangular plate element. In this case, the total mass properties of the element are lumped evenly at the four nodal locations, resulting in a diagonal matrix of the form

$$\underset{\sim}{m}_L = \begin{bmatrix} \hat{\underset{\sim}{m}} & & & 0 \\ & \hat{\underset{\sim}{m}} & & \\ & & \hat{\underset{\sim}{m}} & \\ 0 & & & \hat{\underset{\sim}{m}} \end{bmatrix} \tag{A.30}$$

where $\hat{\underset{\sim}{m}}$ is the diagonal matrix;

$$\hat{\underset{\sim}{m}} = \rho \begin{bmatrix} \frac{abh}{4} & & & & 0 \\ & \frac{abh}{4} & & & \\ & & \frac{abh}{4} & & \\ & & & \frac{abh^3}{48} & \\ 0 & & & & \frac{abh^3}{48} \end{bmatrix} \tag{A.31}$$

### A.1.3.2 Element Equivalent Loads Corresponding to Nonlinear Effects

For the present plane-stress case, the expression for the variation of the work of the stresses within the element is given by+

$$(\delta U)_n = \int_{V_n} \left( \sigma_{xx}\, \delta\tilde{\epsilon}_{xx} + \sigma_{yy}\, \delta\tilde{\epsilon}_{yy} + \sigma_{xy}\, \delta\tilde{\epsilon}_{xy} \right) dV \tag{A.32}$$

where $\sigma_{xx}$ and $\sigma_{yy}$ are the inplane normal stresses in the x and y directions and $\sigma_{xy}$ is the inplane shearing stress. Considering initially-isotropic material, the elastic-plastic stress-strain relation can be expressed by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \tilde{\epsilon}_{xx} - \tilde{\epsilon}^{p}_{xx} \\ \tilde{\epsilon}_{yy} - \tilde{\epsilon}^{p}_{yy} \\ \tilde{\epsilon}_{xy} - \tilde{\epsilon}^{p}_{xy} \end{Bmatrix} \tag{A.33a}$$

or

$$\underset{\sim}{\sigma} = \underset{\sim}{C} \left( \tilde{\underset{\sim}{\epsilon}} - \tilde{\underset{\sim}{\epsilon}}^{p} \right) \tag{A.33b}$$

+For a proper energy evaluation, the $\sigma_{ij}$ stresses represent Second Piola-Kirchhoff stress components since the $\tilde{\epsilon}_{ij}$ are Green strain components [14].

When the stress-strain relation, Eq. A.33, is introduced into Eq. A.32, and then employing the strain-displacement relations of Eqs. A.7 and A.9, the expression for $(\delta U)_n$ will be in the form:

$$(\delta U)_n = \delta \underset{\sim}{q}^T \left( \underset{\sim}{k}\ \underset{\sim}{q} - \underset{\sim}{f}^{NL} \right) \tag{A.34}$$

in which $\underset{\sim}{k}$ is the element linear elastic stiffness matrix and $\underset{\sim}{f}^{NL}$ is the equivalent element force vector corresponding to nonlinear material and geometric effects, and is given by

$$\begin{aligned}
\{f^{NL}\} = &-\frac{Eh}{2(1-\nu^2)} \int_0^a \int_0^b [D_1\ D_2\ D_3] \begin{Bmatrix} (\theta^2+\chi^2) + \nu(\psi^2+\chi^2) \\ (\psi^2+\chi^2) + \nu(\theta^2+\chi^2) \\ (1-\nu)\,\theta\,\psi \end{Bmatrix} dx\,dy \\
&-\frac{Eh}{(1-\nu^2)} \int_0^a \int_0^b [D_7\ D_8\ D_9] \begin{Bmatrix} \theta(\epsilon_{xx}+\nu\epsilon_{yy}) + \left(\frac{1-\nu}{2}\right)\psi\,\epsilon_{xy} \\ \psi(\epsilon_{yy}+\nu\epsilon_{xx}) + \left(\frac{1-\nu}{2}\right)\theta\,\epsilon_{xy} \\ (1+\nu)\,\chi\,(\epsilon_{xx}+\epsilon_{yy}) \end{Bmatrix} dx\,dy \\
&+\int_0^a \int_0^b \left( [D_1\ D_2\ D_3] \begin{Bmatrix} \int_h \sigma_{xx}^p\,dz \\ \int_h \sigma_{yy}^p\,dz \\ \int_h \sigma_{xy}^p\,dz \end{Bmatrix} - [D_4\ D_5\ D_6] \begin{Bmatrix} \int_h z\,\sigma_{xx}^p\,dz \\ \int_h z\,\sigma_{yy}^p\,dz \\ \int_h z\,\sigma_{xy}^p\,dz \end{Bmatrix} \right) dx\,dy \\
&+\int_0^a \int_0^b [D_7\ D_8\ D_9] \begin{Bmatrix} \theta \int_h \sigma_{xx}^p\,dz + \psi \int_h \sigma_{xy}^p\,dz \\ \psi \int_h \sigma_{yy}^p\,dz + \theta \int_h \sigma_{xy}^p\,dz \\ \chi \int_h (\sigma_{xx}^p + \sigma_{yy}^p)\,dz \end{Bmatrix} dx\,dy
\end{aligned} \tag{A.35}$$

where

$$\begin{Bmatrix} \sigma_{xx}^p \\ \sigma_{yy}^p \\ \sigma_{xy}^p \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \tilde{\epsilon}_{xx}^p \\ \tilde{\epsilon}_{yy}^p \\ \tilde{\epsilon}_{xy}^p \end{Bmatrix} \tag{A.36a}$$

or

$$\underset{\sim}{\sigma}^p = \underset{\sim}{C}\ \underset{\sim}{\tilde{\epsilon}}^p \tag{A.36b}$$

In Eq. A.35 the first two integrals represent equivalent nodal loads arising from large deflections, and the third and fourth integrals represent equivalent nodal loads corresponding to plastic strains and associated, respectively, with the linear and nonlinear terms in the strain-displacement relations; for convenience of presentation, all of these equivalent-load-producing nonlinear effects have been collected into a single expression.

The integrals in Eq. A.35 are evaluated for each time step in the transient response prediction process because the integrands (which are functions of displacements, strains, and plastic strains) change with time as the deformation progresses; these integrations are evaluated numerically by using Gaussian quadrature, so that the stresses, strains, and strain increments need to be evaluated at a selected number of spanwise and depthwise integration stations (monitoring points in the Gaussian quadrature).

At any instant in time $t_m$, the nodal generalized displacement parameters $\underset{\sim}{q}_m$ can be calculated, and one may then employ the assumed displacement functions (Eqs. A.3) together with the strain-displacement relations (Eqs. A.9, and Eqs. A.11 through A.13) to obtain the strains and strain increments at any station within the element. Then the stresses and plastic strains are evaluated; the procedure is described briefly in the following.

The strain-hardening behavior of the material is accounted for by employing the mechanical-sublayer material-behavior model (see Subsection A.4). For each mechanical sublayer, the stresses $\underset{\sim}{\sigma}_{m-1}$ at time $t_{m-1}$ are known. Trial (superscript T) total stresses $\underset{\sim}{\sigma}_m^T$ at time $t_m$ may then be calculated by assuming that the strain increments $\Delta\underset{\sim}{\tilde{\epsilon}}_m$ from time $t_{m-1}$ to $t_m$ are entirely elastic

$$\underset{\sim}{\sigma}_m^T = \underset{\sim}{\sigma}_{m-1} + \underset{\sim}{C}\,\Delta\underset{\sim}{\tilde{\epsilon}}_m \tag{A.37}$$

The trial stresses are then substituted into the Huber-Mises-Hencky yield condition which for plane stress is

$$F_m^T = F(\underset{\sim}{\sigma}_m^T, \sigma_o) = \left[\left(\sigma_{xx}^T\right)_m^2 + \left(\sigma_{yy}^T\right)_m^2 - \left(\sigma_{xx}^T\right)_m\left(\sigma_{yy}^T\right)_m + 3\left(\sigma_{xy}^T\right)_m^2 - \sigma_o^2\right] \tag{A.38}$$

where $\sigma_o$ is the known (static or strain-rate dependent) yield stress of the

mechanical sublayer being processed.

If $F_m^T < 0$, the stress state lies within or on the yield surface. In this case, no plastic flow occurs within this sublayer during this time step; the stress state at time $t_m$ thus arises from wholly elastic behavior (in this sublayer) over the increment from $t_{m-1}$ to $t_m$ and thus

$$\underset{\sim}{\sigma}_m = \underset{\sim}{\sigma}_m^T \tag{A.39a}$$

and

$$\underset{\sim}{\epsilon}_m^p = \underset{\sim}{\epsilon}_{m-1}^p \tag{A.39b}$$

If $F_m^T \geq 0$ plastic yielding has occurred. By the flow rule of plasticity, the plastic strain increments may be related to the deviatoric stress state through a real, non-negative scalar proportionality term, $\Delta\lambda$. For the plane stress case this relation is

$$\left\{ \Delta \underset{\sim}{\epsilon}^p \right\}_m = \Delta\lambda \left\{ \begin{matrix} 2\sigma_{xx} - \sigma_{yy} \\ 2\sigma_{yy} - \sigma_{xx} \\ 6\sigma_{xy} \end{matrix} \right\}_{m-1} \tag{A.40}$$

In Eq. A.40, the (known) stress state at time $t_{m-1}$ is used. The use of this stress state instead of the trial state at time $t_m$ has been found [13] to be more accurate than the latter for finite incremental changes involving states of two-dimensional stress. The actual stresses are then calculated from

$$\underset{\sim}{\sigma}_m = \underset{\sim}{\sigma}_m^T - \Delta\lambda^* \left\{ \begin{matrix} \sigma_{xx} - \dfrac{(1-2\nu)}{3(1-\nu)} (\sigma_{xx} + \sigma_{yy}) \\ \sigma_{yy} - \dfrac{(1-2\nu)}{3(1-\nu)} (\sigma_{xx} + \sigma_{yy}) \\ \sigma_{xy} \end{matrix} \right\}_{m-1} \tag{A.41a}$$

or

$$\underset{\sim}{\sigma}_m = \underset{\sim}{\sigma}_m^T - \Delta\lambda^* \, \underset{\sim}{\hat{\sigma}}_{m-1} \tag{A.41b}$$

where
$$\Delta\lambda^* = \frac{3E}{(1+\nu)} \Delta\lambda \tag{A.41c}$$

The plastic flow parameter $\Delta\lambda$ or $\Delta\lambda^*$ is evaluated from the condition that the actual stress state must lie on the yield surface. Thus, the following condition must be satisfied:

$$F_m = F(\sigma_m, \sigma_o) = \left[(\sigma_{xx})_m^2 + (\sigma_{yy})_m^2 - (\sigma_{xx})_m(\sigma_{yy})_m + 3(\sigma_{xy})_m^2 - \sigma_o^2\right] = 0 \tag{A.42}$$

Substituting Eq. A.41a into Eq. A.42 yields the following relation for $\Delta\lambda^*$

$$\Delta\lambda^* = \frac{C}{B + \sqrt{B^2 - AC}} = \frac{B - \sqrt{B^2 - AC}}{A} \tag{A.43a}$$

where

$$A = (\hat{\sigma}_{xx})_{m-1}^2 + (\hat{\sigma}_{yy})_{m-1}^2 - (\hat{\sigma}_{xx})_{m-1}(\hat{\sigma}_{yy})_{m-1} + 3(\sigma_{xy})_{m-1}^2$$

$$\begin{aligned} B = {} & (\sigma_{xx}^T)_m(\hat{\sigma}_{xx})_{m-1} + (\sigma_{yy}^T)_m(\hat{\sigma}_{yy})_{m-1} - \tfrac{1}{2}(\sigma_{xx}^T)_m(\hat{\sigma}_{yy})_{m-1} \\ & - \tfrac{1}{2}(\sigma_{yy}^T)_m(\hat{\sigma}_{yy})_{m-1} + 3(\sigma_{xy}^T)_m(\sigma_{xy})_{m-1} \end{aligned} \tag{A.43b}$$

$$C = F_m^T$$

Also, the following requirements must be satisfied:

$$\begin{aligned} & B^2 - AC \geq 0 \\ & B + \sqrt{B^2 - AC} > 0 \end{aligned} \tag{A.43c}$$

With $\Delta\lambda^*$ (and therefore $\Delta\lambda$) determined, the stress state at time $t_m$ can be obtained from Eq. A.41a and the plastic strain increments from $t_{m-1}$ to $t_m$ can be obtained from Eq. A.40. This process is carried out for each mechanical sublayer at each spanwise and depthwise Gaussian station. Once the sublayer stresses and plastic strains in each mechanical sublayer have been determined, the stresses and plastic strains at that Gaussian station can be found from a weighted sum of the sublayer results; details of the mechanical sublayer model are given in Subsection A.4.

During the operation of the solution process for intense loading problems, instances of large strain increments can occur which sometimes may lead to an imaginary (or negative) value of $\Delta\lambda$; that is, the conditions of Eq. A.43c are violated. Since the time-step size for that particular instance cannot be economically reduced, a subincrement procedure to circumvent this difficulty as discussed in Ref. 13 can be and is used. The basic finite-time increment, $\Delta t$, is divided into a number, say L, of equal subincrements; the size of the subincrement is chosen to be sufficiently small so that a positive real value of $\Delta\lambda$ for each subincrement can be derived successively. The value of the finite-strain increments $(\Delta\tilde{\underset{\sim}{\varepsilon}})_m$ as calculated by Eq. A.11 during the time interval $t_{m-1}$ to $t_m$ are also divided into L equal parts, $(\Delta\tilde{\underset{\sim}{\varepsilon}})_m/L$. It is assumed that during each subincrement of length $\Delta t/L$ this change in strain is approximately correct. Then, by employing the previously mentioned procedure, a valid value for $\Delta\lambda$ along with stress increments and plastic strain increments are calculated for each subinterval, and in the meanwhile, the stresses and plastic strains are kept updated. The process is continued until either (a) the information needed at time $t_m$ is calculated or (b) a complex (or negative) $\Delta\lambda$ is encountered. In the latter case, the process is repeated from time $t_{m-1}$ using a larger value of L. If the stresses at time $t_m$ can be derived successfully, the solution procedure continues with L henceforth set to unity until an imaginary $\Delta\lambda$ is again encountered.

#### A.1.3.3 Element Stiffness Matrix

The preceding discussion pertains to the calculation of the equivalent element nodal force vector $\underset{\sim}{f}^{NL}$ in Eq. A.34. The remaining element property matrix defined from the variation of the work of the stresses is the linear elastic element stiffness matrix, $\underset{\sim}{k}$ (see Eq. A.34).

By substituting Eq. A.33b into Eq. A.32, the element stiffness matrix is seen to be given from the expression

$$\delta \underset{\sim}{q}^T \underset{\sim}{k} \underset{\sim}{q} = \int_{V_n} \delta \underset{\sim}{\tilde{\epsilon}}^T \underset{\sim}{C} \underset{\sim}{\tilde{\epsilon}} \, dV \tag{A.44}$$

Equation A.7 is substituted in Eq. A.44 and the integration through the thickness is performed to give

$$\delta \underset{\sim}{q}^T \underset{\sim}{k} \underset{\sim}{q} = \int_0^a \int_0^b \lfloor \delta \underset{\sim}{\epsilon}^T \; \delta \underset{\sim}{\chi}^T \rfloor \, \underset{\sim}{\hat{C}} \begin{Bmatrix} \underset{\sim}{\epsilon} \\ \underset{\sim}{\chi} \end{Bmatrix} dx\,dy \tag{A.45}$$

where

$$\underset{\sim}{\hat{C}} = \frac{E h}{1-\nu^2} \begin{bmatrix} 1 & & & & & \text{SYMMETRIC} \\ \nu & 1 & & & & \\ 0 & 0 & \frac{1-\nu}{2} & & & \\ 0 & 0 & 0 & \frac{h^2}{12} & & \\ 0 & 0 & 0 & \frac{\nu h^2}{12} & \frac{h^2}{12} & \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-\nu) h^2}{24} \end{bmatrix} \tag{A.46}$$

For computational convenience and efficiency, the stiffness matrix is first formed in the normalized (barred) system and then transformed back to the original x,y coordinate system. To this end, Eq. A.21 (which represents a transformation of strains and curvatures from the x,y system to the normalized system) is substituted in Eq. A.45 to give

$$\delta \underset{\sim}{q}^T \underset{\sim}{k} \underset{\sim}{q} = ab \int_0^1 \int_0^1 \delta \left\lfloor \frac{\partial \bar{u}}{\partial \xi} \; \frac{\partial \bar{v}}{\partial \eta} \; \frac{\partial \bar{u}}{\partial \eta} \; \frac{\partial \bar{v}}{\partial \xi} \; \frac{\partial \bar{\theta}}{\partial \xi} \; \frac{\partial \bar{\Psi}}{\partial \eta} \; 2\bar{\chi} \right\rfloor \underset{\sim}{\bar{C}} \begin{Bmatrix} \frac{\partial \bar{u}}{\partial \xi} \\ \frac{\partial \bar{v}}{\partial \eta} \\ \frac{\partial \bar{u}}{\partial \eta} \\ \frac{\partial \bar{v}}{\partial \xi} \\ \frac{\partial \bar{\theta}}{\partial \xi} \\ \frac{\partial \bar{\Psi}}{\partial \eta} \\ 2\bar{\chi} \end{Bmatrix} \tag{A.47a}$$

where

$$\bar{\underset{\sim}{C}} = \underset{\sim}{A}^T \; \hat{\underset{\sim}{C}} \; \underset{\sim}{A} \tag{A.47b}$$

The derivatives indicated in Eq. A.47a are obtained by differentiation of the interpolation assumptions given by Eqs. A.17a through A.17f:

$$\frac{\partial \bar{u}}{\partial \xi} = \bar{\alpha}_2 + \eta\,\bar{\alpha}_4$$

$$\frac{\partial \bar{v}}{\partial \eta} = \bar{\alpha}_7 + \xi\,\bar{\alpha}_8$$

$$\frac{\partial \bar{u}}{\partial \eta} = \bar{\alpha}_3 + \xi\,\bar{\alpha}_4$$

$$\frac{\partial \bar{v}}{\partial \xi} = \bar{\alpha}_6 + \eta\,\bar{\alpha}_8$$

$$\frac{\partial \bar{\theta}}{\partial \xi} = 2\bar{\alpha}_{13} + 2\eta\bar{\alpha}_{15} + 2\eta^2\bar{\alpha}_{17} + 6\xi\bar{\alpha}_{18} + 6\xi\eta\,\bar{\alpha}_{20} + 6\xi\eta^2\,\bar{\alpha}_{22} + 2\eta^3\bar{\alpha}_{23} + 6\,\xi\eta^3\,\bar{\alpha}_{24}$$

$$\frac{\partial \bar{\Psi}}{\partial \eta} = 2\,\bar{\alpha}_{14} + 2\xi\bar{\alpha}_{16} + 2\,\xi^2\,\bar{\alpha}_{17} + 6\eta\bar{\alpha}_{19} + 6\,\xi\eta\,\bar{\alpha}_{21} + 6\,\xi^3\,\bar{\alpha}_{22} + 6\,\xi^2\eta\;\bar{\alpha}_{23} + 6\;\xi^3\eta\;\bar{\alpha}_{24}$$

$$2\bar{\chi} = 2\bar{\alpha}_{12} + 4\xi\bar{\alpha}_{15} + 4\eta\,\bar{\alpha}_{16} + 8\xi\eta\;\bar{\alpha}_{17} + 6\,\xi^2\bar{\alpha}_{20} + 6\,\eta^2\,\bar{\alpha}_{21} + 12\,\xi^2\eta\;\bar{\alpha}_{22} + 12\,\xi\eta^2\;\bar{\alpha}_{23} + 18\;\xi^3\eta^2\;\bar{\alpha}_{24} \tag{A.48}$$

Equations A.48 can be written in matrix form as

$$\begin{Bmatrix} \dfrac{\partial \bar{u}}{\partial \xi} \\ \dfrac{\partial \bar{v}}{\partial \eta} \\ \dfrac{\partial \bar{u}}{\partial \eta} \\ \dfrac{\partial \bar{v}}{\partial \xi} \\ \hdashline \dfrac{\partial \bar{\theta}}{\partial \xi} \\ \dfrac{\partial \bar{\Psi}}{\partial \eta} \\ 2\bar{\chi} \end{Bmatrix} = \left[D\right] \begin{Bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \bar{\alpha}_3 \\ \vdots \\ \\ \bar{\alpha}_{24} \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{D}_1 & 0 \\ 0 & \underset{\sim}{D}_2 \end{bmatrix} \begin{Bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \vdots \\ \bar{\alpha}_8 \\ \hdashline \bar{\alpha}_9 \\ \bar{\alpha}_{10} \\ \vdots \\ \bar{\alpha}_{24} \end{Bmatrix} \tag{A.49}$$

where the 4 by 8 matrix $\mathbf{D}_1$ and the 3 by 16 matrix $\mathbf{D}_2$ are obtained from Eqs. A.48. Note that the first 4 columns of $\mathbf{D}_2$ are zero. When Eq. A.19 is substituted into Eq. A.49 and the result substituted into Eq. A.47a, the following expression is obtained:

$$\delta \mathbf{q}^T \mathbf{k}\, \mathbf{q} = \delta \bar{\mathbf{q}}^T \underbrace{\left( \bar{\mathbf{G}}^{-1^T} \underbrace{\left[ ab \int_0^1 \int_0^1 \mathbf{D}^T \bar{\mathbf{C}}\, \mathbf{D}\, d\xi\, d\eta \right]}_{\hat{\mathbf{k}}} \bar{\mathbf{G}}^{-1} \right)}_{\bar{\mathbf{k}}} \bar{\mathbf{q}} \tag{A.50}$$

where $\hat{\mathbf{k}}$ may be viewed as the "kernel" stiffness matrix in terms of the generalized displacement parameters $\bar{\alpha}_1, \bar{\alpha}_2, \bar{\alpha}_3, \ldots, \bar{\alpha}_{24}$ and is given by

$$\hat{\mathbf{k}} = ab \int_0^1 \int_0^1 \mathbf{D}^T \bar{\mathbf{C}}\, \mathbf{D}\, d\xi\, d\eta \tag{A.51a}$$

and $\bar{\mathbf{k}}$ is the "kernel" stiffness matrix in terms of generalized nodal displacement parameters $\bar{\mathbf{q}}$ in the normalized system and is given by

$$\bar{\mathbf{k}} = (\bar{\mathbf{G}}^{-1})^T\, \hat{\mathbf{k}}\, \bar{\mathbf{G}}^{-1} \tag{A.51b}$$

Finally, the normalized nodal displacement $\bar{\mathbf{q}}$ can be related to the generalized nodal displacement parameters $\mathbf{q}$ in the form

$$\bar{\mathbf{q}} = \mathbf{T}\, \mathbf{q} \tag{A.52a}$$

where the transformation matrix $\mathbf{T}$ is a diagonal matrix and can be written in the form

$$\mathbf{T} = \begin{bmatrix} [t] & & & \\ & [t] & & 0 \\ & & [t] & \\ 0 & & & [t] \end{bmatrix} \tag{A.52b}$$

and the diagonal sub-matrix $\mathbf{t}$ is obtained from Eqs. A.15 and is given by

$$\underset{\sim}{t} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ & & & a & & \\ & 0 & & & b & \\ & & & & & ab \end{bmatrix} \qquad \text{(A.52c)}$$

The linear elastic element stiffness matrix, $\underset{\sim}{k}$, is defined by substituting Eq. A.52a into Eq. A.50 to give

$$\underset{\sim}{k} = \underset{\sim}{T}^{T} \, \underset{\sim}{\bar{k}} \, \underset{\sim}{T} \qquad \text{(A.53)}$$

In practice $\underset{\sim}{k}$ (Eq. A.51a) is first formed with the indicated area integration performed numerically by using Gaussian quadrature. Next $\underset{\sim}{\bar{k}}$ is formed from Eq. A.51b. It is important to recall that $\underset{\sim}{\bar{G}}^{-1}$ is formed directly from Eqs. A.18 (no inversion is performed) and that $\underset{\sim}{\bar{G}}^{-1}$ is independent of element geometry, and is thus identical for all elements; in the PLATE program $\underset{\sim}{\bar{G}}^{-1}$ is defined through a DATA statement. Finally, $\underset{\sim}{k}$ is formed from Eq. A.53. Since $\underset{\sim}{T}$ is a diagonal matrix, the matrix multiplications indicated in Eq. A.53 simply scale the rows and columns of $\underset{\sim}{\bar{k}}$ to obtain $\underset{\sim}{k}$; this more efficient scaling operation is used in place of the matrix multiplications in Eq. A.53.

#### A.1.3.4 Element Nodal Loads Corresponding to External Forces

The element nodal force vector corresponding to prescribed externally applied forces is derived from the variation of the work done by these forces, $(\delta W)_n$, over the nth element. Since the user is required to write a subroutine which generates the force vector for the PLATE program, a more detailed discussion of this element property vector will be provided to enhance the user's understanding of the concepts involved.

The nodal force vector may be obtained by one of two general methodologies. In the first approach, the variation of the work of the external forces is evaluated (by integration) using both the spatial distribution of the external forces as well as the spatial distribution of the pertinent displacements (as given by the interpolation functions of Eqs. A.3); a nodal force vector derived in this fashion will be terned a consistent (or work-equivalent) force vector.

In the second approach, the interpolated spatial distribution of the displacements is ignored; instead, the displacement behavior is assumed to be either (1) constant over each quarter (or half-side) of the plate and equal to the value of the displacement at the node associated with that quarter (or half-side) of the plate, or (2) constant over the whole plate element (or element side) and equal to the average of the four (or two) nodal values; a nodal force vector derived in this fashion will be termed a <u>lumped</u> force vector. The formulation of a consistent nodal force vector will be discussed first.

Three types of force distribution are evident: (1) line forces (tractions) distributed along element boundaries, (2) area forces which are distributed over the midsurface of the plate (e.g. transverse loading on a plate), and (3) volume forces which are distributed throughout the plate element. The first type is rigorously included in all statements of Virtual Work and the third type may be treated in the same fashion as a distributed body force. The second type may be included as a special case of a distributed body force in which no variation occurs through the thickness.

The work (at a discrete time $t_m$) of the external forces for a typical nth element, $(W)_n$, is given by

$$(W)_n = \int_{S_{\sigma_n}} \left( F_x^L \tilde{u} + F_y^L \tilde{v} + F_z^L \tilde{w} \right)_{t_m} dA + \int_{A_{\sigma_n}} \left( F_x^A u + F_y^A v + F_z^A w \right)_{t_m} dA + \int_{V_n} \left( F_x^B \tilde{u} + F_y^B \tilde{v} + F_z^B \tilde{w} \right)_{t_m} dV \qquad \text{(A.54)}$$

where

$F_x^L, F_u^L, F_z^L$ = Components of the edge (line) force in the x,y, and z directions, which are in general a function of the element side coordinate and thickness ($lb/in^2$).

$F_x^A, F_y^A, F_z^A$ = Components of the force acting over the plate surface in the x, y, and z directions. Each component may, in general, be a function of x and y only ($lb/in^2$).

$F_x^B, F_y^B, F_z^B$ = Components (in the x, y, and z directions of the body force, or any analogous force which is distributed throughout the element volume (lb/in$^3$).

The distribution of the forces is assumed to be known. Equations A.1 may be substituted into Eq. A.54 and the result written in the following matrix form

$$(W)_n = \int_0^{\ell} \int_{-\frac{h}{2}}^{\frac{h}{2}} \lfloor u \; v \; w \; \theta \; \psi \rfloor \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & 0 & 0 \\ 0 & -z & 0 \end{bmatrix} \begin{Bmatrix} F_x^L \\ F_y^L \\ F_z^L \end{Bmatrix}_{t_m} ds\, dz$$

$$+ \int_0^a \int_0^b \lfloor u \; v \; w \rfloor \begin{Bmatrix} F_x^A \\ F_y^A \\ F_z^A \end{Bmatrix} dx\, dy$$

$$+ \int_0^a \int_0^b \int_{-\frac{h}{2}}^{\frac{h}{2}} \lfloor u \; v \; w \; \theta \; \psi \rfloor \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & 0 & 0 \\ 0 & -z & 0 \end{bmatrix} \begin{Bmatrix} F_x^B \\ F_y^B \\ F_z^B \end{Bmatrix} dx\, dy\, dz \tag{A.55}$$

where $\ell$ in the first integral is the length of the element side on which the force is applied. It is generally more convenient to perform calculations in the normalized coordinate system, so that Eq. A.55 should be written as

$$(W)_n = \ell \int_0^1 \int_{-\frac{h}{2}}^{\frac{h}{2}} \lfloor \bar{u} \; \bar{v} \; \bar{w} \; \bar{\theta} \; \bar{\psi} \rfloor \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{z}{a} & 0 & 0 \\ 0 & -\frac{z}{b} & 0 \end{bmatrix} \begin{Bmatrix} F_x^L \\ F_y^L \\ F_z^L \end{Bmatrix} ds\, dz$$

$$+ ab \int_0^1 \int_0^1 \lfloor \bar{u} \; \bar{v} \; \bar{w} \rfloor \begin{Bmatrix} F_x^A \\ F_y^A \\ F_z^A \end{Bmatrix} d\xi\, d\eta$$

$$+ ab \int_0^1 \int_0^1 \lfloor \bar{u} \; \bar{v} \; \bar{w} \; \bar{\theta} \; \bar{\psi} \rfloor \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{z}{a} & 0 & 0 \\ 0 & -\frac{z}{b} & 0 \end{bmatrix} \begin{Bmatrix} F_x^B \\ F_y^B \\ F_z^B \end{Bmatrix} d\xi\, d\eta\, dz \tag{A.56}$$

For the first integral in Eq. A.56, displacements along the boundary are interpolated by using Eqs. A.17a through A.17e to yield a matrix relation in the form

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \\ \bar{\theta} \\ \bar{\psi} \end{Bmatrix} = \left[ N^{L}(s) \right] \begin{Bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{\alpha}_{24} \end{Bmatrix} \qquad \text{(A.57)}$$

The matrix $\underset{\sim}{N}^{L}(s)$ is obtained by evaluating the interpolation assumptions along the element boundary on which the force is applied. The following table may be used to put the interpolation of Eqs. A.17a through A.17e into the proper form:

| Element Side Number | Replace $\xi$ by | Replace $\eta$ by |
|---|---|---|
| 1 | s | 0 |
| 2 | 1 | s |
| 3 | s | 1 |
| 4 | 0 | s |

The force distribution in the first integral of Eq. A.56 must also be given as functions $F_x^L(s,z)$, $F_y^L(s,z)$, and $F_z^L(s,z)$.

The displacement in the second integral of Eq. A.56 may be expressed in terms of displacement parameters $\bar{\alpha}_1$, $\bar{\alpha}_2$, ... $\bar{\alpha}_{24}$ in matrix form

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} = \left[ N^{3}(\xi,\eta) \right] \begin{Bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \vdots \\ \bar{\alpha}_{24} \end{Bmatrix} \qquad \text{(A.58)}$$

where the interpolation matrix $N^3(\xi,\eta)$, is obtained by writing Eqs. A.17a, A.17b, and A.17c in matrix form. Also, the force distribution should be given as functions $F_x^A(\xi,\eta)$, $F_y^A(\xi,\eta)$, and $F_z^A(\xi,\eta)$.

The displacements in the final integral of Eq. A.56 may be interpolated in terms of $\bar{\alpha}_1$, $\bar{\alpha}_2$, ... $\bar{\alpha}_{24}$ in the matrix form

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \\ \bar{\theta} \\ \bar{\Psi} \end{Bmatrix} = \left[ N(\xi,\eta) \right] \begin{Bmatrix} \bar{\alpha}_1 \\ \bar{\alpha}_2 \\ \vdots \\ \bar{\alpha}_{24} \end{Bmatrix} \tag{A.59}$$

where $\underset{\sim}{N}(\xi,\eta)$ is obtained by writing Eqs. A.17a through A.17e in matrix form. The force distributions should be given as functions $F_x^B(\xi,\eta,z)$, $F_y^B(\xi,\eta, z)$, and $F_z^B(\xi,\eta,z)$.

If Eqs. A.57, A.58, and A.59 are substituted into Eq. A.56, the work of the external forces can be put in the form

$$(W)_n = \lfloor \bar{\alpha}_1 \ \bar{\alpha}_2 \ \cdots \ \bar{\alpha}_{24} \rfloor \underset{\sim}{\hat{f}} \tag{A.60}$$

where the vector $\underset{\sim}{\hat{f}}$ is given by

$$\underset{\sim}{\hat{f}} = \ell \int_0^1 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ N^L(s) \right]^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{z}{a} & 0 & 0 \\ 0 & -\frac{z}{b} & 0 \end{bmatrix} \begin{Bmatrix} F_x^L(s,z) \\ F_y^L(s,z) \\ F_z^L(s,z) \end{Bmatrix}_{t_m} ds\, dz$$

$$+ ab \int_0^1 \int_0^1 \left[ N^3(\xi,\eta) \right]^T \begin{Bmatrix} F_x^A(\xi,\eta) \\ F_y^A(\xi,\eta) \\ F_z^A(\xi,\eta) \end{Bmatrix}_{t_m} d\xi\, d\eta$$

$$+ ab \int_0^1 \int_0^1 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ N(\xi,\eta) \right]^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{z}{a} & 0 & 0 \\ 0 & -\frac{z}{b} & 0 \end{bmatrix} \begin{Bmatrix} F_x^B(\xi,\eta,z) \\ F_y^B(\xi,\eta,z) \\ F_z^B(\xi,\eta,z) \end{Bmatrix}_{t_m} d\xi\, d\eta\, dz \tag{A.61}$$

In order to transform Eq. A.60 into a form containing the generalized nodal displacements, $\underset{\sim}{q}$ the displacement parameters $\bar{\alpha}_1$, $\bar{\alpha}_2$, ... $\bar{\alpha}_{24}$ are first related to the normalized nodal displacements, $\underset{\sim}{\bar{q}}$, by Eq. A.19, and the normalized nodal displacements $\underset{\sim}{\bar{q}}$ are then related to the generalized nodal displacements, $\underset{\sim}{q}$, by Eq. A.52a. The final form of Eq. A.60 is then

$$(W)_n = \underset{\sim}{q}^T \underset{\sim}{f}_c \tag{A.62}$$

where $\underset{\sim}{f}_c$ is the consistent element nodal force vector corresponding to externally-applied forces and is related to $\hat{\underset{\sim}{f}}$ (Eq. A.61) by

$$\underset{\sim}{f}_c = \underset{\sim}{T} \left(\bar{\underset{\sim}{G}}^{-1}\right)^T \hat{\underset{\sim}{f}} \tag{A.63}$$

Once the spatial distribution of the forces over the element are known, they are substituted into Eq. A.61 and the integration performed to give $\hat{\underset{\sim}{f}}$; in many cases, the force distributions will be sufficiently simple so that these integrations can be performed analytically and a simple algorithm written to generate $\hat{\underset{\sim}{f}}$ for each element. Alternatively, $\hat{\underset{\sim}{f}}$ may be evaluated by using numerical integration (see, for example, Ref. 10 for a detailed discussion of numerical integration applied to the finite-element method).

No mention has yet been made of the time domain. As will be shown in Subsection A.3, in order to obtain the timewise solution of the governing equations of motion for the structure, the nodal force vector corresponding to externally-applied forces must be specified at discrete time instants, $t_m$. Thus Eqs. A.54 through A.63 refer to the work of the externally-applied forces <u>at a particular instant in time</u> when the <u>spatial</u> distributions of the forces are $F_x^L(s,z)$, $F_y^L(s,z)$, $F_z^L(s,z)$, $F_x^A(\xi,\eta)$, $F_y^A(\xi,\eta)$, $F_z^A(\xi,\eta)$, etc. In general, these spatial distributions may change at each discrete time instant and thus $\underset{\sim}{f}_c$ must be re-evaluated for each element at each time instant using the spatial distributions of the external forces at <u>that time instant</u>.

Some simplification of this calculation may be realized if the spatial and temporal dependence of the external forces can be uncoupled. For simplicity, this simplification will be illustrated only for the second integral of $\hat{\underset{\sim}{f}}$ (Eq. A.61); but it should be noted that the development holds for all integrals required to calculate $\hat{\underset{\sim}{f}}$. Assume, for example, that

$$\begin{Bmatrix} F_x^A \\ F_y^A \\ F_z^A \end{Bmatrix}_t = \begin{bmatrix} g_1(t) & 0 & 0 \\ 0 & g_2(t) & 0 \\ 0 & 0 & g_3(t) \end{bmatrix} \begin{Bmatrix} F_x^A(\xi,\eta) \\ F_y^A(\xi,\eta) \\ F_z^A(\xi,\eta) \end{Bmatrix} \tag{A.64a}$$

where $g_1(t)$, $g_2(t)$, and $g_3(t)$ are specified functions of time and $F_x^A(\xi,\eta)$, $F_y^A(\xi,\eta)$, and $F_z^A(\xi,\eta)$ are independent of time. In this case, the second integral of Eq. A.61 would become

$$ab\int_0^1\int_0^1 \left[\overset{3}{N}(\xi,\eta)\right]^T \begin{bmatrix} g_1(t_m) & 0 & 0 \\ 0 & g_2(t_m) & 0 \\ 0 & 0 & g_3(t_m) \end{bmatrix} \begin{Bmatrix} F_x^A(\xi,\eta) \\ F_y^A(\xi,\eta) \\ F_z^A(\xi,\eta) \end{Bmatrix} d\xi\, d\eta \tag{A.64b}$$

The area integral need be performed only once; the result is then appropriately scaled by $g_1(t_m)$, $g_2(t_m)$, and $g_3(t_m)$ at each time instant.

The greatest simplification of the calculation of $\underset{\sim}{f}_c$ comes when the uncoupled timewise variation of the external forces is the same for all force components, i.e., when all force components may be expressed in the form

$$\begin{Bmatrix} F_x^A(\xi,\eta,t) \\ F_y^A(\xi,\eta,t) \\ F_z^A(\xi,\eta,t) \end{Bmatrix} = \begin{Bmatrix} F_x^A(\xi,\eta) \\ F_y^A(\xi,\eta) \\ F_z^A(\xi,\eta) \end{Bmatrix} g(t) \tag{A.65}$$

In this case, the scalar $g(t_m)$ may be taken outside the integrals in Eq. A.61. It is easy to see that the element force vector $(\underset{\sim}{f}_c)_{t_m}$ evaluated at any time instant $t_m$ is given by

$$\left(\underset{\sim}{f}_c\right)_{t_m} = g(t_m)\ \underset{\sim}{\hat{f}}_c \tag{A.66}$$

where $\underset{\sim}{f}_c$ is given by Eq. A.63 and $\underset{\sim}{\hat{f}}$ is obtained from Eq. A.61 and need be calculated only once for each transient analysis problem. It should be noted that the use of Eq. A.65 implies that the shape of the applied force distribution is identical at all time steps, and that the magnitude of the applied force distribution varies as some prescribed function of time, $g(t)$.

The second (lumped) approach to the formulation of the nodal force vector utilizes the same force distribution as the first (consistent) approach, but a simplified assumption is used for the displacement distribution within the element; the resulting force vectors will be loosely termed lumped force vectors (as a result of the lumping or averaging schemes which will be used to describe

the displacement distributions). Two types of displacement distribution averaging, and the resulting nodal force vectors (to be denoted by $\underset{\sim}{f}_{\ell 1}$ , and $\underset{\sim}{f}_{\ell 2}$), will be described. For the present development, there is no need to introduce normalized coordinates; thus, the distribution of the components of force at time $t_m$ will be specified in the x, y, z coordinate system.

In the first lumping scheme, the displacement behavior within an element is assumed to be constant over the element and equal to the algebraic average of the current nodal displacements for the element. Thus the distribution of the displacements u, v, w, θ, and ψ in Eq. A.55 are given by

$$\begin{Bmatrix} u \\ v \\ w \\ \theta \\ \psi \end{Bmatrix} = \frac{1}{4} \sum_{i=1}^{4} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \\ \psi_i \end{Bmatrix} \tag{A.67}$$

According to the nodal degree of freedom numbering scheme given in Eq. A.4, Eq. A.67 can be written in matrix form as

$$\begin{Bmatrix} u \\ v \\ w \\ \theta \\ \psi \end{Bmatrix} = \frac{1}{4} \left[ \underset{\sim}{b} \;\; \underset{\sim}{b} \;\; \underset{\sim}{b} \;\; \underset{\sim}{b} \right] \left\{ q \right\} = \frac{1}{4} \underset{\sim}{B} \; \underset{\sim}{q} \tag{A.68}$$

where b is the Boolean matrix

$$\underset{\sim}{b} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{A.68a}$$

Equation A.68 applies to the (averaged uniform) distribution of displacements within the element and is in a form appropriate for substitution into the last integral in Eq. A.55. For the second integral in Eq. A.55, only u, v, and w are needed; in this case $\underset{\sim}{B}$ in Eq. A.68 is replaced by $\underset{\sim}{B}^3$ where $\underset{\sim}{B}^3$ is the first three rows of $\underset{\sim}{B}$. For the first integral in Eq. A.55, the averaged uniform displacement distribution along the loaded element side depends only on the current displacements at the nodes along that side. In this case, the averaging factor in Eq. A.68 is 1/2 instead of 1/4, and the form of $\underset{\sim}{B}^5$ (replacing $\underset{\sim}{B}$ in Eq. A.68) depends on the element side being loaded:

$$
\begin{aligned}
&\text{Side 1} \quad : \quad \underset{\sim}{B}^5 = \begin{bmatrix} \underset{\sim}{b} & \underset{\sim}{b} & 0 & 0 \end{bmatrix} \\
&\text{Side 2} \quad : \quad \underset{\sim}{B}^5 = \begin{bmatrix} 0 & \underset{\sim}{b} & \underset{\sim}{b} & 0 \end{bmatrix} \\
&\text{Side 3} \quad : \quad \underset{\sim}{B}^5 = \begin{bmatrix} 0 & 0 & \underset{\sim}{b} & \underset{\sim}{b} \end{bmatrix} \\
&\text{Side 4} \quad : \quad \underset{\sim}{B}^5 = \begin{bmatrix} \underset{\sim}{b} & 0 & 0 & \underset{\sim}{b} \end{bmatrix}
\end{aligned}
\tag{A.69}
$$

Equation A.68 (or the equivalent for the first and second integrals) is substituted into the expression for the work of the external forces for a typical nth element at time $t_m$ to give

$$
\begin{aligned}
(W)_n = {} & \underset{\sim}{q}^T \underset{\sim}{B}^{5^T} \frac{1}{2} \int_0^{\ell} \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & 0 & 0 \\ 0 & -z & 0 \end{bmatrix} \begin{Bmatrix} F_x^L(s,z) \\ F_y^L(s,z) \\ F_z^L(s,z) \end{Bmatrix}_{t_m} ds\,dz \\
& + \underset{\sim}{q}^T \underset{\sim}{B}^{3^T} \frac{1}{4} \int_0^a \int_0^b \begin{Bmatrix} F_x^A(x,y) \\ F_y^A(x,y) \\ F_z^A(x,y) \end{Bmatrix}_{t_m} dx\,dy \\
& + \underset{\sim}{q}^T \underset{\sim}{B}^T \frac{1}{4} \int_0^a \int_0^b \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & 0 & 0 \\ 0 & -z & 0 \end{bmatrix} \begin{Bmatrix} F_x^B(x,y,z) \\ F_y^B(x,y,z) \\ F_z^B(x,y,z) \end{Bmatrix}_{t_m} dx\,dy\,dz
\end{aligned}
\tag{A.70}
$$

The following vectors may be defined

$$\hat{\underset{\sim}{f}}_L = \frac{1}{2}\int_0^{\ell}\int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} F_x^L(s,z) \\ F_y^L(s,z) \\ F_z^L(s,z) \\ -zF_x^L(s,z) \\ -zF_y^L(s,z) \end{Bmatrix}_{t_m} ds\,dz \tag{A.71a}$$

$$\hat{\underset{\sim}{f}}_A = \frac{1}{4}\int_0^{a}\int_0^{b} \begin{Bmatrix} F_x^A(x,y) \\ F_y^A(x,y) \\ F_z^A(x,y) \end{Bmatrix}_{t_m} dx\,dy \tag{A.71b}$$

$$\hat{\underset{\sim}{f}}_B = \frac{1}{4}\int_0^{a}\int_0^{b}\int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} F_x^B(x,y,z) \\ F_y^B(x,y,z) \\ F_z^B(x,y,z) \\ -zF_x^B(x,y,z) \\ -zF_y^B(x,y,z) \end{Bmatrix}_{t_m} \tag{A.71c}$$

Then Eq. A.70 may be expressed in the form

$$(W)_n = \underset{\sim}{q}^T \underset{\sim}{f}_{\ell 1} \tag{A.72}$$

where $\underset{\sim}{f}_{\ell 1}$ is the first type of lumped nodal load vector and is given by

$$\underset{\sim}{f}_{\ell 1} = \underset{\sim}{B}^{5^T} \hat{\underset{\sim}{f}}_L + \underset{\sim}{B}^{3^T} \hat{\underset{\sim}{f}}_A + \underset{\sim}{B}^T \hat{\underset{\sim}{f}}_B \tag{A.73}$$

The operations shown in Eq. A.73 correspond to the proper positioning and addition of the vectors $\hat{\underset{\sim}{f}}_L$, $\hat{\underset{\sim}{f}}_A$, and $\hat{\underset{\sim}{f}}_B$ into the nodal loads vector $\underset{\sim}{f}_{\ell 1}$. The five entries in the $\hat{\underset{\sim}{f}}_L$ vector are added into the five locations corresponding

to the first five degrees of freedom at each of the nodes located along this element edge. The three terms in $\hat{\underset{\sim}{f}}_A$ are added into rows 1 through 3, 7 through 9, 13 through 15, and 19 through 21 in $\underset{\sim}{f}_{\ell 1}$ (corresponding to displacements u, v, and w at each node). The five terms in $\hat{\underset{\sim}{f}}_B$ are added into rows 1 through 5, 7 through 11, 13 through 17, and 19 through 23 in $\underset{\sim}{f}_{\ell 1}$ (corresponding to displacements u, v, w, $\theta$, and $\psi$ at each node).

The second type of lumped loading is established through an alternate assumption for the displacement behavior in the element. First the undeformed rectangular plate element is subdivided into four equal-size regions; using the notation $R_i$ to denote subregion i (associated with node i of the plate), it is now assumed that the displacements within each subregion are constant and equal to the value of the displacements at the node associated with that subregion. If one thus defines

$$\underset{\sim}{F}^L = \begin{Bmatrix} F_x^L(s,z) \\ F_y^L(s,z) \\ F_z^L(s,z) \\ -z\,F_x^L(s,z) \\ -z\,F_y^L(s,z) \end{Bmatrix} \tag{A.74a}$$

$$\underset{\sim}{F}^A = \begin{Bmatrix} F_x^A(x,y) \\ F_y^A(x,y) \\ F_z^A(x,y) \end{Bmatrix} \tag{A.74b}$$

$$\underset{\sim}{F}^B = \begin{Bmatrix} F_x^B(x,y,z) \\ F_y^B(x,y,z) \\ F_z^B(x,y,z) \\ -z\,F_x^B(x,y,z) \\ -z\,F_y^B(x,y,z) \end{Bmatrix} \tag{A.74c}$$

Next by subdividing the line, area, and volume integrals of Eq. A.55, the work of the external forces can be defined in the form

$$(W)_n = \underset{\sim}{q}^T \underset{\sim}{f}_{\ell 2} \tag{A.75}$$

where $\underset{\sim}{f}_{\ell 2}$ is the lumped nodal load vector of the second type and is given by

$$\underset{\sim}{f}_{\ell 2} = \left\{\begin{array}{c} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{s_1} \underset{\sim}{F}^L dA \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{s_2} \underset{\sim}{F}^L dA \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{s_3} \underset{\sim}{F}^L dA \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{s_4} \underset{\sim}{F}^L dA \\ \hline 0 \end{array}\right\} + \left\{\begin{array}{c} \int_{R_1} \underset{\sim}{F}^A dx\,dy \\ \hline 0 \\ 0 \\ 0 \\ \hline \int_{R_2} \underset{\sim}{F}^A dx\,dy \\ \hline 0 \\ 0 \\ 0 \\ \hline \int_{R_3} \underset{\sim}{F}^A dx\,dy \\ \hline 0 \\ 0 \\ 0 \\ \hline \int_{R_4} \underset{\sim}{F}^A dx\,dy \\ \hline 0 \\ 0 \\ 0 \end{array}\right\} + \left\{\begin{array}{c} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{R_1} \underset{\sim}{F}^B dx\,dy\,dz \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{R_2} \underset{\sim}{F}^B dx\,dy\,dz \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{R_3} \underset{\sim}{F}^B dx\,dy\,dz \\ \hline 0 \\ \hline \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{R_4} \underset{\sim}{F}^B dx\,dy\,dz \\ \hline 0 \end{array}\right\} \tag{A.76}$$

where it should be noted that the length $s_1$ refers to the half-side lengths associated with node 1. Clearly the first vector corresponds to line loadings, the second vector to loading distributed over the mid-surface of the plate, and the third vector to loading distributed over the plate volume.

The difference between $\underset{\sim}{f}_{\ell 1}$ and $\underset{\sim}{f}_{\ell 2}$ may be summarized as follows: at a node, the lumped load is equal either to (1) one-quarter of the total load integrated over the whole plate (for $\underset{\sim}{f}_{\ell 1}$), or (2) to the value of the load integrated over one-quarter of the plate. For a constant load (in space), these two forms are equivalent. Also, it should be expected that the second form of lumping, $\underset{\sim}{f}_{\ell 2}$, will be the more accurate of the two lumping schemes.

The preceding discussion pertains only to the spatial distributions of the external force, and the forces defined should be viewed as being derived on the basis of the force distributions at a particular instant in time. The discussion of time dependence for the consistent loading vector and corresponding possible simplification also applies to the two alternate lumping schemes and is, therefore, not repeated here.

### A.1.3.5 Element Stiffness Matrix Corresponding to Elastic Restoring Springs

In the PLATE program, linear elastic line restoring springs may be specified. These line restoring springs are permitted to act only along element boundaries. The variation of the work done by the restoring springs, $\delta W_{sp}$, is given by (writing $-\delta W_{sp}$ hereinafter for convenience)

$$-\delta W_{sp} = \int_{\text{side } i} \left( k_x u \delta u + k_y v \delta v + k_z w \delta w + k_\theta \theta \delta\theta + k_\psi \psi \delta\psi \right) ds \qquad \text{(A.77)}$$

where $k_x$, $k_y$, and $k_z$ are the line translational spring constants (units of lb/in per in of span) corresponding to springs oriented in the x, y, and z directions, respectively; $k_\theta$ and $k_\psi$ are the line <u>torsional</u> spring constants (units of in-lb/radian per in of span) corresponding to rotation about the y and x axes, respectively; and the integration of Eq. A.77 extends over the i th side of the element (i=1,2,3, or 4).

For convenience, the normalized coordinate system (Subsection A.1.2) and associated normalized nodal degrees of freedom are used. Then Eq. A.77 may be written in terms of normalized quantities in the matrix form

$$-\delta W_{sp} = \ell_i \int_0^1 \lfloor \delta\bar{u}\ \delta\bar{v}\ \delta\bar{w}\ \delta\bar{\theta}\ \delta\bar{\psi} \rfloor \underbrace{\begin{bmatrix} k_x & & & & \\ & k_y & & \bigcirc & \\ & & k_z & & \\ & \bigcirc & & \frac{k_\theta}{a^2} & \\ & & & & \frac{k_\psi}{b^2} \end{bmatrix}}_{[\bar{C}_{sp}]} \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \\ \bar{\theta} \\ \bar{\psi} \end{Bmatrix} \qquad \text{(A.78)}$$

where $\ell_i$ is the length of element side i. The normalized displacements, $\bar{u}$, $\bar{v}$, $\bar{w}$, $\bar{\theta}$, and $\bar{\psi}$ must be expressed in terms of the normalized nodal generalized displacements at the two nodes (i and i+1) bounding element side i. This relation for side 1 (for example) is given by

$$\begin{aligned}
\bar{u} &= \bar{u}_1(1-s) + \bar{u}_2 s \\
\bar{v} &= \bar{v}_1(1-s) + \bar{v}_2 s \\
\bar{w} &= \bar{w}_1(1-3s^2+2s^3) + \bar{w}_2(3s^2-2s^3) + \bar{\theta}_1(s-2s^2+s^3) + \bar{\theta}_2(-s^2+s^3) \\
\bar{\theta} &= \bar{\theta}_1(1-4s+3s^2) + \bar{\theta}_2(-2s+3s^2) + \bar{w}_1(-6s+6s^2) + \bar{w}_2(6s-6s^2) \\
\bar{\Psi} &= \bar{\Psi}_1(1-3s^2+2s^3) + \bar{\Psi}_2(3s^2-2s^3) + \bar{\chi}_1(s-2s^2+s^3) + \bar{\chi}_2(-s^2+s^3)
\end{aligned} \tag{A.79}$$

and for side 4 (where $\xi=0$), the relation is

$$\begin{aligned}
\bar{u} &= \bar{u}_1(1-s) + \bar{u}_2 s \\
\bar{v} &= \bar{v}_1(1-s) + \bar{v}_4 s \\
\bar{w} &= \bar{w}_1(1-3s^2+2s^3) + \bar{w}_4(3s^2-2s^3) + \bar{\Psi}_1(s-2s^2+s^3) + \bar{\Psi}_4(-s^2+s^3) \\
\bar{\theta} &= \bar{\theta}_1(1-3s^2+2s^3) + \bar{\theta}_4(3s^2-2s^3) + \bar{\chi}_1(s-2s^2+s^3) + \bar{\chi}_4(-s^2+s^3) \\
\bar{\Psi} &= \bar{\Psi}_1(1-4s+3s^2) + \bar{\Psi}_4(-2s+3s^2) + \bar{w}_1(-6s+6s^2) + \bar{w}_4(6s-6s^2)
\end{aligned} \tag{A.80}$$

In general, these relations are written in matrix form:

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \\ \bar{\theta} \\ \bar{\Psi} \end{Bmatrix} = \left[ N^i \right] \begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \vdots \\ \bar{u}_{i+1} \\ \bar{v}_{i+1} \\ \vdots \\ \bar{\Psi}_{i+1} \end{Bmatrix} \tag{A.81}$$

When Eq. A.81 is substituted into Eq. A.78, $-\delta W_{sp}$ becomes

$$-\delta W_{sp} = \delta \lfloor \bar{u}_i \; \bar{v}_i \; \bar{w}_i \cdots \bar{u}_{i+1} \; \bar{v}_{i+1} \cdots \bar{\Psi}_{i+1} \rfloor \underbrace{\ell_i \int_0^1 [N^i]^T [\bar{C}_{sp}][N^i]\, ds}_{[\bar{k}_{sp}]} \begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \vdots \\ \bar{u}_{i+1} \\ \bar{v}_{i+1} \\ \vdots \\ \bar{\Psi}_{i+1} \end{Bmatrix} \tag{A.82}$$

The normalized nodal generalized displacements are then expressed in terms of nodal displacements $u_i$, $v_i$ ... as

$$\begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{w}_i \\ \vdots \\ \bar{u}_{i+1} \\ \bar{v}_{i+1} \\ \vdots \\ \bar{\psi}_{i+1} \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{t} & 0 \\ 0 & \underset{\sim}{t} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \cdot \\ u_{i+1} \\ v_{i+1} \\ \vdots \\ \psi_{i+1} \end{Bmatrix} \equiv \begin{bmatrix} T \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \vdots \\ u_{i+1} \\ v_{i+1} \\ \vdots \\ \psi_{i+1} \end{Bmatrix} \tag{A.83}$$

where the diagonal matrix, $\underset{\sim}{t}$, is defined in Eq. A.52c. Substituting Eq. A.83 into Eq. A.82 yields

$$-\delta W_{sp} = \delta \begin{bmatrix} u_i & v_i & \cdots & u_{i+1} & v_{i+1} & \cdots & \psi_{i+1} \end{bmatrix} \begin{bmatrix} k_{sp}^{i} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \cdot \\ \cdot \\ u_{i+1} \\ v_{i+1} \\ \vdots \\ \psi_{i+1} \end{Bmatrix} \tag{A.84}$$

where $[k_{sp}^{i}]$ is the compacted stiffness matrix corresponding to line elastic restoring springs on element side i, and is given by

$$\begin{bmatrix} k_{sp} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}^T \begin{bmatrix} \bar{k}_{sp} \end{bmatrix} \begin{bmatrix} T \end{bmatrix} \tag{A.85}$$

The compacted matrix $[\bar{k}_{sp}]$ has been obtained analytically and is given in Table A.1a for element sides 1 and 3, and in Table A.1b for element sides 2 and 4. The variation of work, $-\delta W_{sp}$, can be related to all of the element nodal displacements, $\underset{\sim}{q}$ by

$$-\delta W_{sp} = \delta\, \underset{\sim}{q}^T \begin{bmatrix} k_{sp} \end{bmatrix} \underset{\sim}{q} \tag{A.86}$$

by "assembling" (relocating and adding in) the entries in the compacted matrices $\underset{\sim}{k}_{sp}$ or $\underset{\sim}{k}_{sp}^{1}$ into the element spring-stiffness matrix, $\underset{\sim}{k}_{sp}$. To aid in this assembly process, the rows and columns of $\underset{\sim}{\bar{k}}_{sp}$ given in Table A.1 are labelled with the appropriate nodal displacements.

## A.2 Formulations for Stiffened Rectangular Plate Elements

The PLATE program accommodates integrally-stiffened flat plates where the stiffeners must be oriented in the x and/or y directions. Other assumptions related to geometry and deformation behavior of the stiffeners are given in Subsection 2.1.

In the PLATE program, the stiffened plate element is developed by superimposing an unstiffened plate element and a compatibly derived beam element. The formulation of straight beam elements by the assumed-displacement model is a straightforward process and is detailed in most texts on finite-element analysis methods. For the present case, the stiffener (or stiffeners) located on a particular element will be assumed to contribute only to the kinetic energy (mass matrix) and the internal work of the stresses (stiffness matrix and equivalent nodal loads corresponding to nonlinear effects). The stiffener is assumed to have no effect on the work of the external forces or the work of the line restoring springs; thus, the developments in Subsections A.1.3.4 and A.1.3.5 apply to both unstiffened and stiffened plates.

Before presenting details of the formulation of the stiffener contributions, some comments regarding the interpolation assumption for the stiffeners should be made. In order to calculate the property matrices for a stiffener, the geometry (rectangular cross-section width and height) and location of the stiffener ($\eta_s$ for x-direction stiffeners, and $\xi_s$ for y-direction stiffeners) and the z-location (top or bottom of the plate) must be known.

Because the stiffener is assumed to be perfectly bonded to the plate, the displacement interpolation assumption of Eqs. A.17 can be utilized to determine the distribution of any component of displacement within the stiffener in terms of $\bar{\alpha}_1, \bar{\alpha}_2 \ldots \bar{\alpha}_{24}$. For x-direction stiffeners, $\eta=\eta_s$ and $\xi$ is the variable; for y-direction stiffeners, $\xi=\xi_s$ and $\eta$ is the variable; in both cases, the z-integrations will extend over the height of the stiffener (note that the reference z=0 is located at the midsurface of the plate). Once

the element stiffness and mass matrices are defined in terms of $\bar{\alpha}_1, \bar{\alpha}_2 \ldots \bar{\alpha}_{24}$, Eqs. A.19 and A.52a are used to transform these into a final form which is fully compatible with the stiffness and mass matrices of the unstiffened plate element (i.e. expressed in terms of the nodal displacements of the plate element). The stiffness (or mass) matrix for the stiffened plate is then obtained by adding together the stiffness (or mass) matrices of the stiffener and the unstiffened plate element to which it is attached.

A.2.1 Stiffener Mass Matrix

The consistent mass matrix for a stiffener is derived from the expression for the kinetic energy, $(T)_s$, which for a typical stiffener element is

$$(T)_s = \int_0^{\ell} \int_{A_s} \frac{1}{2} \rho_s \left[ (\dot{u} - z\dot{\theta})^2 + (\dot{v} - z\dot{\psi})^2 + \dot{w}^2 \right] dA\, ds \qquad \text{(A.87)}$$

where $A_s$ and $\rho_s$ are the stiffener cross-sectional area and mass per unit volume. Introducing normalized coordinates and normalized displacements and integrating over the cross-section of the stiffener, Eq. A.87 can be written in the following matrix form:

$$(T)_s = \frac{1}{2} \rho_s \ell_s \int_0^1 \lfloor \dot{\bar{u}}\ \dot{\bar{v}}\ \dot{\bar{w}}\ \dot{\bar{\theta}}\ \dot{\bar{\psi}} \rfloor \left[ \tilde{m}^s \right] \begin{Bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{w}} \\ \dot{\bar{\theta}} \\ \dot{\bar{\psi}} \end{Bmatrix} ds \qquad \text{(A.88)}$$

where $\ell_s$ is the length of the stiffener (= a for x-direction stiffeners, =b for y-direction stiffeners), and

$$\left[ \tilde{m}^s \right] = \begin{bmatrix} A_s & & & & \text{SYMM.} \\ 0 & A_s & & & \\ 0 & 0 & A_s & & \\ \frac{-A_s \lambda_s}{a} & 0 & 0 & \frac{I_s}{a^2} & \\ 0 & \frac{-A_s \lambda_s}{b} & 0 & 0 & \frac{I_s}{b^2} \end{bmatrix} \qquad \text{(A.89)}$$

In Eq. A.89, $I_s$ is the cross-sectional area moment of inertia of the stiffener, and $\lambda_s$ is the distance along the z axis from the plate reference surface to the centroid of the stiffener. It should be noted that for x-direction stiffeners, ds (Eq. A.88) is equal to $d\xi$, and is equal to $d\eta$ for y-direction stiffeners.

By substituting Eq. A.25 (evaluated at the stiffener location) into Eq. A.88 and applying Eqs. A.19 and A.27, the expression for the kinetic energy can be written in terms of the nodal generalized velocities of the plate element to which the stiffener is attached as:

$$(T)_s = \frac{1}{2} \dot{\underset{\sim}{q}}^T \underset{\sim}{m}_c^s \dot{\underset{\sim}{q}} \tag{A.90}$$

where $\underset{\sim}{m}_c^s$ is the consistent element mass matrix for the stiffener and is given by

$$\underset{\sim}{m}_c^s = \rho_s \ell_s A_s \underset{\sim}{T}^T \left(\underset{\sim}{\bar{G}}^{-1}\right)^T \int_0^1 \underset{\sim}{N}^T(\xi,\eta_s)\, \tilde{\underset{\sim}{m}}^s \underset{\sim}{N}(\xi,\eta_s)\, d\xi\, \underset{\sim}{\bar{G}}^{-1} \underset{\sim}{T} \tag{A.91a}$$

for x-direction stiffeners and

$$\underset{\sim}{m}_c^s = \rho_s \ell_s A_s \underset{\sim}{T}^T \left(\underset{\sim}{\bar{G}}^{-1}\right)^T \int_0^1 \underset{\sim}{N}^T(\xi_s,\eta)\, \tilde{\underset{\sim}{m}}^s N(\xi_s,\eta)\, d\eta\, \underset{\sim}{\bar{G}}^{-1} \underset{\sim}{T} \tag{A.91b}$$

for y-direction stiffeners.

A lumped (diagonal) mass matrix can also be formed for the stiffener element by proportionally lumping the total mass properties of the stiffener to the four corner nodes of the plate element to which it is attached. The lumped mass matrix is of the form

$$\underset{\sim}{m}_L^s = \begin{bmatrix} f_1 \hat{\underset{\sim}{m}}^s & & & \bigcirc \\ & f_2 \hat{\underset{\sim}{m}}^s & & \\ & & f_3 \hat{\underset{\sim}{m}}^s & \\ \bigcirc & & & f_4 \hat{\underset{\sim}{m}}^s \end{bmatrix} \tag{A.92}$$

where

$$\hat{\underset{\sim}{m}}^s = \begin{bmatrix} \frac{A_s \ell_s}{2} & & & & & \\ & \frac{A_s \ell_s}{2} & & & 0 & \\ & & \frac{A_s \ell_s}{2} & & & \\ & & & \frac{I_s \ell_s}{2} & & \\ & 0 & & & \frac{I_s \ell_s}{2} & \\ & & & & & 0 \end{bmatrix} \tag{A.92a}$$

and where the multiplying factors $f_1$, $f_2$, $f_3$, and $f_4$ depend on the type and location of the stiffener and are assigned values as follows:

| Stiffener | $f_1$ | $f_2$ | $f_3$ | $f_4$ |
|---|---|---|---|---|
| x-direction | $(1-\eta_s)$ | $(1-\eta_s)$ | $\eta_s$ | $\eta_s$ |
| y-direction | $(1-\xi_s)$ | $\xi_s$ | $\xi_s$ | $(1-\xi_s)$ |

A.2.2 Element Equivalent Loads Corresponding to Nonlinear Effects

For the present beam-type stiffeners, the expression for the variation of the work of the stresses within the element is

$$\delta U = w_s \int_z \int_0^{\ell_s} \sigma \, \delta\epsilon \, ds \, dz \tag{A.93}$$

where $w_s$ is the width of the stiffener and where $\sigma$ and $\varepsilon$ are the stress and strain in the direction of the stiffener. The corresponding stress-strain relation is

$$\sigma = E_s \left( \tilde{\epsilon} - \tilde{\epsilon}^p \right) = E_s \left( \epsilon - z\mathcal{K} - \tilde{\epsilon}^p \right) \tag{A.94}$$

where $E_s$ is Young's modulus of the sth stiffener. When Eq. A.94 is substituted into Eq. A.93, the expression for $\delta U$ reads

$$\delta U = w_s \int_z \int_0^{\ell_s} E_s \left( \epsilon - z\mathcal{K} - \tilde{\epsilon}^p \right) \delta \left( \epsilon - z\mathcal{K} \right) ds\, dz \qquad \text{(A.95)}$$

The pertinent strain-displacement relations of Eqs. A.8 and A.9 ($\varepsilon_{xx}$ and $\kappa_{xx}$ for x-direction stiffeners, and $\varepsilon_{yy}$ and $\kappa_{yy}$ for y-direction stiffeners) are substituted into Eq. A.95 to yield

$$\delta U = \delta \underset{\sim}{q}^T \underset{\sim}{k}_s \underset{\sim}{q} - \delta \underset{\sim}{q}^T \underset{\sim}{f}_s^{NL} \qquad \text{(A.96)}$$

where $\underset{\sim}{k}_s$ is the stiffness matrix of the s th stiffener (and is discussed in the next subsection) and $\underset{\sim}{f}_s^{NL}$ is the vector of equivalent nodal forces for the s th stiffener corresponding to nonlinear geometric and material effects. For x-direction stiffeners $\underset{\sim}{f}_s^{NL}$ is given by

$$\begin{aligned}
\left\{ f_s^{NL} \right\} = &-E_s A_s \int_0^a \begin{bmatrix} D_1(\xi,\eta_s) \\ D_4(\xi,\eta_s) \\ D_7(\xi,\eta_s) \end{bmatrix}^T \begin{Bmatrix} \dfrac{\theta^2}{2} \\ -\dfrac{\lambda_s \theta^2}{2} \\ \theta\, \epsilon_{xx} - \lambda_s \theta \mathcal{K}_{xx} \end{Bmatrix} dx \\
&+ E_s w_s \int_0^a \begin{bmatrix} D_1(\xi,\eta_s) \\ D_4(\xi,\eta_s) \end{bmatrix}^T \begin{Bmatrix} \int_z \tilde{\epsilon}_{xx}^p \, dz \\ -\int_z z\, \tilde{\epsilon}_{xx}^p \, dz \end{Bmatrix} dx \qquad \text{(A.97)} \\
&+ E_s w_s \int_0^a \left\{ D_7(\xi,\eta_s) \right\} \left( \theta \int_z \tilde{\epsilon}_{xx}^p \, dz \right) dx
\end{aligned}$$

and for y-direction stiffeners is given by

$$\begin{aligned}
\left\{ f_s^{NL} \right\} = &-E_s A_s \int_0^b \begin{bmatrix} D_2(\xi_s,\eta) \\ D_5(\xi_s,\eta) \\ D_8(\xi_s,\eta) \end{bmatrix}^T \begin{Bmatrix} \dfrac{\psi^2}{2} \\ -\dfrac{\lambda_s \psi^2}{2} \\ \psi\, \epsilon_{yy} - \lambda_s \psi \mathcal{K}_{yy} \end{Bmatrix} dy \\
&+ E_s w_s \int_0^b \begin{bmatrix} D_2(\xi_s,\eta) \\ D_5(\xi_s,\eta) \end{bmatrix}^T \begin{Bmatrix} \int_z \tilde{\epsilon}_{yy}^p \, dz \\ -\int_z z\, \tilde{\epsilon}_{yy}^p \, dz \end{Bmatrix} dy \qquad \text{(A.98)} \\
&+ E_s w_s \int_0^b \left\{ D_8(\xi_s,\eta) \right\} \left( \psi \int_z \tilde{\epsilon}_{yy}^p \, dz \right) dy
\end{aligned}$$

The calculation of $\underset{\sim}{f}_s^{NL}$ for a stiffener follows essentially the same steps as for the plate elements except that the integrations now extend only along the stiffener direction and through-the-thickness. The reader is invited to consult Subsection A.1.3.2 for a general discussion.

A.2.3 Element Stiffness Matrix

The stiffness matrix contributions for a stiffener correspond to the term $\delta \underset{\sim}{q}^T \underset{\sim}{k}_s \underset{\sim}{q}$ in Eq. A.96 and can be derived from the linear term in the strain-displacement relation. From Eq. A.95 and A.96

$$\delta \underset{\sim}{q}^T \underset{\sim}{k}_s \underset{\sim}{q} = w_s \int_z \int_0^{l_s} E_s (\epsilon - z\mathcal{K})\, \delta(\epsilon - z\mathcal{K})\, ds\, dz \tag{A.99}$$

or, since $E_s$ is assumed to be constant over the stiffener, and integrating over the stiffener thickness:

$$\delta \underset{\sim}{q}^T \underset{\sim}{k}_s \underset{\sim}{q} = E_s \int_0^{l_s} \delta \lfloor \epsilon \quad \mathcal{K} \rfloor \begin{bmatrix} A_s & -A_s\lambda_s \\ -A_s\lambda_s & I_s \end{bmatrix} \begin{Bmatrix} \epsilon \\ \mathcal{K} \end{Bmatrix} ds \tag{A.100}$$

The interpolations used for the normal strain, $\varepsilon$, and curvature, $\kappa$, must be compatible with that used for the plate element since perfect bonding is assumed at the plate/stiffener interface. Thus, the strain and curvature are evaluated from the displacement interpolations of Eqs. A.17. For an x-direction stiffener:

$$\epsilon \equiv \epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{1}{a}\frac{\partial \bar{u}}{\partial \xi} = \frac{\bar{\alpha}_2 + \eta_s \bar{\alpha}_4}{a} \tag{A.101a}$$

$$\mathcal{K} \equiv \mathcal{K}_{xx} = \frac{\partial \theta}{\partial x} = \frac{1}{a^2}\frac{\partial \bar{\theta}}{\partial \xi} = \frac{1}{a^2}\Big(2\bar{\alpha}_{13} + 2\eta_s \bar{\alpha}_{15} + 2\eta_s^2 \bar{\alpha}_{17}$$

$$+ 6\xi \bar{\alpha}_{18} + 6\xi\eta_s \bar{\alpha}_{20} + 6\xi\eta_s^2 \bar{\alpha}_{22} + 2\eta_s^3 \bar{\alpha}_{23} + 6\xi\eta_s^3 \bar{\alpha}_{24}\Big) \tag{A.101b}$$

and for a y-direction stiffener:

$$\epsilon \equiv \epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial \bar{v}}{\partial \eta} = \frac{\bar{\alpha}_7 + \xi_s \bar{\alpha}_8}{b} \tag{A.102a}$$

$$\begin{aligned} \mathcal{K} \equiv \mathcal{K}_{yy} = \frac{\partial \psi}{\partial y} = \frac{1}{b^2}\frac{\partial \bar{\psi}}{\partial \eta} = \frac{1}{b^2}\Big( & 2\bar{\alpha}_{14} + 2\xi_s \bar{\alpha}_{16} + 2\xi_s^2 \bar{\alpha}_{17} \\ & + 6\eta \bar{\alpha}_{19} + 6\eta \xi_s \bar{\alpha}_{21} + 2\xi_s^3 \bar{\alpha}_{22} + 6\eta \xi_s^2 \bar{\alpha}_{23} + 6\eta \xi_s^3 \bar{\alpha}_{24}\Big) \end{aligned} \tag{A.102b}$$

or in matrix form

$$\begin{Bmatrix} \epsilon \\ \mathcal{K} \end{Bmatrix} = \underset{\sim}{D}\ \underset{\sim}{\bar{\alpha}} \tag{A.103}$$

where $\underset{\sim}{D}$ is obtained from Eqs. A.101 for x-direction stiffeners or from Eqs. A.102 for y-direction stiffeners.

When Eq. A.103 is substituted into Eq. A.100, the resulting expression is

$$\delta \underset{\sim}{q}^T \underset{\sim}{k}_s \underset{\sim}{q} = \delta \underset{\sim}{\bar{\alpha}}^T \underset{\sim}{\bar{k}} \underset{\sim}{\bar{\alpha}} \tag{A.104}$$

where

$$\underset{\sim}{\bar{k}} = E_s \int_0^{\ell_s} \underset{\sim}{D}^T \begin{bmatrix} A_s & -A_s \lambda_s \\ -A_s \lambda_s & I_s \end{bmatrix} \underset{\sim}{D}\, ds \tag{A.105}$$

Finally, the stiffness matrix is obtained by relating $\underset{\sim}{\bar{\alpha}}$ to $\underset{\sim}{q}$ through Eqs. A.19 and A.27 and is given by

$$\underset{\sim}{k}_s = \underset{\sim}{T}^T \left(\underset{\sim}{\bar{G}}^{-1}\right)^T \underset{\sim}{\bar{k}}\ \underset{\sim}{\bar{G}}^{-1} \underset{\sim}{T} \tag{A.106}$$

The stiffness matrix for both x- and y-direction stiffeners is, therefore, related to the generalized nodal displacement parameters of the plate element to which they are attached. As a result, the stiffness matrix of a stiffened plate element can be formed by adding the stiffness matrix of the unstiffened plate and the stiffness matrix of each stiffener. Alternatively, unstiffened plate and stiffener elements can be processed separately. In the PLATE program, the latter approach is used.

## A.3 Formation and Solution of the Governing Dynamic Equations for the Structure

The governing equations of motion for the discretized plate structure are obtained by applying the Principle of Virtual Work and D'Alembert's Principle to give

$$\sum_{n=1}^{n_p} \delta \underset{\sim}{q}^T \left( \underset{\sim}{m}\, \ddot{\underset{\sim}{q}} + \underset{\sim}{k}\, \underset{\sim}{q} + \underset{\sim}{k}_s\, \underset{\sim}{q} - \underset{\sim}{f} - \underset{\sim}{f}^{NL} \right)$$
$$+ \sum_{n=1}^{n_s^x} \delta \underset{\sim}{q}^T \left( \underset{\sim}{m}_s\, \ddot{\underset{\sim}{q}} + \underset{\sim}{k}_s\, \underset{\sim}{q} - \underset{\sim}{f}_s^{NL} \right) + \sum_{n=1}^{n_s^y} \delta \underset{\sim}{q}^T \left( \underset{\sim}{m}_s\, \ddot{\underset{\sim}{q}} + \underset{\sim}{k}_s\, \underset{\sim}{q} - \underset{\sim}{f}_s^{NL} \right) = 0 \qquad \text{(A.107)}$$

where $n_p$, $n_s^x$, $n_s^y$ are the number of plate elements, x-direction stiffeners, and y-direction stiffeners, respectively. The nodal generalized displacements, $\underset{\sim}{q}$, (and accelerations, $\ddot{\underset{\sim}{q}}$) for each element are then related to the global generalized displacements, $\underset{\sim}{q}^*$, by

$$\underset{\sim}{q} = \underset{\sim}{J}\, \underset{\sim}{q}^* \qquad \text{(A.108)}$$

where $\underset{\sim}{J}$ is a Boolean matrix. The indicated summations are then performed; this operation is the usual element "assembly" procedure. After assembly, the following matrix equation results, which governs the dynamic, large-deflection, elastic-plastic response of the plate structure;

$$\underset{\sim}{M}\, \ddot{\underset{\sim}{q}}^* + \underset{\sim}{K}\, \underset{\sim}{q}^* = \underset{\sim}{F} + \underset{\sim}{F}^{NL} \qquad \text{(A.109)}$$

where

- $\underset{\sim}{q}^*, \ddot{\underset{\sim}{q}}^*$ = vector of generalized nodal displacements and accelerations, respectively, for the assembled structure.
- $\underset{\sim}{M}$ = assembled mass matrix for the structure.
- $\underset{\sim}{K}$ = assembled elastic stiffness matrix for the structure, including the effective stiffness supplied by the line restoring springs.
- $\underset{\sim}{F}$ = vector of nodal forces for the structure corresponding to prescribed externally-applied forces.
- $\underset{\sim}{F}^{NL}$ = vector of nodal equivalent forces for the structure corresponding to geometric and material nonlinear effects.

The solution of the dynamic equation of motion (Eq. A.109) can be accomplished by applying a direct timewise integration scheme. In this scheme, the time derivatives of the nodal displacement vector ($\ddot{\underset{\sim}{q}}^*$ and $\dot{\underset{\sim}{q}}^*$) are expressed at a discrete time instant in terms of the nodal displacements at several nearby discrete time instants. When substituted into the governing equation of motion, a recurrence relation is obtained from which displacements can be calculated at each discrete time instant.

In the PLATE program, the (implicit) Houbolt operator is used, so that the accelerations $\ddot{\underset{\sim}{q}}^*$ at time $t_{m+1}$ are expressed by a 4-point backward-difference formula:

$$\ddot{\underset{\sim}{q}}^*_{m+1} = \frac{1}{(\Delta t)^2}\left(2\,\underset{\sim}{q}^*_{m+1} - 5\,\underset{\sim}{q}^*_{m} + 4\,\underset{\sim}{q}^*_{m-1} - \underset{\sim}{q}^*_{m-2}\right) + O(\Delta t^2) \qquad \text{(A.110)}$$

The velocities $\dot{\underset{\sim}{q}}^*_{m+1}$ at time $t_{m+1}$ can be expressed by the following 3-point backward-difference formula having the same truncation error as $\ddot{\underset{\sim}{q}}^*_{m+1}$

$$\dot{\underset{\sim}{q}}^*_{m+1} = \frac{1}{2\,\Delta t}\left(3\,\underset{\sim}{q}^*_{m+1} - 4\,\underset{\sim}{q}^*_{m} + \underset{\sim}{q}^*_{m-1}\right) + O(\Delta t^2) \qquad \text{(A.111)}$$

For convenience, (particularly, as will be seen in Appendix B, for impact analysis) the terms in Eq. A.110 can be regrouped so that $\ddot{\underset{\sim}{q}}^*_{m+1}$ at time $t_{m+1}$ can also be related to $\dot{\underset{\sim}{q}}^*_{m}$ at time $t_m$;

$$\begin{aligned}\ddot{\underset{\sim}{q}}^*_{m+1} &= \frac{\left[2\left(\underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_{m}\right)\right] + \left[-3\,\underset{\sim}{q}^*_{m} + 4\,\underset{\sim}{q}^*_{m-1} - \underset{\sim}{q}^*_{m-2}\right]}{(\Delta t)^2} \\ &= \frac{2}{(\Delta t)^2}\left(\underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_{m}\right) - \frac{2}{\Delta t}\,\dot{\underset{\sim}{q}}^*_{m}\end{aligned} \qquad \text{(A.112)}$$

Equation A.109 can be written to express dynamic equilibrium at time $t_{m+1}$ as

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{q}}^*_{m+1} + \underset{\sim}{K}\,\underset{\sim}{q}^*_{m+1} = \underset{\sim}{F}_{m+1} + \underset{\sim}{F}^{NL}_{m+1} \qquad \text{(A.113)}$$

Equation A.112 is then substituted into Eq. A.113 and the terms regrouped to give

$$\left[\frac{2}{(\Delta t)^2} \underset{\sim}{M} + \underset{\sim}{K}\right] \underset{\sim}{q}^*_{m+1} = \underset{\sim}{F}_{m+1} + \underset{\sim}{F}^{NL}_{m+1} + \frac{2}{\Delta t} \underset{\sim}{M}\left(\dot{\underset{\sim}{q}}^*_m + \frac{1}{\Delta t} \underset{\sim}{q}^*_m\right) \quad \text{(A.114)}$$

The recurrence relation given by Eq. A.114 can be solved at each time step for the unknown displacements $\underset{\sim}{q}^*_{m+1}$ at time $t_{m+1}$, based on the knowledge of $\underset{\sim}{F}_{m+1}$, $\underset{\sim}{F}^{NL}_{m+1}$, $\dot{\underset{\sim}{q}}^*_m$, and $\underset{\sim}{q}^*_m$. Once $\underset{\sim}{q}^*_{m+1}$ is determined, the velocities, $\dot{\underset{\sim}{q}}^*_{m+1}$ can be obtained from Eq. A.111, and the solution advanced to the next time instant. This process is repeated until some specified termination point is reached.

Several comments concerning the solution by Eq. A.114 are needed. The quantities $\underset{\sim}{F}_{m+1}$, $\dot{\underset{\sim}{q}}^*_m$, and $\underset{\sim}{q}^*_m$ on the right side of Eq. A.114 are known (or user specified in the case of $\underset{\sim}{F}_{m+1}$, $\dot{\underset{\sim}{q}}^*_o$, and $\underset{\sim}{q}^*_o$) at time $t_{m+1}$, but the vector of equivalent forces corresponding to nonlinear effects, $\underset{\sim}{F}^{NL}_{m+1} = \underset{\sim}{F}^{NL}_{m+1}(\underset{\sim}{q}^*_{m+1})$ is a function of $\underset{\sim}{q}^*_{m+1}$ and is thus not known. Consequently, either some form of extrapolation or extrapolation with iteration is required to calculate $\underset{\sim}{F}^{NL}_{m+1}$. In the PLATE program, <u>a linear extrapolation is used with no iteration</u> so that $\underset{\sim}{F}^{NL}_{m+1}$ in Eq. A.114 is replaced by

$$\underset{\sim}{F}^{NL}_{m+1} \doteq 2\,\underset{\sim}{F}^{NL}_{m} - \underset{\sim}{F}^{NL}_{m-1} \quad \text{(A.115)}$$

Equation A.114 may be used for times $t_{M+1}$ such that $m+1 \geq 2$. For earlier times, however, a starting procedure must be used; that chosen procedure is as follows [4]. Use the central-difference operator to write:

$$\ddot{\underset{\sim}{q}}_o = \frac{\underset{\sim}{q}_1 - 2\,\underset{\sim}{q}_o + \underset{\sim}{q}_{-1}}{(\Delta t)^2} + O(\Delta t)^2 \quad \text{(A.115a)}$$

and a mixed operator to obtain

$$\dot{\underset{\sim}{q}}_o = \frac{2\,\underset{\sim}{q}_1 + 3\,\underset{\sim}{q}_o - 6\,\underset{\sim}{q}_{-1} + \underset{\sim}{q}_{-2}}{6\,\Delta t} + O(\Delta t)^2 \quad \text{(A.115b)}$$

From Eq. A.115a, one obtains

$$\underset{\sim}{q}_{-1} = (\Delta t)^2 \ddot{\underset{\sim}{q}}_o + 2\,\underset{\sim}{q}_o - \underset{\sim}{q}_1 \quad \text{(A.115c)}$$

Applying Eq. A.115c to Eq. A.115b, one finds

$$\underset{\sim}{q}_{-2} = 6(\Delta t)^2 \ddot{\underset{\sim}{q}}_o + 6\Delta t \dot{\underset{\sim}{q}}_o + 9\underset{\sim}{q}_o - 8\underset{\sim}{q}_{-1} \tag{A.115d}$$

Since $\ddot{\underset{\sim}{q}}_1$ is given by the Houbolt operator as

$$\ddot{\underset{\sim}{q}}_1 = \frac{2\underset{\sim}{q}_1 - 5\underset{\sim}{q}_o + 4\underset{\sim}{q}_{-1} - \underset{\sim}{q}_{-2}}{(\Delta t)^2} \tag{A.115e}$$

one can use Eqs. A.115c, A.115d, and A.115e to write the finite-difference equations of motion at time $t_1$ (i.e. m+l=1) as

$$\left(6\underset{\sim}{M} + (\Delta t)^2 \underset{\sim}{K}\right)\underset{\sim}{q}_1 = (\Delta t)^2\left(\underset{\sim}{F}_1 + \underset{\sim}{F}_o^{NL}\right) + \underset{\sim}{M}\left(2(\Delta t)^2 \ddot{\underset{\sim}{q}}_o + 6(\Delta t)\dot{\underset{\sim}{q}}_o + 6\underset{\sim}{q}_o\right) \tag{A.116}$$

where $\underset{\sim}{F}_1^{NL}$ has been approximated (replaced) by its value at t=0: $\underset{\sim}{F}_o^{NL}$ and $\ddot{\underset{\sim}{q}}_o$ is found from the basic equation of motion written at time t=0:

$$\underset{\sim}{M}\ddot{\underset{\sim}{q}}_o + \underset{\sim}{K}\underset{\sim}{q}_o = \underset{\sim}{F}_o + \underset{\sim}{F}_o^{NL} \tag{A.117}$$

Since $\underset{\sim}{q}_o$, $\underset{\sim}{F}_o$ and $\underset{\sim}{F}_o^{NL}$ are all prescribed, one can find $\ddot{\underset{\sim}{q}}_o$. Then, one can calculate $\underset{\sim}{q}_1$ from Eq. A.116 since all quantities on the right-hand side of Eq. A.116 are either prescribed or otherwise known. Now one can evaluate $\underset{\sim}{F}_1^{NL}$. Next, one can find $\dot{q}_1$ by using the Houbolt operator:

$$\dot{\underset{\sim}{q}}_1 = \frac{3\underset{\sim}{q}_1 - 4\underset{\sim}{q}_o + \underset{\sim}{q}_{-1}}{2\Delta t} \tag{A.118}$$

where $\underset{\sim}{q}_{-1}$ is given by Eq. A.115c. This ends the starting procedure.

Thereafter one uses Eq. A.114 as modified by Eq. A.115, followed by Eq. A.111 for times $t_2$, $t_3$, $t_4$, ... to obtain the transient response solution.

The matrices $\underset{\sim}{M}$ and $\underset{\sim}{K}$, and the time-step size, $\Delta t$, are held constant throughout the timewise solution. In order to solve Eq. A.114 for $\underset{\sim}{q}^*_{m+1}$, the triple-factoring form of Gauss-Jordan elimination is used. The matrix sum $[\frac{2}{\Delta t^2} \underset{\sim}{M} + \underset{\sim}{K}]$ is thus formed and factored prior to the first time step. At each time step, $\underset{\sim}{q}^*_{m+1}$ is obtained by a back-substitution operation. The increased computation time and storage realized when using a consistent (fully populated) element mass matrix formulation instead of a lumped (diagonal) element mass matrix formulation is also evidenced by Eq. A.114. In the case of a lumped mass formulation, the storage requirements for $\underset{\sim}{M}$ are the same as for a vector of length equal to the total number of degrees of freedom, whereas if a consistent mass formulation is used, $\underset{\sim}{M}$ requires the same storage space as $\underset{\sim}{K}$ which could be as high as 2 to 3 orders of magnitude greater than the total number of degrees of freedom. Also, the matrix product $\underset{\sim}{M}\dot{\underset{\sim}{q}}^*_m$, for example, will require more computation time when a consistent mass formulation is used.

## A.4 Description of the Mechanical-Sublayer Material Model

In the present analysis, the strain-hardening behavior of the material is characterized by using the mechanical sublayer material model [6,7][+]. In this model, the uniaxial stress-strain curve (tension or compression ) is first approximated by (n+1) piecewise-linear segments defined at the coordinates $(\sigma_k, \varepsilon_k)$, k = 1,2,3 ... n as shown in Fig. A.1a. Next the material is conceived of as behaving at any material point as n equally-strained "sublayers" of elastic, perfectly-plastic behavior, with each sublayer having the same elastic modulus, E, but an appropriately different static yield stress, $\sigma_{ok}$, (see Fig. A.1b) given by

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[+]See page 6 for pertinent comments and additional references.

$$\sigma_{ok} = E\,\epsilon_k \qquad (k = 1, 2, 3, \ldots, n) \tag{A.119a}$$

for application to uniaxial conditions, but by

$$\sigma_{ok} = E\,\epsilon_k + \frac{E(\frac{1}{2}-\nu)}{(1+\nu)} \sum_{j=1}^{k} \left(1 - \frac{E_j}{E}\right)[\epsilon_j - \epsilon_{j-1}] \tag{A.119b}$$

for application to multiaxial conditions [7,14]. The stress value, $\sigma_k$, associated with the kth sublayer can be defined uniquely by the strain history and the value of the strain and strain-rate present at that point. The stress, $\sigma$, at that material point corresponding to strain, $\varepsilon$, is then obtained as a weighted average of the stresses $\sigma_k(\varepsilon)$ in each sublayer

$$\sigma(\epsilon) = \sum_{k=1}^{n} C_k\,\sigma_k(\epsilon) \tag{A.120}$$

where the weighting factor $C_k$ for the kth sublayer for uniaxial applications is given by

$$C_k = \frac{E_k - E_{k+1}}{E} \tag{A.120a}$$

and that for application to multiaxial conditions is given by

$$C_k = 1 - \frac{\frac{2}{3}(1+\nu)\,E_{k+1}}{E - \frac{2}{3}(\frac{1}{2}-\nu)\,E_{k+1}} - \sum_{j=1}^{k-1} C_j \tag{A.120b}$$

and where

$$E_1 \equiv E\,, \quad E_k = \frac{\sigma_k - \sigma_{k-1}}{\epsilon_k - \epsilon_{k-1}} \quad (k = 2, 3, \ldots, n)\,;\; E_{n+1} = 0 \qquad \text{(A.120 )}$$

Two special subcases are evident; for elastic, perfectly-plastic materials only one sublayer is required, and for elastic, linear strain-hardening materials two sublayers are required with the value of the yield stress in the second sublayer taken sufficiently high so that the behavior in that sublayer remains elastic.

The relation between sublayer stresses and material point stress given by Eq. A.120 is written in terms of uniaxial stresses. For the plane-stress state in the plate elements, it is assumed that the uniaxial stresses in Eq. A.120 may be replaced by the corresponding stress vectors so that

$$\underset{\sim}{\sigma} = \sum_{k=1}^{n} C_k \; \underset{\sim}{\sigma}_k(\underset{\sim}{\epsilon}) \tag{A.121}$$

where $\underset{\sim}{\sigma}_k$ is the vector of stresses in the kth sublayer and $\underset{\sim}{\sigma}$ is the vector of stresses at the material point.

Strain-rate effects may also be easily accommodated in the mechanical sublayer model, by assuming that the yield stress in each sublayer follows the relation

$$\sigma_{k_y} = \sigma_{ok}\left(1 + \left|\frac{\dot{\epsilon}}{D}\right|^{\frac{1}{p}}\right) \tag{A.122}$$

where D and p are material constants and $\sigma_{y_k}$ is the current yield stress in sublayer k corresponding to a strain-rate $\dot{\epsilon}$, and $\sigma_{ok}$ is the static yield stress of sublayer k. Figures A.2a and A.2b illustrate the strain-rate effects for an elastic, perfectly-plastic material, and for a strain-hardening material which is represented by the mechanical sublayer model, respectively.

It should be noted that the material strain-rate parameters D and p are assumed in the PLATE code to be identical for each mechanical sublayer for a given material type. More comprehensive strain-rate behavior could be accommodated in the future by assuming different values of D and p in each sublayer. Also, the equivalent uniaxial strain rate $\dot{\epsilon}$ for the thin plate under the Kirchhoff assumption of no transverse normal or shear strain rate is calculated by[+]

$$\dot{\epsilon} = \frac{2}{3}\;\frac{1}{\Delta t}\left[\left(\Delta\tilde{\epsilon}_{xx}\right)^2 + \left(\Delta\tilde{\epsilon}_{yy}\right)^2 - \Delta\tilde{\epsilon}_{xx}\,\Delta\tilde{\epsilon}_{yy} + \frac{3}{4}\left(\Delta\tilde{\epsilon}_{xy}\right)^2\right]^{\frac{1}{2}} \tag{A.123}$$

where $\Delta(\;)$ are the increments of ( ) over the current time step. The value of each incremental strain can be obtained from Eqs. A.12, and the value of $\dot{\epsilon}$ calculated from Eq. A.123 is then used in Eq. A.122.

[+]See pp. 35-45 of Ref. 28.

Note, as depicted in Fig. A.1c, that unloading, reversed loading and straining, reloading, etc. are accounted for very simply and efficiently (computationally) by this model. As noted in Subsection 2.1, this mechanical-sublayer material model accounts for nonlinear strain hardening, anisotropic strain hardening, the formation of corners on the yield surface, and the Bauschinger effect -- all of these are features that ductile metals exhibit. No other material model devised to date appears to represent these features as faithfully and efficiently as does the mechanical-sublayer model.

## A.5 Evaluation of the Elastic Strain Energy

The elastic strain energy per unit volume is given, in general, for isotropic material by:

$$\bar{U} = \frac{1}{2E}\left[\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2\right] - \frac{\nu}{E}\left[\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx}\right] + \frac{1}{2G}\left[\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2\right] \tag{A.124}$$

### A.5.1 Plate Element Evaluation

For the plane stress (plate element) state where $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$, this reduces to:

$$\bar{U} = \frac{1}{2E}\left[\sigma_{xx}^2 + \sigma_{yy}^2\right] - \frac{\nu}{E}\left[\sigma_{xx}\sigma_{yy}\right] + \frac{1}{2G}\left[\sigma_{xy}^2\right] \tag{A.125}$$

Hence, for a plate element of dimensions a, b, h in directions x, y, and z respectively, the elastic strain energy in the plate element becomes

$$U = \iiint \bar{U}\, dV = \int_{-\frac{a}{2}}^{+\frac{a}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \int_{-\frac{h}{2}}^{+\frac{h}{2}} \bar{U}\; dx\, dy\, dz$$

$$= \frac{abh}{8} \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} \left[\frac{1}{2E}\left(\sigma_{xx}^2 + \sigma_{yy}^2\right) - \frac{\nu}{E}\sigma_{xx}\sigma_{yy} + \frac{1}{2G}\sigma_{xy}^2\right] d\bar{x}\, d\bar{y}\, d\bar{z} \tag{A.126}$$

where $\bar{x} = x/(a/2)$, $\bar{y} = y/(b/2)$, and $\bar{z} = z/(h/2)$. The quantity U for each plate element is evaluated by Gaussian quadrature with 3 x-direction, 3 y-direction, and 4 depthwise Gaussian stations at each of the 9 spanwise Gaussian stations.

### A.5.2 Evaluation for Stiffener Elements

Each stiffener behaves in a Bernoulli-Euler fashion. Hence, only the axial stress ($\sigma_{xx}$ for an x-direction stiffener or $\sigma_{yy}$ for a y-direction stiffener) contributes to the elastic strain energy. Accordingly for an x-direction stiffener of length $\ell_s$, width $w_s$, and height $h_s$, the elastic strain energy is given by

$$U = \iiint \bar{U}\, dx\, dy\, dz \equiv w_s \int_{-\frac{\ell_s}{2}}^{+\frac{\ell_s}{2}} \int_{-\frac{h_s}{2}}^{+\frac{h_s}{2}} \frac{\sigma_{xx}^2}{2E}\, dx\, dz$$

$$= \frac{w_s\, \ell_s\, h_s}{4} \int_{-1}^{+1} \int_{-1}^{+1} \frac{\sigma_{xx}^2}{2E}\, d\bar{\xi}\, d\bar{\zeta} \tag{A.127}$$

where $\bar{\xi} = x/(\ell_s/2)$ and $\bar{\zeta} = z/(h_s/2)$. In the computer program, U is evaluated by Gaussian quadrative by using 4 depthwise Gaussian stations at each of 3 spanwise Gaussian stations.

## A.6 Comments on Estimating the Plastic Work

For calculation convenience, the plastic work in the entire structure is evaluated at each time instant in an <u>indirect</u> fashion. For example, if the structure were subjected to only a prescribed <u>initial</u> velocity distribution, one would evaluate the associated initial kinetic energy ($KE_o$); thereafter at any time instant, one would evaluate (a) the total kinetic energy (KE) (b) the total elastic strain energy EE. Then the Plastic Work (PW) is estimated from

$$PW = (KE)_o - KE - EE \tag{A.128}$$

On the other hand if the structure were subjected only to a prescribed distribution and time history of externally applied forces, the work done on the structure by those forces from time zero until the present instant of time is computed and saved; this is called the work input (WI). In this case, the plastic work is estimated from:

$$PW = WI - KE - EE \tag{A.129}$$

The work input (WI) to the structure by, for example, the externally applied loads {F} may be evaluated by using

$$WI = \int_0^t \lfloor F \rfloor \{\dot{q}\}\, dt \tag{A.130}$$

where for evaluations up to time instant $t_n = n\Delta t$, one may use the trapezoidal rule to write

$$WI(t_n) = \frac{\Delta t}{2}(H_0 + H_1) + \frac{\Delta t}{2}(H_1 + H_2) + \cdots + \frac{\Delta t}{2}(H_{n-1} + H_n) \tag{A.131}$$

where $H_0 = \lfloor F_0 \rfloor \{\dot{q}_0\}$ = power input at t=0

$H_1 = \lfloor F_1 \rfloor \{\dot{q}_1\}$ = power input at t = 1$\Delta$t

$\vdots$

$WI(0) = 0$

This is the procedure employed in the computer program.

$$
\begin{array}{ll}
 & \begin{array}{cccccccccccc} \text{Side 3} \rightarrow u_4 & v_4 & w_4 & \theta_4 & \psi_4 & \chi_4 & u_3 & v_3 & w_3 & \theta_3 & \psi_3 & \chi_3 \\ \text{Side 1} \rightarrow u_1 & v_1 & w_1 & \theta_1 & \psi_1 & \chi_1 & u_2 & v_2 & w_2 & \theta_2 & \psi_2 & \chi_2 \end{array} \\
\begin{array}{c} u_4 \; u_1 \\ v_4 \; v_1 \\ w_4 \; w_1 \\ \theta_4 \; \theta_1 \\ \psi_4 \; \psi_1 \\ \chi_4 \; \chi_1 \\ u_3 \; u_2 \\ v_3 \; v_2 \\ w_3 \; w_2 \\ \theta_3 \; \theta_2 \\ \psi_3 \; \psi_2 \\ \chi_3 \; \chi_2 \end{array} &
\left[ \begin{array}{cccccccccccc}
\frac{k_x}{3} & & & & & & & & & & & \\
0 & \frac{k_y}{3} & & & & & & & & & & \\
0 & 0 & \frac{12}{35} k_z + \frac{6 k_\theta}{6 a^2} & & & & & & \text{SYMMETRIC} & & & \\
0 & 0 & \frac{11 k_z}{210} + \frac{k_\theta}{10 a^2} & \frac{k_z}{105} + \frac{2 k_\theta}{15 a^2} & & & & & & & & \\
0 & 0 & 0 & 0 & \frac{13 k_\psi}{35 b^2} & & & & & & & \\
0 & 0 & 0 & 0 & \frac{11 k_\psi}{210 b^2} & \frac{k_\psi}{105 b^2} & & & & & & \\
\frac{k_x}{6} & 0 & 0 & 0 & 0 & 0 & \frac{k_x}{3} & & & & & \\
0 & \frac{k_y}{6} & 0 & 0 & 0 & 0 & 0 & \frac{k_y}{3} & & & & \\
0 & 0 & \frac{k_z}{70} - \frac{6 k_\theta}{5 a^2} & \frac{13 k_z}{420} - \frac{k_\theta}{10 a^2} & 0 & 0 & 0 & 0 & \frac{13}{35} k_z + \frac{6 k_\theta}{5 a^2} & & & \\
0 & 0 & \frac{-13 k_z}{420} + \frac{k_\theta}{10 a^2} & \frac{-k_z}{140} - \frac{k_\theta}{30 a^2} & 0 & 0 & 0 & 0 & \frac{-11 k_z}{210} - \frac{k_\theta}{10 a^2} & \frac{k_z}{105} + \frac{2 k_\theta}{15 a^2} & & \\
0 & 0 & 0 & 0 & \frac{k_\psi}{70 b^2} & \frac{13 k_\psi}{420 b^2} & 0 & 0 & 0 & 0 & \frac{13 k_\psi}{35 b^2} & \\
0 & 0 & 0 & 0 & \frac{-13 k_\psi}{420 b^2} & \frac{-k_\psi}{140 b^2} & 0 & 0 & 0 & 0 & \frac{-11 k_\psi}{210 b^2} & \frac{k_\psi}{105 b^2}
\end{array} \right] \times a
\end{array}
$$

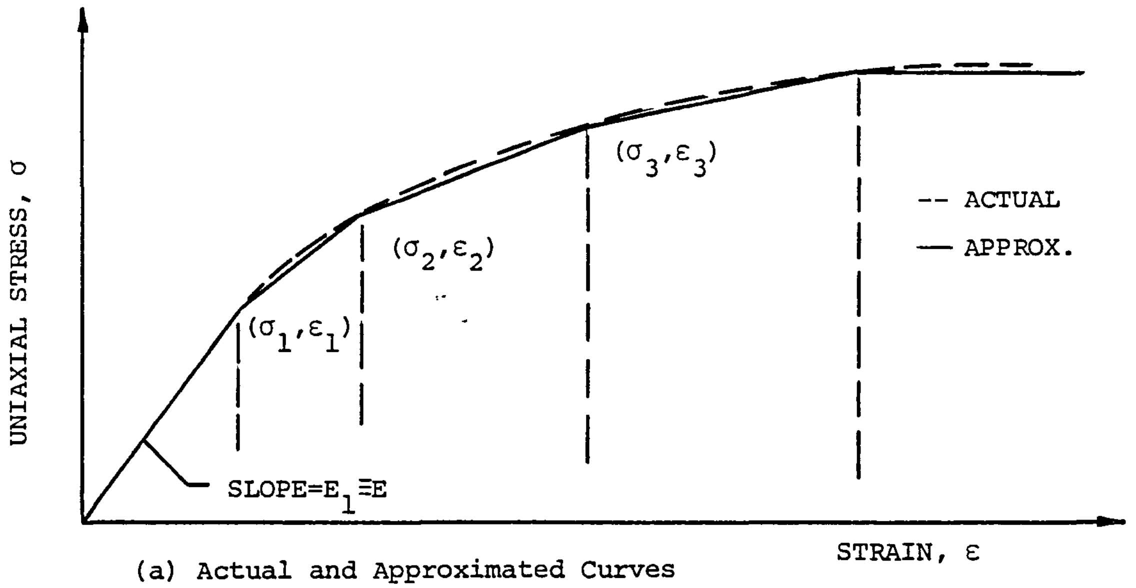
(a) $[\bar{k}_s]$ for Plate Sides 1 and 3

TABLE A.1 COMPACTED INTERMEDIATE STIFFNESS MATRIX $[\bar{k}_s]$ CORRESPONDING TO LINE ELASTIC RESTORING SPRINGS

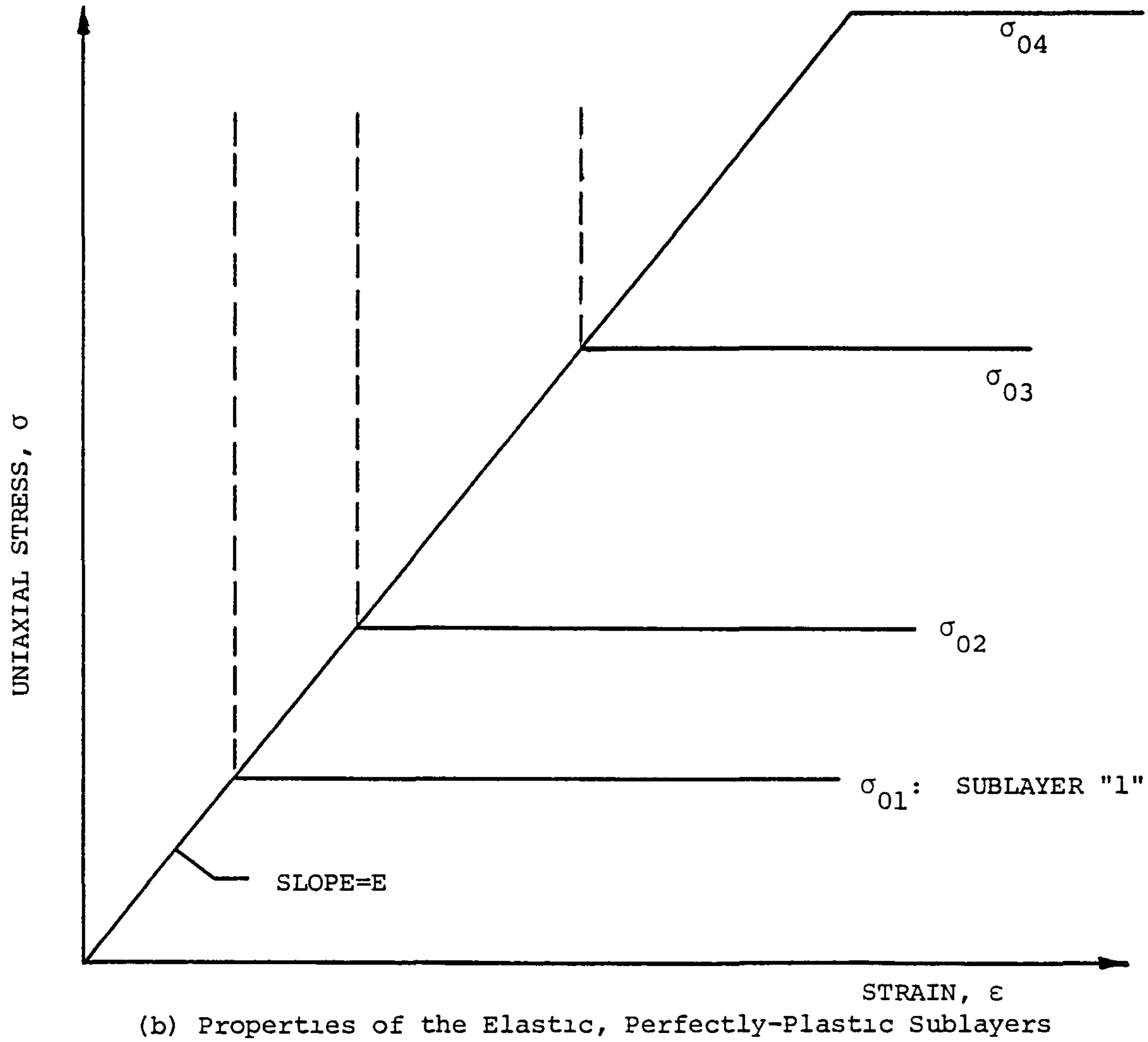
$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc}
\text{Side 2} \rightarrow u_2 & v_2 & w_2 & \theta_2 & \psi_2 & \chi_2 & u_3 & v_3 & w_3 & \theta_3 & \psi_3 & \chi_3 \\
\text{Side 4} \rightarrow u_1 & v_1 & w_1 & \theta_1 & \psi_1 & \chi_1 & u_4 & v_4 & w_4 & \theta_4 & \psi_4 & \chi_4
\end{array} \\
\begin{array}{cc}
\downarrow & \downarrow \\
u_2 & u_1 \\ v_2 & v_1 \\ w_2 & w_1 \\ \theta_2 & \theta_1 \\ \psi_2 & \psi_1 \\ \chi_2 & \chi_1 \\ u_3 & u_4 \\ v_3 & v_4 \\ w_3 & w_4 \\ \theta_3 & \theta_4 \\ \psi_3 & \psi_4 \\ \chi_3 & \chi_4
\end{array} &
\left[\begin{array}{cccccccccccc}
\frac{k_x}{3} & & & & & & & & & & & \\
0 & \frac{k_y}{3} & & & & & & & & & & \\
0 & 0 & \frac{13k_z}{35}+\frac{6k_y}{5b^2} & & & & & & & & & \\
0 & 0 & 0 & \frac{13k_\theta}{35a^2} & & & & & & & & \\
0 & 0 & \frac{11k_z}{210}+\frac{k_y}{10b^2} & 0 & \frac{k_z}{105}+\frac{2k_y}{15b^2} & & & \text{SYMMETRIC} & & & & \\
0 & 0 & 0 & \frac{11k_\theta}{210a^2} & 0 & \frac{k_\theta}{105a^2} & & & & & & \\
\frac{k_x}{6} & 0 & 0 & 0 & 0 & 0 & \frac{k_x}{3} & & & & & \\
0 & \frac{k_y}{6} & 0 & 0 & 0 & 0 & 0 & \frac{k_y}{3} & & & & \\
0 & 0 & \frac{k_z}{70}-\frac{6k_y}{5b^2} & 0 & \frac{13k_z}{420}-\frac{k_y}{10b^2} & 0 & 0 & 0 & \frac{13k_z}{35}+\frac{6k_y}{5b^2} & & & \\
0 & 0 & 0 & \frac{k_\theta}{70a^2} & 0 & \frac{13k_\theta}{420a^2} & 0 & 0 & 0 & \frac{13k_\theta}{35a^2} & & \\
0 & 0 & \frac{-13k_z}{420}+\frac{k_y}{10b^2} & 0 & \frac{-k_z}{140}-\frac{k_y}{30b^2} & 0 & 0 & 0 & \frac{-11k_z}{210}-\frac{k_y}{10b^2} & 0 & \frac{k_z}{105}+\frac{2k_y}{15b^2} & \\
0 & 0 & 0 & \frac{-13k_\theta}{420a^2} & 0 & -\frac{k_\theta}{140a^2} & 0 & 0 & 0 & \frac{-11k_\theta}{210a^2} & 0 & \frac{k_\theta}{105a^2}
\end{array}\right] \times b
\end{array}
$$

(b) $[\bar{k}_s]$ for Plate Sides 2 and 4

TABLE A.1 CONCLUDED

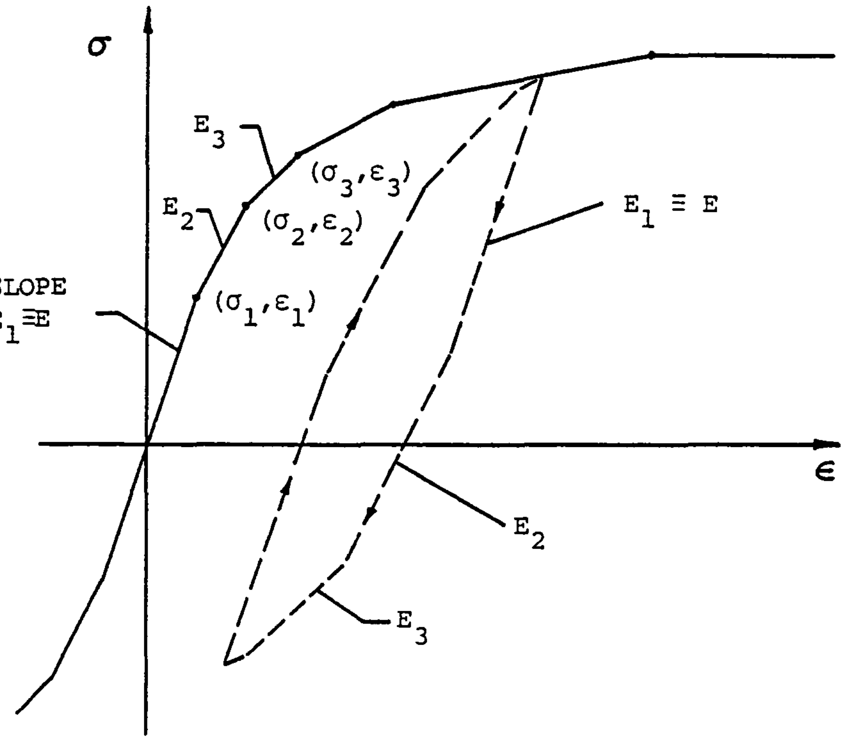



(a) Actual and Approximated Curves



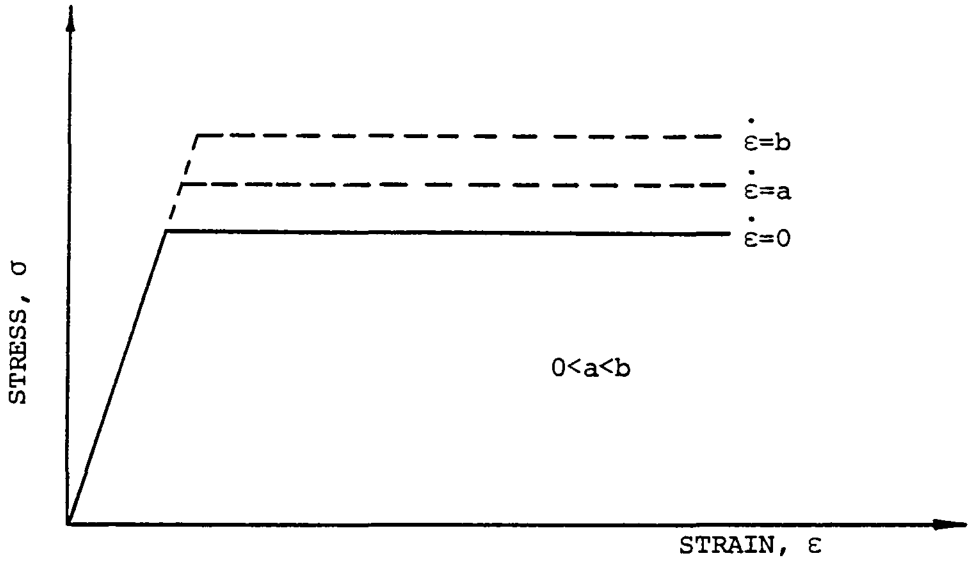

(b) Properties of the Elastic, Perfectly-Plastic Sublayers

FIG. A.1 APPROXIMATION OF A UNIAXIAL STRESS-STRAIN CURVE BY THE MECHANICAL-SUBLAYER MODEL

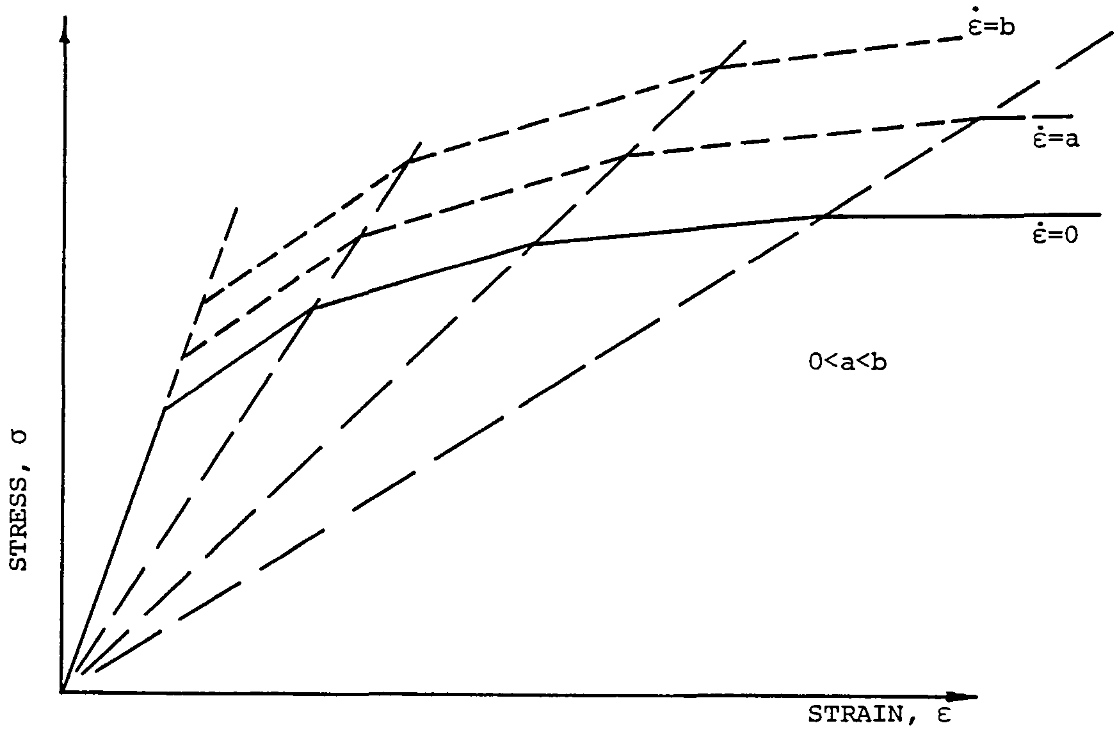



(c) Schematic of Loading, Unloading, and Reloading Paths

FIG. A.1 CONCLUDED




(a) Elastic, Perfectly-Plastic Material




(b) Special Strain Hardening Material

FIG. A.2 SCHEMATIC OF STRAIN-RATE DEPENDENT UNIAXIAL STRESS-STRAIN CURVES

APPENDIX B

GOVERNING EQUATIONS ON WHICH THE CIVM-PLATE PROGRAM IS BASED

## B.1 Introduction

The CIVM-PLATE program is designed to predict the large-deflection, elastic-plastic, transient responses of stiffened and/or unstiffened initially-flat plates subjected to impact by an idealized rigid spherical fragment having specified translational and/or rotational velocity components prior to impact. Much of the theoretical basis of the CIVM-PLATE program is identical to that of the PLATE program; this is evidenced by the large number of computer subroutines shared by the PLATE and CIVM-PLATE program (see Sections 3 and 6). The key difference between the two programs is the cause of structural response. In the PLATE program, structural response is a result of prescribed initial nodal velocities or prescribed externally-applied forces. In the CIVM-PLATE program, structural response is a result of fragment/plate impact and interaction; no other form of "external loading" is permitted.

The governing equations used to formulate the element property matrices for unstiffened and stiffened plate elements also apply to the CIVM-PLATE program and the reader is invited to consult Subsections A.1 and A.2 for a discussion of these developments. However, the following exceptions should be noted. Since no externally-applied forces are allowed in the CIVM-PLATE program, the development of nodal force vectors given in Subsection A.1.3.4 is not required in the CIVM-PLATE program. Also, in order that the mass properties of the plate used in the transient structural response calculations be consistent with those assumed in the impact-interaction calculations, only lumped (diagonal) mass matrices (Eqs. A.30 and A.31, and A.92) are used in the CIVM-PLATE program.

To predict the transient response of the plate structure, programming logic must be introduced which (1) detects the time and location on the plate of a plate/fragment collision, (2) predicts the interaction between the plate and fragment during the collision, and (3) moves the transient structural

response calculations ahead in time. The formulation of the equations governing the collision inspection/interaction analysis is given in Subsection B.2, and a description of the solution of the governing dynamic equations of motion of the rigid fragment and the structure is given in Subsection B.3. Finally, the step-by-step program flow of the collision inspection/interaction analysis is discussed in Subsection B.4.

A detailed list of the assumptions which apply to the collision inspection/interaction analysis as well as a terse description of each of these procedures is given in Subsection 2.4. It is recommended that the information in that subsection as well as Fig. 7 be reviewed.

## B.2 Plate/Fragment Collision Inspection/Interaction Analysis

### B.2.1 Inspection for Location of Plate/Fragment Collision

The first task in the collision inspection/interaction analysis is to determine if a plate/fragment collision has occurred during the current time step and, if so, to determine the position on the plate where the collision occurs; this is a problem of geometry. Previous inspection schemes for two-dimensional beam/ring impact codes such as CIVM-JET 4B [3] and CIVM-JET 5B [4] have been based on the approximation that the deformed beam/ring element is straight (for the sake of inspection calculations only); these schemes attempt not only to determine the point of collision but also to calculate the approximate instant of the time of collision. Such techniques require a knowledge (or assumption) of the structure nodal and fragment centroidal location as a function of time within each time step.

The calculation of the instant of collision is deemed not practical at present for plate/fragment impact problems. Instead, trial positions for the plate nodes and the centroid of the spherical fragment at the end of a time step are calculated. Based on these trial positions and various simplifying assumptions on the space occupancy of the plate elements between nodes, the space occupancy of the plate elements and the spherical fragment are compared; if any overlap of the plate and fragment are detected, a collision is said to have occurred during that time step. No attempt is made to calculate the precise instant in time of plate/fragment collision. Instead, the collision

is assumed to have occurred at the beginning of the time step, and the corrections invoked as a result of the impact interaction analysis are enforced at the beginning of the time step.

The inspection for plate fragment overlap is based on current trial positions of the plate nodes and fragment centroid at the end of each time step; time as a variable does not enter into these calculations. The inspection for position (on the plate) of plate/fragment overlap can be accomplished by inspecting for overlap in each element. However, this potentially time-consuming inspection process has been simplified in the CIVM-PLATE program. First, the distance from the fragment centroid to each nodal position is calculated and the node closest to the fragment centroid is identified. The plate/fragment overlap (if any) should be found in one of the elements connected to the identified node. At most, four plate elements can be connected to the identified node and thus the inspection is carried out element by element over these four (or fewer) elements.

The inspection procedure within each element must be simplified further. Prior to initial impact each plate element is flat, of rectangular planform, and of uniform thickness. After deformation the space occupancy of a single element is complicated and is a combination of the plate initial coordinates plus the assumed deformation shapes given by Eqs. A.3. Inspection using this complicated surface has been judged to be impractical and inefficient. Instead, the following simplified geometry has been used; as discussed in Subsection 2.4, the plate element is subdivided into two triangles using the convention shown in Fig. 6. A planar triangular region is then defined by the current location of the three nodes associated with the triangle. The inspection for each element is thus divided into two inspections, each over a triangular region. The remainder of this subsection is devoted to the presentation of the equations which may be used to determine whether overlap between the spherical fragment and a triangular planar region, and, if so, to calculate the amount of this overlap.

Assume a triangular planar region of uniform thickness, h, in global x,y,z space with the midsurface defined by the coordinates $(x_i, y_i)$, i=1,2,3 of the corners of the triangle. The idealized rigid spherical fragment of radius R is located by the coordinates $(x_f, y_f, z_f)$ of its centroid (Fig. 5) relative to the same global x,y,z system. The nodal numbering sequence for either triangular inspection subregion has been chosen (see Fig. B.1) in such a way that a vector, $\vec{V}_n$, which is normal to the triangular region (and therefore normal to the plane defined by the triangular region) can be defined by the vector cross-product

$$\vec{V}_n = \vec{V}_{12} \times \vec{V}_{13} \tag{B.1}$$

where $\vec{V}_{12}$ is the vector from node 1 to node 2 and $\vec{V}_{13}$ is the vector from node 1 to node 3 of the triangle. The vector $\vec{V}_n$ will be directed toward the <u>outer surface</u> of the plate, and it is assumed that impact always occurs on the <u>inner surface</u> of the plate. The vectors $\vec{V}_{12}$ and $\vec{V}_{13}$ can be defined in terms of the global coordinates of the nodes of the triangle. Using the notation $x_{ij}=x_i-x_j$ etc. to define the difference between nodal coordinates, the vector $\vec{V}_n$ can be evaluated from Eq. B.1 and is given by

$$\vec{V}_n = A\,\hat{i} + B\,\hat{j} + C\,\hat{k} \tag{B.2}$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the global x,y, and z directions and

$$\begin{aligned} A &= y_{21}\, z_{31} - y_{31}\, z_{21} \\ B &= x_{31}\, z_{21} - x_{21}\, z_{31} \\ C &= x_{21}\, y_{31} - x_{31}\, y_{21} \end{aligned} \tag{B.2a}$$

A coordinate transformation is required (for the later stages of collision inspection and for all phases of collision interaction to a system $\bar{x}\ \bar{y}\ \bar{z}$ such that the $\bar{z}$ axis (or N axis, as used in the impact interaction analysis) is normal to plane of impact (in the triangular region where overlap occurs) and the $\bar{x}$ and $\bar{y}$ axes lie in the impacted plane. This coordinate transformation will now be derived.

The vector $\vec{V}_n$ is normal to triangular plane and is directed along the $\bar{z}$ axis. The unit vector $\hat{\bar{k}}$ in the $\bar{z}$ direction can then be defined from Eq. B.2 as

$$\hat{\bar{k}} = \frac{A}{\ell_3}\hat{i} + \frac{B}{\ell_3}\hat{j} + \frac{C}{\ell_3}\hat{k} \quad \text{(B.3)}$$

where $\ell_3$ is the length of the vector $\vec{V}_n$ and is given by

$$\ell_3 = \left[A^2 + B^2 + C^2\right]^{\frac{1}{2}} \quad \text{(B.3a)}$$

In order to define the $\bar{x}$ and $\bar{y}$ coordinates which lie in the plane of the triangle, one of these directions may be chosen arbitrarily; in the present case, it is assumed that the $\bar{x}$ axis is directed from node 1 to node 2 of the triangle. The vector $\vec{V}_{12}$ is thus directed along the $\bar{x}$ axis and the unit vector $\hat{\bar{i}}$ along the $\bar{x}$ axis can be defined as

$$\hat{\bar{i}} = \frac{x_{21}}{\ell_1}\hat{i} + \frac{y_{21}}{\ell_1}\hat{j} + \frac{z_{21}}{\ell_1}\hat{k} \quad \text{(B.4)}$$

where $\ell_1$ is the length of the vector $\vec{V}_{12}$ given by

$$\ell_1 = \left[x_{21}^2 + y_{21}^2 + z_{21}^2\right]^{\frac{1}{2}} \quad \text{(B.4a)}$$

The unit vector $\hat{\bar{j}}$ along the $\bar{y}$ axis can then be defined in terms of $\hat{\bar{k}}$ and $\hat{\bar{i}}$ by the vector cross-product

$$\hat{\bar{j}} = \hat{\bar{k}} \times \hat{\bar{i}} \quad \text{(B.5)}$$

which upon substitution of Eqs. B.3 and B.4 becomes

$$\hat{\bar{j}} = \frac{(Bz_{21} - Cy_{21})}{\ell_1\ell_3}\hat{i} + \frac{(Cx_{21} - Az_{21})}{\ell_1\ell_3}\hat{j} + \frac{(Ay_{21} - Bx_{21})}{\ell_1\ell_3}\hat{k} \quad \text{(B.5a)}$$

The coordinate transformation matrix can now be defined. Consider any vector $\vec{V}$ having components $(V_x, V_y, V_z)$ in the xyz coordinate system, and components $(\bar{V}_x, \bar{V}_y, \bar{V}_z)$ in the $\bar{x}\,\bar{y}\,\bar{z}$ coordinate system. The vector $\bar{V}$ may be written in terms of components in either coordinate system and therefore the following equality must hold:

$$\bar{V}_x\hat{\bar{i}} + \bar{V}_y\hat{\bar{j}} + \bar{V}_z\hat{\bar{k}} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k} \quad \text{(B.6)}$$

The relation between components $(\bar{V}_x, \bar{V}_y, \bar{V}_z)$ and $(V_x, V_y, V_z)$ is obtained by first dotting both sides of Eq. B.6 by $\hat{\bar{i}}$, then dotting both sides of Eq. B.6 by $\hat{\bar{j}}$, and finally, dotting both sides of Eq. B.6 by $\hat{\bar{k}}$. The three equations thus obtained can be written in matrix form as

$$\begin{Bmatrix} \bar{V}_x \\ \bar{V}_y \\ \bar{V}_z \end{Bmatrix} = \begin{bmatrix} \hat{i}\cdot\hat{\bar{i}} & \hat{j}\cdot\hat{\bar{i}} & \hat{k}\cdot\hat{\bar{i}} \\ \hat{i}\cdot\hat{\bar{j}} & \hat{j}\cdot\hat{\bar{j}} & \hat{k}\cdot\hat{\bar{j}} \\ \hat{i}\cdot\hat{\bar{k}} & \hat{j}\cdot\hat{\bar{k}} & \hat{k}\cdot\hat{\bar{k}} \end{bmatrix} \begin{Bmatrix} V_x \\ V_y \\ V_z \end{Bmatrix} \tag{B.7}$$

or

$$\underset{\sim}{\bar{V}} = \underset{\sim}{T}\,\underset{\sim}{V} \tag{B.7a}$$

The matrix $\underset{\sim}{T}$ is the required transformation matrix to obtain components of a vector in the $\bar{x}\ \bar{y}\ \bar{z}$ coordinate system in terms of components in the xyz coordinate system. The inverse of Eq. B.7a can be obtained by noting that $\underset{\sim}{T}$ is an orthogonal matrix (having the property that $\underset{\sim}{T}^{-1} = \underset{\sim}{T}^{T}$) so that

$$\underset{\sim}{V} = \underset{\sim}{T}^{T}\,\underset{\sim}{\bar{V}} \tag{B.7b}$$

The unit vector dot products in Eq. B.7 can be evaluated from Eqs. B.3, B.4, and B.5a giving $\underset{\sim}{T}$ in terms of known nodal coordinates (and other previously-defined quantities related to nodal coordinates) by

$$\underset{\sim}{T} = \begin{bmatrix} \dfrac{x_{21}}{\ell_1} & \dfrac{y_{21}}{\ell_1} & \dfrac{z_{21}}{\ell_1} \\[2ex] \dfrac{(B z_{21} - C y_{21})}{\ell_1 \ell_3} & \dfrac{(C x_{21} - A z_{21})}{\ell_1 \ell_3} & \dfrac{(A y_{21} - B x_{21})}{\ell_1 \ell_3} \\[2ex] \dfrac{A}{\ell_3} & \dfrac{B}{\ell_3} & \dfrac{C}{\ell_3} \end{bmatrix} \tag{B.8}$$

The perpendicular distance, $d_{fp}$, between the fragment centroid and the plane of the triangle can be calculated by noting that by applying the above coordinate transformation, the triangular region now lies in a $\bar{z}$ = constant

plane. The distance $d_{fp}$ can be calculated as the difference between the $\bar{z}$ location of the fragment centroid (obtained by transformation of the fragment centroid coordinates $x_f$, $y_f$, and $z_f$ using the last row of $\underset{\sim}{T}$ and Eq. B.7a) and the $\bar{z}$ location of any triangle node (here the $\bar{z}$ location of node 1 is used, obtained by using the last row of $\underset{\sim}{T}$ and Eq. B.7a). Adopting the convention that a <u>negative value of $d_{fp}$ implies that the fragment centroid is below the plate</u> (in a negative $\bar{z}$ direction from the plate), $d_{fp}$ is given by

$$
\begin{aligned}
d_{fp} &= \bar{z}_f - \bar{z}_1 = T_{31}(x_f - x_1) + T_{32}(y_f - y_1) + T_{33}(z_f - z_1) \\
&= \frac{A(x_f - x_1) + B(y_f - y_1) + C(z_f - z_1)}{\left[A^2 + B^2 + C^2\right]^{\frac{1}{2}}}
\end{aligned}
\tag{B.9}
$$

Using the above-mentioned convention for $d_{fp}$, the penetration distance, $d_o$ (amount of plate/fragment overlap), can be calculated as

$$
d_o = \left(r_f + \frac{h}{2}\right) + d_{fp} \tag{B.10}
$$

where $r_f$ is the radius of the rigid spherical fragment and h is the (constant) thickness of the plate.

Impact can occur only if $d_o$ is <u>positive</u>. If $d_o \leq 0$, no further inspection is required; overlap cannot occur. If $d_o$ is positive, this implies that overlap occurs at some point located by coordinates $(x_n, y_n, z_n)$ in the plane which includes the triangular region. The coordinates $(x_n, y_n, z_n)$ of the point on the plane which is along the normal line from $(x_f, y_f, z_f)$ to the plane and at which the overlap $d_o$ is obtained must be found. The vector from $(x_f, y_f, z_f)$ to $x_n, y_n, z_n)$ has a magnitude of $d_{fp}$ and is directed in the positive $\bar{z}$ direction if $d_{fp}$ is negative. The components of this vector in the xyz coordinate system can be obtained by applying Eq. B.7b to give

$$
\begin{aligned}
\left(d_{fp}\right)_x &= -\frac{A\, d_{fp}}{\ell_3} \\
\left(d_{fp}\right)_y &= -\frac{B\, d_{fp}}{\ell_3} \\
\left(d_{fp}\right)_z &= -\frac{C\, d_{fp}}{\ell_3}
\end{aligned}
\tag{B.11}
$$

Hence, the coordinates $(x_n, y_n, z_n)$ can be obtained by adding these components to the fragment centroidal position as

$$x_n = x_f + \left(d_{fp}\right)_x = x_f - \frac{A\, d_{fp}}{\ell_3}$$
$$y_n = y_f + \left(d_{fp}\right)_y = y_f - \frac{B\, d_{fp}}{\ell_3} \qquad \text{(B.12)}$$
$$z_n = z_f + \left(d_{fp}\right)_z = z_f - \frac{C\, d_{fp}}{\ell_3}$$

where A, B, and C are given by Eqs. B.2a, $\ell_3$ by Eq. B.3a, and $d_{fp}$ is calculated from Eq. B.9. An inspection must be performed to determine if the point $(x_n, y_n, z_n)$ falls within the triangular inspection region.

The transformation given by Eq. B.7a can be applied to the coordinates of the corners of the triangle yielding $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$, $(\bar{x}_2, \bar{y}_2, \bar{z}_2)$, and $(\bar{x}_3, \bar{y}_3, \bar{z}_3)$ and to the point of maximum overlap $(x_n, y_n, z_n)$ to give $(\bar{x}_n, \bar{y}_n, \bar{z}_n)$ where $\bar{z}_1 = \bar{z}_2 = \bar{z}_3 = \bar{z}_n$. Triangular coordinates $(L_1, L_2, L_3)$ can be defined by (see, for example, Ref. 10)

$$L_1 = \left(a_1 + b_1 \bar{x} + c_1 \bar{y}\right) / (2A^*)$$
$$L_2 = \left(a_2 + b_2 \bar{x} + c_2 \bar{y}\right) / (2A^*) \qquad \text{(B.13)}$$
$$L_3 = \left(a_3 + b_3 \bar{x} + c_3 \bar{y}\right) / (2A^*)$$

where

$$a_1 = \bar{x}_2 \bar{y}_3 - \bar{x}_3 \bar{y}_2 \qquad b_1 = \bar{y}_2 - \bar{y}_3 \qquad c_1 = \bar{x}_3 - \bar{x}_2$$
$$a_2 = \bar{x}_3 \bar{y}_1 - \bar{x}_1 \bar{y}_3 \qquad b_2 = \bar{y}_3 - \bar{y}_1 \qquad c_2 = \bar{x}_1 - \bar{x}_3 \qquad \text{(B.13a)}$$
$$a_3 = \bar{x}_1 \bar{y}_2 - \bar{x}_2 \bar{y}_1 \qquad b_3 = \bar{y}_1 - \bar{y}_2 \qquad c_3 = \bar{x}_2 - \bar{x}_1$$

and $A^*$ is the area of the triangle and is given by

$$A^* = \frac{1}{2}\left(\bar{x}_2\bar{y}_3 + \bar{x}_1\bar{y}_2 + \bar{x}_3\bar{y}_1 - \bar{x}_2\bar{y}_1 - \bar{x}_3\bar{y}_2 - \bar{x}_1\bar{y}_3\right) \tag{B.13b}$$

It can be verified easily that for all values of $(\bar{x},\bar{y})$ within the triangle, the values of the triangular coordinates $(L_1,L_2,L_3)$ will be between 0 and 1. To check if point $(\bar{x}_n,\bar{y}_n)$ falls inside the triangular region, the coordinates $(\bar{x}_n,\bar{y}_n)$ are substituted into Eqs. B.13. If the point $(\bar{x}_n\bar{y}_n)$ is within the triangle, the following conditions must be satisfied:

$$\begin{aligned} 0 &\leq L_1(\bar{x}_n,\bar{y}_n) \leq 1 \\ 0 &\leq L_2(\bar{x}_n,\bar{y}_n) \leq 1 \\ 0 &\leq L_3(\bar{x}_n,\bar{y}_n) \leq 1 \end{aligned} \tag{B.14}$$

If any of Eqs. B.14 is violated, the point of maximum plane/fragment overlap is not within the triangular region and no plate/fragment overlap occurs in this triangular inspection region.

If Eqs. B.14 are satisfied, a plate/fragment impact has occurred during the current time step at point $(x_n,y_n)$ (or at point $(x_n,y_n,z_n)$) in this triangular inspection region with an overlap (penetration) of $d_o$ given by Eq. B.10. This procedure is followed for each of the two triangular inspection regions for each element in the region of interest. As a result of this inspection, it is possible that either (a) no positive penetrations or (b) one or more positive penetrations will be found. If no positive penetrations are found (i.e. $d_o \leq 0$ or Eqs. B.14 are not satisfied for all triangular inspection regions), fragment/plate impact has not occurred during the current time cycle.

If fragment/plate impact occurs during the current cycle, one or more positive $d_o$ (with Eqs. B.14 also satisfied) values will be found. If only one positive $d_o$ is detected, impact is said to occur at $(x_n,y_n,z_n)$ as described above. However, it should be recalled that no attempt is made in the present program to "subdivide" the time step, $\Delta t$, in order to identify precise times of impact, etc. Such schemes introduce unwanted complications and additional computations, as they tend to engender an excessive number of "special cases" and corresponding checks. Therefore, if more than one positive overlap is

detected during the inspection, no attempt is made to identify which "impact occurred first. Instead, the 2 or more impacts are assumed to have occurred simultaneously, and their effects are imposed (as described earlier) at the beginning of the time cycle.

The developments in this subsection have concentrated on a typical triangular inspection region. For purposes of impact correction (see the collision interaction analysis to be described in Subsection B.2.2), the point of impact and the plane of impact must be defined (along with the corresponding coordinate transformation matrix). If only one overlap occurs, the point and plane of impact, and transformation are as described earlier. If 2 or more overlaps occur, a modified definition is needed and is described in the following.

It is assumed that all points of overlap and associated planes of impact for the cycle have been identified. If more than one overlap has occurred, effective plate/fragment impact is said to occur at the location $(x_a, y_a, z_a)$ which is computed as the algebraic average of all points of plate/fragment overlap. The effective plane of impact and associated coordinate transformation must also be defined. The unit outward normal, $\hat{k}_a$, to the effective plane of impact is given by

$$\hat{k}_a = \frac{1}{\ell_3}\left(A_a \hat{i} + B_a \hat{j} + C_a \hat{k}\right) \tag{B.15a}$$

where $A_a$, $B_a$, and $C_a$ are the sum of the x, y, and z components, respectively, of the unit outward normals to each plane of overlap. Also,

$$\ell_3 = \sqrt{A_a^2 + B_a^2 + C_a^2} \tag{B.15b}$$

so that $\hat{k}_a$ is a unit vector. Note that this is analogous to defining the "average" plane of impact.

In order to define the necessary coordinate transformation matrix (analogous to Eqs. B.7-B.8), the inplane $\bar{x}$-$\bar{y}$ coordinate directions must be identified. Therefore, define a vector $\bar{C}_{\bar{x}}$ as

$$\bar{C}_{\bar{x}} = (x_i - x_a)\hat{i} + (y_i - y_a)\hat{j} + (z_i - z_a)\hat{k} \tag{B.15c}$$

where $(x_1, y_1, z_1)$ is some as yet unspecified location. Now require that $\bar{C}_{\bar{x}}$ define the $\bar{x}$ direction and, therefore, lie in a plane having the unit outward normal, $\hat{k}_a$. Thus, $(x_1, y_i, z_1)$ should be chosen such that

$$\hat{k}_a \cdot \bar{C}_{\bar{x}} = 0 \tag{B.15d}$$

Clearly, two of the coordinates can be chosen arbitrarily and the third determined from Eq. B.15d. The following choices satisfy Eq. B.15d:

$$\begin{aligned} (x_i - x_a) &= 1 \\ (y_i - y_a) &= 1 \\ (z_i - z_a) &= -\frac{(A_a + B_a)}{C_a} \equiv -d_a \end{aligned} \tag{B.15e}$$

so that the unit vector, $\hat{i}$, in the $\bar{x}$ direction is given by

$$\hat{i}_a = \frac{1}{\ell_1}(\hat{i} + \hat{j} - d_a \hat{k}) \tag{B.15f}$$

where

$$\ell_1 = \sqrt{2 + d_a^2} \tag{B.15g}$$

The unit vector, $\hat{j}_a$, in the $\bar{y}$ direction is then given by

$$\hat{j}_a = \hat{k}_a \times \hat{i}_a = \frac{1}{\ell_2}\left[(-B_a d_a - C_a)\hat{i} + (C_a + A_a d_a)\hat{j} + (A_a - B_a)\hat{k}\right] \tag{B.15h}$$

where

$$\ell_2 = \ell_1 \ell_3 \tag{B.15i}$$

With the coordinate directions of the effective plane of impact defined above, the coordinate transformation matrix, $\underset{\sim}{T}$, can be defined via Eqs. B.7 and B.7a with $\hat{i}$, $\hat{j}$, $\hat{k}$ in B.7 replaced by $\hat{i}_a$, $\hat{j}_a$, $\hat{k}_a$, respectively, defined in Eqs. B.15a, B.15f, and B.15h. The result is

$$\underset{\sim}{T} = \begin{bmatrix} \frac{1}{\ell_1} & \frac{1}{\ell_1} & -\frac{d_a}{\ell_1} \\ \frac{(-B_a d_a - C_a)}{\ell_2} & \frac{(C_a + A_a d_a)}{\ell_2} & \frac{(A_a - B_a)}{\ell_2} \\ \frac{A_a}{\ell_3} & \frac{B_a}{\ell_3} & \frac{C_a}{\ell_3} \end{bmatrix} \tag{B.15j}$$

It should be noted that the identification of an effective point of impact for multiple overlaps during a time step is used only in the general whole-plate analysis. For the symmetry cases, maximum positive penetration determines the point and plane of impact. Also note that the present effective point and plane of impact reduces to the actual point and plane of impact for cases where only one overlap is found. Finally, in subsequent discussion and descriptions, the terminologies "point of maximum penetration", "point of impact", and "point of effective impact" may be used interchangeably.

### B.2.2 Collision Interaction Analysis

Momentum and energy considerations are utilized to predict the interaction between the rigid fragment and the plate once a plate/fragment collision (overlap) is detected. First the region of the plate affected by the impact is defined; those nodes which fall in this region of the plate are termed the impact-affected nodes. The impact interaction analysis to be

described in this subsection is then used to predict the changes in velocity of the fragment and of the impact-affected nodes. This procedure is termed the Collision-Imparted-Velocity-Method (CIVM). The CIVM has been previously utilized in 2-D beam/ring codes [1,2,3,4], and is extended here for the present 3-D plate impact analysis.

As discussed in the previous subsection, no attempt is made to identify the precise instant in time (during the time step, $\Delta t$) at which impact occurs. Instead, when plate/fragment overlap is detected at the end of a time step, the velocity corrections (as a result of impact) are based on the positions and velocities at the beginning of the time step and corrections are imposed at the beginning of the time step.

Before calculating velocity corrections, the impact-affected region of the plate must be defined. For present purposes, the impact affected region of the plate (see Fig. B.2) is defined by a circle whose origin is at the calculated point of plate/fragment collision (maximum plate/fragment overlap). The effects of the impact are assumed to travel at the elastic longitudinal wave speed so that the radius of the circle, $L_{eff}$, can be defined as the distance traveled in the time step, $\Delta t$,

$$L_{eff} = \left[ \frac{E}{\rho_o (1 - \nu^2)} \right]^{\frac{1}{2}} \Delta t \tag{B.16}$$

where E, $\nu$, and $\rho_o$ are Young's modulus, the Poisson ratio, and the mass density of the plate material. Those plate nodes which fall within this circular region are the impact-affected nodes. Other estimates for $L_{eff}$ could, of course, be selected and used (see Card 70 in Subsection 5.2).

The impulse-momentum relations will be written for the fragment and each impact-affected node in the transformed coordinate system ($\bar{x}, \bar{y}$, N) --- (with N replacing $\bar{z}$ --- defined as the plane of impact (see Subsection B.2.1). Thus, components of velocity and impulse normal (N) and tangent ($\bar{x}, \bar{y}$) to the impact are considered.

For present purposes, a cone-shaped distribution[+] of impulse (centered at the point of impact) is assumed over the impact-affected region of the plate.

[+]This is similar to the procedure used by Gerstle [15,16,17].

The impulse components, $p_{i\bar{x}}$, $p_{i\bar{y}}$, and $p_{iN}$ applied at node i in the impact-affected region are then given by

$$p_{i\bar{x}} = c\left(1 - \frac{|s_i|}{L_{eff}}\right)\tilde{p}_{\bar{x}} \equiv c\,\alpha_i'\,\tilde{p}_{\bar{x}}$$

$$p_{i\bar{y}} = c\left(1 - \frac{|s_i|}{L_{eff}}\right)\tilde{p}_{\bar{y}} \equiv c\,\alpha_i'\,\tilde{p}_{\bar{y}} \qquad \text{(B.17a)}$$

$$p_{iN} = c\left(1 - \frac{|s_i|}{L_{eff}}\right)\tilde{p}_{N} \equiv c\,\alpha_i'\,\tilde{p}_{N}$$

where $|s_i|$ is the distance from the ith impact-affected node to the point of impact (see Fig. B.2) and it is assumed that k nodes fall in the impact-affected region. The components $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ represent the total impulse applied to the plate and in an equal but opposed fashion to the fragment. The constant, c, is determined by requiring that the sum of the nodal impulse components be equal to the total imparted impulse, i.e.

$$\sum_{i=1}^{k} \begin{Bmatrix} p_{i\bar{x}} \\ p_{i\bar{y}} \\ p_{iN} \end{Bmatrix} = \begin{Bmatrix} \tilde{p}_{\bar{x}} \\ \tilde{p}_{\bar{y}} \\ \tilde{p}_{N} \end{Bmatrix} \qquad \text{(B.17b)}$$

substituting any one of Eqs. B.17a into Eq. B.17b yields

$$c = \frac{1}{\sum_{i=1}^{k}\left(1 - \frac{|s_i|}{L_{eff}}\right)} = \frac{1}{\sum_{i=1}^{k} \alpha_i'} \qquad \text{(B.18)}$$

It is assumed that only the translational components of nodal velocity are affected. Denoting by primes the "after-impact" translational and rotational (fragment only) velocities, the following impulse-momentum relations may be written for the fragment and the impact-affected plate nodes:

$\bar{x}$-Direction: Translational

$$m_f\left[v'_{f\bar{x}} - v_{f\bar{x}}\right] = -\tilde{p}_x \qquad \text{fragment}$$

(B.19a)

$$\left.\begin{aligned} m_1\left[v'_{1\bar{x}} - v_{1\bar{x}}\right] &= \alpha_1\,\tilde{p}_{\bar{x}} \\ m_2\left[v'_{2\bar{x}} - v_{2\bar{x}}\right] &= \alpha_2\,\tilde{p}_{\bar{x}} \\ &\vdots \\ m_k\left[v'_{k\bar{x}} - v_{k\bar{x}}\right] &= \alpha_k\,\tilde{p}_{\bar{x}} \end{aligned}\right\} \text{plate impact-affected nodes}$$

$\bar{y}$-Direction: Translational

$$m_f\left[v'_{f\bar{y}} - v_{f\bar{y}}\right] = -\tilde{p}_{\bar{y}} \qquad \text{fragment}$$

(B.19b)

$$\left.\begin{aligned} m_1\left[v'_{1\bar{y}} - v_{1\bar{y}}\right] &= \alpha_1\,\tilde{p}_{\bar{y}} \\ m_2\left[v'_{2\bar{y}} - v_{2\bar{y}}\right] &= \alpha_2\,\tilde{p}_{\bar{y}} \\ &\vdots \\ m_k\left[v'_{k\bar{y}} - v_{k\bar{y}}\right] &= \alpha_k\,\tilde{p}_{\bar{y}} \end{aligned}\right\} \text{plate impact-affected nodes}$$

Normal Direction: Translational

$$m_f\left[v'_{fN} - v_{fN}\right] = -\tilde{p}_N \qquad \text{fragment}$$

(B.19c)

$$\left.\begin{aligned} m_1\left[v'_{1N} - v_{1N}\right] &= \alpha_1\,\tilde{p}_N \\ m_2\left[v'_{2N} - v_{2N}\right] &= \alpha_2\,\tilde{p}_N \\ &\vdots \\ m_k\left[v'_{kN} - v_{kN}\right] &= \alpha_k\,\tilde{p}_N \end{aligned}\right\} \text{plate impact-affected nodes}$$

Rotational

(B.19d)

$$\left.\begin{aligned} I_f\left[\omega'_{f\bar{y}} - \omega_{f\bar{y}}\right] &= -r_f\,\tilde{p}_{\bar{x}} \\ I_f\left[\omega'_{f\bar{x}} - \omega_{f\bar{x}}\right] &= -r_f\,\tilde{p}_{\bar{y}} \\ \omega'_{fN} - \omega_{fN} &= 0 \end{aligned}\right\} \text{fragment}$$

where $m_f$, $I_f$, and $r_f$ are the mass, mass moment of inertia, and radius of the fragment, $m_i$ is the (diagonalized lumped) translational mass of node i, and

$\alpha_i (=c\alpha_i')$ is the proportionality constant associated with node i. The known translational velocity components of node i are $v_{i\bar{x}}$, $v_{i\bar{y}}$, and $v_{iN}$ and the known translational and rotational components of the fragment centroid are $v_{f\bar{x}}$, $v_{f\bar{y}}$, $v_{fN}$, $\omega_{f\bar{x}}$, $\omega_{f\bar{y}}$, and $\omega_{fN}$ (these are the transformed nodal and fragment velocities at the beginning of the time step). Note that the last of Eqs. B.19d simply indicates that there is no mechanism (i.e., friction, etc.) which can change the angular velocity of the fragment about the N axis. Once the impulse components $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, and $\tilde{p}_N$ are determined, the post-impact (primed) nodal and fragment velocities can be calculated from Eqs. B.19.

Analytic expressions for the impulse components $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ at the instant of termination of the impact are obtained by an extension of a graphical technique suggested by Goldsmith [18]. The motion of the system during the impact process is described by an image point, $\bar{P}$, in the $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ space (Fig. B.3). The coordinates of $\bar{P}$ at the termination of the impact process then define the impulse imparted during impact.

The relative velocity of sliding, S', and the relative velocity of approach, A', between the impact-affected plate nodes and the fragment at the point of contact can be defined by

$$\begin{aligned} S' \cos\bar{\theta} &= \left[ v_{f\bar{x}}' - r_f\, \omega_{f\bar{y}}' \right] - \sum_{i=1}^{k} \alpha_i\, v_{i\bar{x}}' \\ S' \sin\bar{\theta} &= \left[ v_{f\bar{y}}' - r_f\, \omega_{f\bar{x}}' \right] - \sum_{i=1}^{k} \alpha_i\, v_{i\bar{y}}' \\ A' &= v_{fN}' - \sum_{i=1}^{k} \alpha_i\, v_{iN}' \end{aligned} \qquad \text{(B.20)}$$

where $\bar{\theta}$ is the angle found by the direction of sliding with the $\bar{x}$ axis (see Fig. B.3). Substituting the impulse-momentum relations (B.19) into Eqs. B.20 yields relations in the form

$$\begin{aligned} S' \cos\bar{\theta} &= S_1 - B_1\, \tilde{p}_{\bar{x}} \\ S' \sin\bar{\theta} &= S_2 - B_2\, \tilde{p}_{\bar{y}} \\ A' &= A_o - B_3\, \tilde{p}_N \end{aligned} \qquad \text{(B.21)}$$

where $A_o$, $S_1$, $S_2$, $B_1$, $B_2$, and $B_3$ are given by

$$
\begin{aligned}
S_1 &= S_o \cos\bar{\theta} = \left[ v_{f\bar{x}} + r_f\, \omega_{f\bar{y}} \right] - \sum_{i=1}^{k} \alpha_i\, v_{i\bar{x}} \\
S_2 &= S_o \sin\bar{\theta} = \left[ v_{f\bar{y}} - r_f\, \omega_{f\bar{x}} \right] - \sum_{i=1}^{k} \alpha_i\, v_{i\bar{y}} \\
A_o &= v_{fN} - \sum_{i=1}^{k} \alpha_i\, v_{iN} \\
B_2 &= B_1 = \left[ \frac{1}{m_f} + \frac{r_f^2}{I_f} \right] + \sum_{i=1}^{k} \frac{\alpha_i^2}{m_i} \\
B_3 &= \frac{1}{m_f} + \sum_{i=1}^{k} \left( \alpha_i^2 / m_i \right)
\end{aligned}
\tag{B.22}
$$

It should be noted that, as defined, $B_1$ and $B_3$ must be non-negative. Also, the relative velocity of approach at the instant of initiation of contact, $A_o$, must be non-negative; if $A_o < 0$, the plate and fragment are moving apart and no collision occurs. If $S_1$ or $S_2$ are positive, the fragment slides initially along the plate segment; this sliding is assumed to occur at the value of limiting friction.

The no-sliding and maximum-approach conditions identify limiting conditions during the collision process. From the first two of Eqs. B.21, it is seen that the locus of no sliding, $S'=0$, is a straight line perpendicular to the $\tilde{p}_{\bar{x}}$ $\tilde{p}_{\bar{y}}$ plane, which intersects the $\tilde{p}_N=0$ plane at:

$$\tilde{p}_{\bar{x}} = \frac{S_1}{B_1} \quad , \quad \tilde{p}_{\bar{y}} = \frac{S_2}{B_1}$$

From the last of Eqs. B.21, the locus of maximum approach, $A'=0$, is seen to be a plane whose normal is in the $\tilde{p}_N$ direction. The plane intersects the $\tilde{p}_N$ axis at the point

$$\tilde{p}_N = \frac{A_o}{B_3}$$

Finally, the line of no sliding, $S'=0$, intersects the plane of maximum approach, $A'=0$, at the point (see Fig. B.3):

$$\tilde{p}_{\bar{x}} = \frac{S_1}{B_1} \quad , \quad \tilde{p}_{\bar{y}} = \frac{S_2}{B_1} \quad , \quad \tilde{p}_N = \frac{A_o}{B_3}$$

In order to obtain the impulse coordinates $\tilde{p}_{\bar{x}}$ $\tilde{p}_{\bar{y}}$ $\tilde{p}_N$ at the instant of termination of the collision, the motion of the image point, $\bar{P}$, in $\tilde{p}_{\bar{x}}$ $\tilde{p}_{\bar{y}}$ $\tilde{p}_N$

space, must be followed. The existence of a value of limiting friction, $\mu$, requires that the first stage of the path of $\bar{P}$ be governed by the relations

$$\frac{d\tilde{p}_{\bar{x}}}{\cos\bar{\theta}} = \frac{d\tilde{p}_{\bar{y}}}{\sin\bar{\theta}} = \mu\, d\tilde{p}_N \tag{B.23}$$

This condition restricts initial motion to be in the plane formed by the line S'=0 and the $\tilde{p}_N$ axis, at an angle

$$\nu = \tan^{-1}\mu \tag{B.24}$$

with respect to the $\tilde{p}_N$ axis. The subsequent motion of $\bar{P}$ will first be defined for the general case where $0<\mu<\infty$, then for the two special cases, $\mu=0$ (perfectly smooth contact surfaces) and $\mu=\infty$ (perfectly rough contact surfaces).

Case I: $0<\mu<\infty$

First the friction angle, $\nu$, is defined according to Eq. B.24. Next, the angle $\Lambda$ formed between the $\tilde{p}_N$ axis and the line from the origin to the intersection of S'=0 and A'=0 (see Fig. B.4) is defined by

$$\Lambda = \tan^{-1}\left[\frac{B_3}{A_0 B_1}\left(S_1^2 + S_2^2\right)^{\frac{1}{2}}\right] \tag{B.25}$$

Initially, the image point moves along the path which subtends an angle $\nu$ with the $\tilde{p}_N$ axis. Subsequent motion is described by the following two subcases.

Subcase Ia: $\mu=\tan\nu<\tan\Lambda$

The subsequent motion of $\bar{P}$ is shown in Fig. B.4a. The line $P_oL$ of initial motion intersects the plane of maximum approach, A'=0, at point $P_1$ before reaching the line of no sliding, S'=0. The intersection point $P_1$ represents the state at the instant of termination of the approach period. This is followed by a restitution period; the process ceases at point P' (path $P_o$-$P_1$-P'), whose coordinates are

$$\tilde{p}_{\bar{x}} = \mu(1+e)\, p_{N_1} \cos\bar{\theta}$$
$$\tilde{p}_{\bar{y}} = \mu(1+e)\, p_{N_1} \sin\bar{\theta} \qquad \text{(B.25)}$$

where $\tilde{p}_N = (1+e)\, p_{N_1}$

$$\cos\bar{\theta} = \frac{S_1}{[S_1^2 + S_2^2]^{\frac{1}{2}}} \qquad \text{(B.25a)}$$
$$\sin\bar{\theta} = \frac{S_2}{[S_1^2 + S_2^2]^{\frac{1}{2}}} \qquad p_{N_1} = \frac{A_0}{B_3}$$

and e is the coefficient of restitution.

Subcase Ib: $\mu=\tan\nu\geq\tan\Lambda$

The subsequent motion of the image point $\bar{P}$ for this subcase is illustrated in Fig. B.4b. The line $P_oL$ of initial motion will intersect the line of no sliding, S'=0, first at the point $P_2$ which marks the end of the initial sliding phase. The image point $\bar{P}$ will then move along the line of no sliding, through the intersection point $P_3$ with the plane of maximum approach, A'=0. This is followed by a restoration period, moving the image point to P' (path $P_o$-$P_2$-$P_3$-P'). The final impulse coordinate values are

$$\tilde{p}_{\bar{x}} = \frac{S_1}{B_1}$$
$$\tilde{p}_{\bar{y}} = \frac{S_2}{B_1} \qquad \text{(B.26)}$$
$$\tilde{p}_N = (1+e)\, p_{N_3} = (1+e)\frac{A_0}{B_3}$$

The special subcases will now be considered.

Case II: $\mu=0$ (perfectly smooth contact surfaces)

This situation is a special case of Subcase Ia in which line $P_oL$ coalesces with the $\tilde{p}_N$ axis. The image point will move along the $\tilde{p}_N$ axis through the plane A'=0 so that after a period of restoration, the final impulse coordinate values are

$$\tilde{p}_{\bar{x}} = \tilde{p}_{\bar{y}} = 0 \quad , \quad \tilde{p}_N = (1+e)\frac{A_0}{B_3} \qquad \text{(B.27)}$$

Case III: $\mu=\infty$ (completely rough contact surfaces)

The motion of $\bar{P}$ is similar to Subcase Ib except that $\bar{P}$ moves initially in the $\tilde{p}_N = 0$ plane to the line $S'=0$. The remaining motion along the line of no sliding to the end of impact is identical to Subcase Ib. The final impulse coordinates are thus

$$
\begin{aligned}
\tilde{p}_{\bar{x}} &= \frac{S_1}{B_1} \\
\tilde{p}_{\bar{y}} &= \frac{S_2}{B_1} \\
\tilde{p}_N &= (1+e)\frac{A_0}{B_3}
\end{aligned}
\tag{B.28}
$$

Once the values of $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, and $\tilde{p}_N$ are determined from Eqs. B.25, or B.26, or B.27, or B.28, the post-impact plate nodal and fragment centroidal velocity components can be calculated from the impulse-momentum relations (Eqs. B.19). For example, the normal components of post-impact velocity would be calculated as

$$
v'_{fN} = v_{fN} - \frac{\tilde{p}_N}{m_f} \qquad \text{fragment}
$$

$$
\left.
\begin{aligned}
v'_{1N} &= v_{1N} + \frac{\alpha_1}{m_1}\tilde{p}_N \\
v'_{2N} &= v_{2N} + \frac{\alpha_2}{m_2}\tilde{p}_N \\
&\ \ \vdots \\
v'_{kN} &= v_{kN} + \frac{\alpha_k}{m_k}\tilde{p}_N
\end{aligned}
\right\} \qquad \text{plate impact-affected nodes}
\tag{B.29}
$$

Similar relations are found for the $\bar{x}$ and $\bar{y}$ components.

It should be noted that the relations in this subsection have been written in the transformed $(\bar{x},\bar{y},N)$ system. When the procedure is linked with the dynamic structural response calculations, it is necessary to transform impact-affected nodal and fragment velocities to the $(\bar{x},\bar{y},N)$ system before calculation of $\tilde{p}_{\bar{x}}, \tilde{p}_{\bar{y}}$, and $\tilde{p}_N$. After calculation of the post-impact nodal and fragment velocities, these quantities will be transformed back to the $(x,y,z)$ coordinate system.

### B.2.3 Collision Inspection/Interaction Analysis for Cases of Single and Double Symmetry

For the general case of impact on a plate, the inspection and interaction analyses discussed in Subsections B.2.1 and B.2.2 must be applied. In some

of these problems, the computıon tımes and computer core storage requirements can become excessive. However, in a number of practical problems, the geometry of the plate, locations of plate/fragment impact, and subsequent interaction may be such that the response of the plate (and fragment) will be symmetric with respect to one or two planes. In these cases, it is necessary to model only one-half (in the case of a single symmetry) or one-quarter (in the case of a double symmetry) of the plate; as a result, considerable savings in computation time and computer core storage can be realized.

In these cases, the collision inspection is simplified and the collision interaction analysis must be modified to reflect the single- and double-symmetry conditions; that is, the resulting plate nodal and fragment centroidal post-impact velocities obtained from the symmetric analysis must be identical to those which would be obtained from the corresponding analysis of the whole plate. Details of these modifications for both single and double symmetry are given next.

B.2.3.1 Inspection/Interaction Analysis for Single Symmetry

Without loss of generality, the following two cases of single symmetry can be identified: (1) the case where the x-axis (xz plane) is the line of symmetry, and (2) where the y axis (yz plane) is the line of symmetry. The case where the x axis is the line of symmetry will be considered first.

In order that the x axis be a line of symmetry, the fragment coordinates and velocity must satisfy

$$\left.\begin{aligned} y_f &= 0 \\ v_{f_y} &= 0 \\ \omega_{f_y} &= 0 \end{aligned}\right\} \quad \text{at } t = 0 \qquad \text{(B.30)}$$

In addition, the results obtained from the collision interaction analysis must satisfy Eqs. B.30 for all time. The collision inspection procedure used in the general case is also used here. When plate/fragment impact is detected, a specialized collision interaction analysis is followed.

In the case of single symmetry (about the xz plane) each impact may be considered as a single impact occurring along the line of symmetry at location

($x_c$,o) with two points of contact located symmetrically with respect to the line of symmetry (i.e. at ($x_c$,$y_c$) abd ($x_c$,$-y_c$)) as shown in Fig. B.5a. Once the point of effective impact is identified, a coordinate transformation is defined such that the $\bar{x}$ axis is along the line between the nodes on the line of symmetry which bound the point of effective impact; the N coordinate is then defined as normal to the line connecting these nodes (see Fig. B.5b). The impact interaction analysis is then performed in the ($\bar{x}$,y,N) system.

In order to perform the impact interaction analysis in the half-plate (HP) model, the general interaction equations for the whole-plate (WP) presented in Subsection B.2.2 are first reduced to the case of symmetry about the $\bar{x}$ axis. Denoting by a superscript "B" those plate nodes falling on the boundary (line of symmetry) and by "I" those nodes falling in the interior of the plate, and noting that $\tilde{p}_y$ must be zero by symmetry, the impulse-momentum equations may be written

$$\left.\begin{aligned} m_f\left[v'_{f\bar{x}} - v_{f\bar{x}}\right] &= -\tilde{p}_{\bar{x}} \\ m_f\left[v'_{fN} - v_{fN}\right] &= -\tilde{p}_N \\ I_f\left[\omega'_{fy} - \omega_{fy}\right] &= -r_f\,\tilde{p}_{\bar{x}} \end{aligned}\right\}\ \text{fragment}$$

$$\left.\begin{aligned} m_i^B\left[v^{B\prime}_{i\bar{x}} - v^B_{i\bar{x}}\right] &= \alpha_i^B\,\tilde{p}_{\bar{x}} \\ m_i^B\left[v^{B\prime}_{iN} - v^B_{iN}\right] &= \alpha_i^B\,\tilde{p}_N \end{aligned}\right\}\ \text{ith plate boundary node} \qquad \text{(B.31)}$$

$$\left.\begin{aligned} m_i^I\left[v^{I\prime}_{i\bar{x}} - v^I_{i\bar{x}}\right] &= \alpha_i^I\,\tilde{p}_{\bar{x}} \\ m_i^I\left[v^{I\prime}_{iN} - v^I_{iN}\right] &= \alpha_i^I\,\tilde{p}_N \end{aligned}\right\}\ \text{ith plate interior node}$$

The impact interaction constants ($A_o$,$S_1$,$S_2$, ...) will be given by

$$S_1 = S_o\cos\bar{\theta}_o = \left[v_{f\bar{x}} + r_f\,\omega_{fy}\right] - \left[\sum \alpha_i^B\,v^B_{i\bar{x}} + \sum \alpha_i^I\,v^I_{i\bar{x}}\right]$$

$$S_2 = S_o\sin\bar{\theta}_o = \left[\cancel{v_{fy}}^{\,0} - r_f\,\cancel{\omega_{f\bar{x}}}^{\,0}\right] - \left[\sum \alpha_i^B\,\cancel{v^B_{iy}}^{\,0} + \underbrace{\sum \alpha_i^I\,v^I_{iy}}_{=0}\right] = 0 \qquad \text{(B.32)}$$

$$A_o = v_{fN} - \left[\sum \alpha_i^B\,v^B_{iN} + \sum \alpha_i^I\,v^I_{iN}\right]$$

$$B_1 = B_2 = \frac{1}{m_f} + \frac{r_f^2}{I_f} + \sum \frac{\alpha_i^{B\,2}}{m_i^B} + \sum \frac{\alpha_i^{I\,2}}{m_i^I}$$

$$B_3 = \frac{1}{m_f} + \sum \frac{\alpha_i^{B\,2}}{m_i^B} + \sum \frac{\alpha_i^{I\,2}}{m_i^I}$$

The impulse coordinates $\tilde{p}_{\bar{x}}$ and $\tilde{p}_N$ can be obtained from Cases I through III in Subsection B.2.2 by substituting $S_2=0$ and by noting that $\tilde{p}_y=0$ for all cases. By examining the resulting equations, it is seen that the solutions for $\tilde{p}_{\bar{x}}$ and $\tilde{p}_N$ are functions of the ratios $A_o/B_3$ and $S_1/B_1$.

It is required to perform an analysis of the half-plate model and obtain the same post-impact velocities for the fragment and impact-affected plate nodes as in the whole-plate analysis. An examination of the expressions for $S_1$ and $A_o$ (Eqs. B.32) show that

$$\begin{aligned} (S_1)_{HP} &= (S_1)_{WP} \\ (A_o)_{HP} &= (A_o)_{WP} \end{aligned} \tag{B.33}$$

provided that the $\alpha_i^I$ terms are multiplied by 2.0 to account for the missing lower-half interior nodes in the summation.

Consider now the parameters $B_1$ and $B_3$ in Eqs. B.32 term by term. In the half-plate analysis, the mass, $m_i^B$, assigned to each boundary node is half that of the corresponding node in the whole-plate analysis, so that

$$\left( \sum \frac{\alpha_i^{B\,2}}{m_i^B} \right)_{HP} = 2 \left( \sum \frac{\alpha_i^{B\,2}}{m_i^B} \right)_{WP} \tag{B.34}$$

The summations over the interior nodes for the half-plate will be half that of the whole plate analysis (interior node masses will be the same for half and whole plate analysis) since only the upper half plate nodes are included. But the sum will increase by a factor of 4 as a result of the redefining of $\alpha_i^I = 2\alpha_i^I$ so that

$$\left( \sum \frac{\alpha_i^{I\,2}}{m_i^I} \right)_{HP} = 2 \left( \sum \frac{\alpha_i^{I\,2}}{m_i^I} \right)_{WP} \tag{B.34b}$$

If the actual fragment mass, $m_f$, and mass moment of inertia, $I_f$, are reduced by a factor of 2 for half-plate analyses, then

$$\left(\frac{1}{m_f}\right)_{HP} = 2\left(\frac{1}{m_f}\right)_{WP} \tag{B.34c}$$

$$\left(\frac{r_f^2}{I_f}\right)_{HP} = 2\left(\frac{r_f^2}{I_f}\right)_{WP} \tag{B.34d}$$

As a result of Eqs. B.34, the parameters $(B_1)_{HP}$ and $(B_3)_{HP}$ for the half-plate are related to the equivalent whole-plate values by

$$\begin{aligned}(B_1)_{HP} &= 2\,(B_1)_{WP}\\ (B_3)_{HP} &= 2\,(B_3)_{WP}\end{aligned} \tag{B.35}$$

so that a combination of Eqs. B.33 and B.35 yields

$$\begin{aligned}\left(\frac{A_o}{B_3}\right)_{HP} &= \frac{1}{2}\left(\frac{A_o}{B_3}\right)_{WP}\\ \left(\frac{S_1}{B_1}\right)_{HP} &= \frac{1}{2}\left(\frac{S_1}{B_1}\right)_{WP}\end{aligned} \tag{B.36}$$

The total imparted impulses $\tilde{p}_{\bar{x}}$ and $\tilde{p}_N$ are functions of the ratios given in Eqs. B.36. Therefore, for all interaction cases where the x-axis is a line of symmetry

$$\begin{aligned}(\tilde{p}_{\bar{x}})_{HP} &= \frac{1}{2}(\tilde{p}_{\bar{x}})_{WP}\\ (\tilde{p}_N)_{HP} &= \frac{1}{2}(\tilde{p}_N)_{WP}\end{aligned} \tag{B.37}$$

The post-impact plate node and fragment velocities for the case of symmetry would be calculated from Eqs. B.31 for the whole-plate analysis. However, it can be easily verified that in the half-plate analysis by redefining (scaling) $\alpha_1^I$, $m_f$, and $I_f$ as discussed, and as a result of the boundary plate node masses being $(m_i^B)_{HP} = \frac{1}{2}(m_i^B)_{WP}$ and the result given by Eq. B.37, all scaling factors cancel in Eqs. B.31. As a result, the post-impact plate node and fragment velocities calculated by the half-plate analysis will be the same as those obtained in the whole-plate analysis.

The relations established and programming logic used for general (whole-plate) impact interaction calculations can be used for the present single symmetry case in which the x-axis is the line of symmetry, provided the following modifications are made:

(1) Scale the fragment mass, $m_f = \frac{1}{2}m_f$, and fragment mass moment of inertia, $I_f = \frac{1}{2}I_f$, prior to the first time step.

(2) Calculate all $\alpha_i$ as if performing a whole plate analysis (Eqs. B.17a and B.18).

(3) Scale $\alpha_i^I$ corresponding to interior nodes by $\alpha_i^I = 2\alpha_i^I$.

(4) Set $S_2 = 0$.

(5) Solve impact-interaction problem in transformed ($\bar{x}$-N) system, using the same logic as in the whole plate analysis. Note that the nodal velocities in the y-direction must remain unchanged and the fragment velocities must satisfy Eqs. B.30.

A second case of single symmetry is one in which the y-axis (yz plane) is the line of symmetry. In this case, the fragment coordinates and velocity must satisfy

$$\begin{aligned} x_f &= 0 \\ v_{fx} &= 0 \\ \omega_{fx} &= 0 \end{aligned} \tag{B.38}$$

at all instants of time. Each impact may be considered as a single impact occurring along the line of symmetry at location $(o,y_c)$ with two points of simultaneous contact located symmetrically with respect to the line of symmetry (i.e. at $(x_c,y_c)$ and $(-x_c,y_c)$) as shown in Fig. B.6a, and directed normal to line of symmetry. Once the point of effective impact is identified, a coordinate transformation is defined such that the $\bar{y}$ axis is along the line between the nodes on the line of symmetry which bounds the point of effective impact; the N direction is defined as normal to the line connecting these nodes (see Fig. B.6b). The impact interaction calculations are performed in the $(x,\bar{y},N)$ system.

The development of an impact interaction analysis for this case of single symmetry follows steps similar to those for the previous single symmetry case (x-axis being the line of symmetry). However, in the present case where the y-axis is the line of symmetry, the impulse $\tilde{p}_x$ must be zero by symmetry; also, the parameter $S_1$ (Eq. B.22) will be zero, while $S_2$ will be nonzero. The impulse coordinates $\tilde{p}_{\bar{y}}$ and $\tilde{p}_N$ will be obtained from Cases I through III in Subsection B.2.2 by substituting $S_1=0$ and $\tilde{p}_x=0$.

It can be shown that the relations established and the program logic used for the general impact interaction calculations can be used to obtain the post-impact fragment and plate-node velocities for the present single-symmetry case (where the y-axis is the line of symmetry) provided the following modifications are made:

(1) Scale the fragment mass, $m_f = \frac{1}{2}m_f$, and fragment mass moment of inertia, $I_f = \frac{1}{2}I_f$, prior to the first time step.

(2) Calculate all $\alpha_i$ as if performing a whole-plate analysis (Eqs. B.17a and B.18).

(3) Scale $\alpha_i^I$ corresponding to interior nodes by $\alpha_i^I = 2\alpha_i^I$.

(4) Set $S_1=0$.

(5) Solve the impact-interaction problem in the transformed $(x,\bar{y},N)$ system, using the same logic as for general impact cases. Note that nodal velocities in the x direction must remain unchanged and the fragment velocities must satisfy Eqs. B.38.

B.2.3.2 Inspection/Interaction Analysis for Double Symmetry*

In cases where both the x and y axes are lines of symmetry and where the initial and all subsequent impacts occur at the center of the plate, double symmetry exists and the analysis may be reduced to a quarter of the plate structure; in the present case it is assumed that the first quadrant (see Fig. B.7) is analyzed and that the origin of the x,y system is at the center of the original whole plate.

In the case of double-symmetry, the following assumptions/simplifications can be made:

---

*This implies either frictionless impact-interaction or a non-spinning spherical fragment.

(1) Impact is treated as a single impact occurring at point (0,0) with 4 points of simultaneous contact, i.e. at $(x_c, y_c)$, $(-x_c, y_c)$, $(-x_c, -y_c)$, and $(x_c, -y_c)$ as shown in Fig. B.7.

(2) The net effective normal direction for the single impact is the global z-axis.

(3) No net impulse should be found in the global x or y directions at any plate nodes. This is based on the assumption that the impact-affected regions for each of the 4 simultaneous contact points overlap completely; in this case, the inplane x and y direction impulses must cancel exactly at all impact-affected plate nodes.

(4) As a consequence of (3), the impulse coordinates $\tilde{p}_x$ and $\tilde{p}_y$ will be zero; as a result, no post-impact velocity changes in the x and y direction will occur.

(5) The fragment coordinates and velocity must satisfy

$$\begin{aligned} x_f &= 0 \qquad & v_{fx} &= 0 \qquad & \omega_{fx} &= 0 \\ y_f &= 0 \qquad & v_{fy} &= 0 \qquad & \omega_{fy} &= 0 \end{aligned} \tag{B.39}$$

at all time instants.

The impact-interaction calculations are performed in the global (x,y,z) coordinate system (i.e. no transformation of coordinates is required). Since $\tilde{p}_x$ and $\tilde{p}_y$ are zero, only $\tilde{p}_z$ need be considered; again, it assumed that the impulse at node i is given by

$$p_{iz} = \alpha_i \tilde{p}_z \tag{B.40}$$

where the $\alpha_i$ are calculated as for the whole-plate analysis. Using superscripts B and I to refer to symmetry boundary and interior nodes, respectively, and noting that the center node is node number 1, the impulse-momentum relations for the double symmetry case are given as

$$
\begin{array}{lll}
\text{Fragment} & : & m_f \left[ v_{fz}' - v_{fz} \right] = -\tilde{p}_z \\
\text{Center Node} & : & m_1 \left[ v_{1z}' - v_{1z} \right] = \alpha_1 \tilde{p}_z \\
\text{Boundary Nodes} & : & m_i^B \left[ v_{iz}^{B\prime} - v_{iz}^B \right] = \alpha_i^B \tilde{p}_z \\
\text{Interior Nodes} & : & m_i^I \left[ v_{iz}^{I\prime} - v_{iz}^I \right] = \alpha_i^I \tilde{p}_z
\end{array}
\qquad \text{(B.41)}
$$

The only remaining nonzero interaction parameters are $A_o$ and $B_3$ (see Eqs. B.22) and are given by

$$
\begin{aligned}
A_o &= v_{fz} - \left[ \alpha_1 v_{1z} + \sum \alpha_i^B v_{iz}^B + \sum \alpha_i^I v_{iz}^I \right] \\
B_3 &= \frac{1}{m_f} + \frac{\alpha_1^2}{m_1} + \sum \frac{\alpha_i^{B\,2}}{m_i^B} + \sum \frac{\alpha_i^{I\,2}}{m_i^I}
\end{aligned}
\qquad \text{(B.42)}
$$

The impulse $\tilde{p}_z$ is obtained from Cases I through III of Subsection B.2.2 with $S_1=S_2=0$, and reduces to the same expression for all cases:

$$
\tilde{p}_z = (1+e)\frac{A_o}{B_3} \qquad \text{(B.43)}
$$

It should be noted that the value of $\tilde{p}_z$ is independent of the coefficient of friction, $\mu$, since no relative sliding takes place.

It is required to perform the interaction on the quarter plate model (QP) and obtain the same post-impact velocities as would be obtained in the whole-plate (WP) analysis. The effects of the quarter-plate model on $A_o$ and $B_3$ must be investigated. The whole plate model would include each $\alpha_i^B v_{iz}^B$ term twice and each $\alpha_i^I v_{iz}^I$ term four times in $A_o$; thus, if the sums in $A_o$ apply only to the quarter-model, the $\alpha_i^B$ and $\alpha_i^I$ terms must be scaled by

$$
\begin{aligned}
\left(\alpha_i^B\right) &= 2\left(\alpha_i^B\right) \\
\left(\alpha_i^I\right) &= 4\left(\alpha_i^I\right)
\end{aligned}
\qquad \text{(B.44)}
$$

so that

$$
\left(A_o\right)_{QP} = \left(A_o\right)_{WP} \qquad \text{(B.45)}
$$

Consider each term in $B_3$:

$\Sigma \frac{\alpha_i^{I2}}{m_i^I}$ : In the quarter model, the sum will be 1/4 that of the whole plate. The scaling of $\alpha_i^I = 4\alpha_i^I$ will increase the sum by a factor of 16, so that

$$\left(\sum \frac{\alpha_i^{I\,2}}{m_i^I}\right)_{QP} = 4\left(\sum \frac{\alpha_i^{I\,2}}{m_i^I}\right)_{WP} \tag{B.46a}$$

$\Sigma \frac{\alpha_i^{B2}}{m_i^B}$ : For the quarter model, the sum will be 1/2 that of the whole plate. Because of the scaling $\alpha_i^B = 2\alpha_i^B$ the sum will increase by a factor of 4. Finally, the nodal masses, $m_i^B$, generated internally for boundary nodes will be 1/2 of the value for a whole plate analysis. Thus,

$$\left(\sum \frac{\alpha_i^{B\,2}}{m_i^B}\right)_{QP} = 4\left(\sum \frac{\alpha_i^{B\,2}}{m_i^B}\right)_{WP} \tag{B.46b}$$

$\frac{\alpha_1^2}{m_1}$ : In the quarter model, the internally-generated mass at node 1 is 1/4 that of the whole plate analysis, so that

$$\left(\frac{\alpha_1^2}{m_1}\right)_{QP} = \left(\frac{\alpha_1^2}{m_1}\right)_{WP} \tag{B.46c}$$

$\frac{1}{m_f}$ : To achieve consistency with Eqs. B.46a - B.46c, the fragment mass used in the quarter-plate analysis is redefined to be one-quarter the actual fragment mass, so that

$$\left(\frac{1}{m_f}\right)_{QP} = 4\left(\frac{1}{m_f}\right)_{WP} \tag{B.46d}$$

As a result of Eqs. B.46 through B.46d, $(B_3)_{QP}$ for the quarter-plate will be

$$\left(B_3\right)_{QP} = 4\left(B_3\right)_{WP} \tag{B.47}$$

and by substituting Eqs. B.45 and B.47 into Eq. B.43

$$\left(\tilde{p}_z\right)_{QP} = \frac{1}{4}\left(\tilde{p}_z\right)_{WP} \tag{B.48}$$

Compare the impulse-momentum equations used to obtain post-impact velocities for the whole-plate analysis and for the quarter-plate analysis:

| Whole-Plate Analysis | Quarter-Plate Analysis |
|---|---|
| $m_f\left[v'_{fz} - v_{fz}\right] = -\tilde{p}_z$ | $\left(\frac{1}{4}m_f\right)\left[v'_{fz} - v_{fz}\right] = -\left(\frac{1}{4}\tilde{p}_z\right)$ |
| $m_1\left[v'_{1z} - v_{1z}\right] = \alpha_1\tilde{p}_z$ | $\left(\frac{1}{4}m_1\right)\left[v'_{1z} - v_{1z}\right] = \alpha_1\left(\frac{1}{4}\tilde{p}_z\right)$ |
| $m_i^B\left[v_{iz}^{B\,\prime} - v_{iz}^B\right] = \alpha_i^B\tilde{p}_z$ | $\left(\frac{1}{2}m_i^B\right)\left[v_{iz}^{B\,\prime} - v_{iz}^B\right] = \left(2\alpha_i^B\right)\left(\frac{1}{4}\tilde{p}_z\right)$ |
| $m_i^I\left[v_{iz}^{I\,\prime} - v_{iz}^I\right] = \alpha_i^I\tilde{p}_z$ | $m_i^I\left[v_{iz}^{I\,\prime} - v_{iz}^I\right] = \left(4\alpha_i^I\right)\left(\frac{1}{4}\tilde{p}_z\right)$ |

The scaling factors in the quarter-plate analysis cancel, so the same post-impact fragment and plate nodal velocities would be found using either analysis.

Thus, the relations established and program logic used for the general impact interaction calculations can be used in the case of a quarter-plate analysis provided the following modifications are made:

(1) Scale the fragment mass by $m_f = \frac{1}{4}m_f$.

(2) Calculate all $\alpha_i$ as if performing a whole plate analysis (Eqs. B.16 and B.18).

(3) Scale $\alpha_i^B$ corresponding to nodes on each symmetry boundary by $\alpha_i^B = 2\alpha_i^B$, and $\alpha_i^I$ corresponding to interior nodes by $\alpha_i^I = 4\alpha_i^I$.

(4) Set $S_1 = S_2 = 0$.

(5) Solve the impact-interaction problem in the global (x,y,z) system (no coordinate transformation), using the same logic as for the general interaction cases. Note that nodal velocities in the x and y directions will remain unchanged and that the fragment velocities will satisfy Eqs. B.39.

## B.3 Timewise Solution of the Governing Equations of Motion for the Fragment and the Plate

### B.3.1 Motion of the Fragment

The motion of the rigid spherical fragment is given in terms of the motion of its centroid. Neglecting the effects of gravity, the governing equations of motion for the fragment are given by

$$
\begin{aligned}
\ddot{x}_f &= 0 \qquad & \ddot{\theta}_x &= 0 \\
\ddot{y}_f &= 0 \qquad & \ddot{\theta}_y &= 0 \\
\ddot{z}_f &= 0 \qquad & \ddot{\theta}_z &= 0
\end{aligned}
\tag{B.49}
$$

Equations B.49 represent constant-velocity motion. Thus the Cartesian coordinates of the fragment at time $t_{m+1}$ can be obtained from the Cartesian coordinates and translational velocities of the fragment at time $t_m$ as

$$
\begin{Bmatrix} x_f \\ y_f \\ z_f \end{Bmatrix}_{m+1} = \begin{Bmatrix} x_f \\ y_f \\ z_f \end{Bmatrix}_{m} + \Delta t \begin{Bmatrix} v_{fx} \\ v_{fy} \\ v_{fz} \end{Bmatrix}
\tag{B.50a}
$$

The angular position of the fragment at time $t_{m+1}$ is given by

$$
\begin{Bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}_{m+1} = \begin{Bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}_{m} + \Delta t \begin{Bmatrix} \omega_{fx} \\ \omega_{fy} \\ \omega_{fz} \end{Bmatrix}
\tag{B.50b}
$$

where $\Delta t = t_{m+1} - t_m$ is the time increment.

The fragment velocities used in Eqs. B.50a and B.50b are the current velocities at time $t_m$. If no impact occurs during the time $t_m$ to $t_{m+1}$, these will be the same velocity values as for the previous time step. However, if impact is detected during the time $t_m$ to $t_{m+1}$, the fragment velocities used in Eqs. B.50a and B.50b will be the updated post-impact fragment velocities obtained from the collision interaction analysis described in Subsection B.2.2.

B.3.2 Motion of the Plate

The governing discretized equations of motion for the plate structure have been derived in Appendix A. For impact response problems, no prescribed, externally-applied loads are present. Thus, the equations of motion of the plate for plate/fragment impact response predictions are obtained from Eq. A.109 by setting $\underset{\sim}{F}=0$ and are

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{q}}^{*} + \underset{\sim}{K}\,\underset{\sim}{q}^{*} = \underset{\sim}{F}^{NL} \tag{B.51}$$

where $\underset{\sim}{M}$, $\underset{\sim}{K}$, $\ddot{\underset{\sim}{q}}^{*}$, $\underset{\sim}{q}^{*}$, and $\underset{\sim}{F}^{NL}$ are as defined following Eq. A.109.

The solution of Eq. B.51 is accomplished by applying a direct timewise integration scheme, which results in a recurrence relation from which displacements can be calculated at each discrete time instant. The (implicit) Houbolt operator is used (as in the PLATE program). Following the steps outlined by Eqs. A.110 through A.114, the following recurrence relation (corresponding to Eq. B.51) is obtained:

$$\left[\frac{2}{(\Delta t)^2}\underset{\sim}{M} + \underset{\sim}{K}\right]\underset{\sim}{q}^{*}_{m+1} = \underset{\sim}{F}^{NL}_{m+1} + \frac{2}{\Delta t}\underset{\sim}{M}\left[\dot{\underset{\sim}{q}}^{*}_{m} + \frac{1}{\Delta t}\underset{\sim}{q}^{*}_{m}\right] \tag{B.52}$$

Equation B.52 is solved for the unknown displacements $\underset{\sim}{q}^{*}_{m+1}$ at time $t_{m+1}$ based on the known quantities $\underset{\sim}{F}^{NL}_{m+1}$, $\dot{\underset{\sim}{q}}^{*}_{m}$, and $\underset{\sim}{q}^{*}_{m}$. It should again be noted that $\underset{\sim}{F}^{NL}_{m+1} = \underset{\sim}{F}^{NL}_{m+1}\,(\underset{\sim}{q}^{*}_{m+1})$; however, linear extrapolation of this quantity is used as given in Eq. A.115 with no iteration.

The form of Eq. B.52 is particularly suited to the present impact analysis. During a time step in which plate/fragment collision occurs, the nodal velocity vector $\dot{\underset{\sim}{q}}^{*}_{m}$ is updated to post-impact nodal velocities according to the collision interaction analysis described in Subsection B.2.2.

The overall solution procedure for a time step is as follows. Given $\underset{\sim}{q}^{*}_{m}$, $\dot{\underset{\sim}{q}}^{*}_{m}$, and $\underset{\sim}{F}^{NL}_{m+1}$ (evaluated by linear extrapolation, and based on $\underset{\sim}{q}^{*}_{m}$), a trial displacement vector, $\overset{T}{\underset{\sim}{q}}{}^{*}_{m+1}$, is evaluated from Eq. B.52. Based on these trial nodal displacements (and similar trial displacements of the fragment), the collision inspection is performed. If no plate/fragment overlap is found, the trial displacements $\overset{T}{\underset{\sim}{q}}{}^{*}_{m+1}$ are the correct nodal displacements. If plate/fragment overlap is detected, the collision interaction analysis is performed and the nodal velocities $\dot{\underset{\sim}{q}}^{*}_{m}$ are updated to their post-impact value (as are the

fragment velocities). A new trial displacement vector, $\overset{T}{\underset{\sim}{q}}{}^{*}_{m+1}$, is calculated from Eq. B.52 and a collision inspection is performed. This process is repeated until no plate/fragment overlap is detected; then the trial displacements, $\overset{T}{\underset{\sim}{q}}{}^{*}_{m+1}$, are said to be the actual displacements.

It should be recalled at this point that the corrections resulting from a plate/fragment impact are applied at the <u>beginning</u> of the time step during which the impact occurs; no attempt is made to identify the precise instant of impact. The advantage of this scheme is the resulting relative simplicity of the inspection/interaction procedure; few, if any, "special cases" need be considered. Impulse-momentum and conservation of kinetic energy are utilized to calculate the instantaneous post-impact nodal (and fragment) velocities which are applied at the beginning of a time step. It can be remarked that this interaction analysis neglects all other system forces/energies in the calculation of velocity. However, since all pertinent system energies are included in Eqs. B.51 and B.52, the <u>subsequent motion</u> of the plate as a result of impact does include all of the system forces/energies.

Once the displacements, $\underset{\sim}{q}{}^{*}_{m+1}$, have been calculated, the nodal velocities, $\dot{\underset{\sim}{q}}{}^{*}_{m+1}$ (needed to calculate $\underset{\sim}{q}{}^{*}_{m+2}$), can be evaluated. If no impacts occurred during the time $t_m$ to $t_{m+1}$, then Eq. A.11 (repeated here for convenience),

$$\dot{\underset{\sim}{q}}{}^{*}_{m+1} = \frac{1}{2\Delta t}\left(3\,\underset{\sim}{q}{}^{*}_{m+1} - 4\,\underset{\sim}{q}{}^{*}_{m} + \underset{\sim}{q}{}^{*}_{m-1}\right) \tag{B.53}$$

can be used; recall that this difference formula has the same truncation error as the Houbolt expression for $\ddot{\underset{\sim}{q}}{}^{*}_{m+1}$. If impact has occurred during this time step, a modified expression must be used:

$$\dot{\underset{\sim}{q}}{}^{*}_{m+1} = \frac{2}{\Delta t}\left(\underset{\sim}{q}{}^{*}_{m+1} - \underset{\sim}{q}{}^{*}_{m}\right) - \dot{\underset{\sim}{q}}{}^{*}_{m} \tag{B.54}$$

where $\dot{\underset{\sim}{q}}{}^{*}_{m}$ is the vector of post-impact (corrected) nodal velocities. The derivation and possible implications of the use of Eq. B.54 are now discussed.

The modified velocity expression given by Eq. B.54 can be derived in a manner analogous to the starting sequence required in the PLATE program (Eqs. A.116 - A.118). If $\underset{\sim}{q}{}^{*}_{m+1}$ is calculated from the <u>central difference</u> expression for $\dot{\underset{\sim}{q}}_{m}$:

$$\dot{\tilde{q}}_m = \frac{1}{2\,\Delta t}\left( \tilde{q}^{*}_{m+1} - \tilde{q}^{*}_{m-1} \right) \qquad \text{(B.55)}$$

and substituted into the Houbolt velocity expression (Eq. B.53) the modified velocity expression of Eq. B.54 is obtained. It should be noted that the velocity $\dot{\tilde{q}}^{*}_{m}$ in Eq. B.54 is the vector of post-impact (corrected) nodal velocities.

The unconditional stability of the Houbolt operator, represented by Eqs. A.112 and B.53, is well established for linear systems. For nonlinear systems, this operator is conditionally stable.+ For cycles in which no plate/fragment impacts occur, the Houbolt operator is used; whereas, for cycles in which impact occurs, the modified Houbolt operator, represented by Eqs. A.112 and B.54, is used. Numerical experience with the CIVM-PLATE program shows the following trends: (1) early in the timewise response, the impacts are widely spaced in time and the Houbolt operator is, in effect, used for nearly all cycles, (2) however, as the fragment velocity decreases and nears zero (prior to rebound), impacts may be found during nearly every cycle, so that the modified Houbolt operator dominates. In view of the fact that a large segment of timewise response could be governed by the modified Houbolt operator, it is desirable to obtain some idea of the stability characteristics of this operator.

The discussion of stability of the modified Houbolt operator is simplified if the operator can be shown to be analogous to some other operator. Consider the difference expressions corresponding to the Generalized Acceleration Method [8,19] as written here, for simplicity, for a one degree-of-freedom system:

$$x_{m+1} = x_m + (\Delta t)\,\dot{x}_m + (\Delta t)^2\left[\left(\tfrac{1}{2}-\beta\right)\ddot{x}_m + \beta\,\ddot{x}_{m+1}\right] \qquad \text{(B.56a)}$$

$$\dot{x}_{m+1} = \dot{x}_m + (\Delta t)\left[(1-\gamma)\,\ddot{x}_m + \gamma\,\ddot{x}_{m+1}\right] \qquad \text{(B.56b)}$$

where $\beta$ and $\gamma$ are parameters to be specified. A variety of difference expressions (both explicit and implicit) can be obtained by the choice of $\beta$ and $\gamma$. For the present case, substitution of $\beta=1/2$ into Eq. B.56a yields, after rearranging,

+This is the case if iteration is not used within a given time step $\Delta t$: iteration is not used here.

$$\ddot{x}_{m+1} = \frac{2(x_{m+1} - x_m)}{(\Delta t)^2} - \frac{2}{\Delta t}\dot{x}_m \tag{B.57}$$

which is identical to Eq. A.112 of the Houbolt operator. Substituting Eq. B.57 into Eq. B.56a, with $\gamma=1$, yields after rearranging:

$$\dot{x}_{m+1} = \frac{2(x_{m+1} - x_m)}{\Delta t} - \dot{x}_m \tag{B.58}$$

which is identical to the velocity expression (Eq. B.54) of the modified Houbolt operator. Thus, the modified Houbolt operator used for cycles in which impact occurs corresponds to a Generalized Acceleration Method with $\beta=1/2$ and $\gamma=1$. For linear systems, this algorithm can be shown to be unconditionally stable [19]; however, no data are available for nonlinear systems.

To obtain a comparison of the Houbolt and modified Houbolt operators for nonlinear systems, consider the one degree-of-freedom nonlinear spring/mass system with a cubic hardening spring governed by the equation

$$\ddot{x} + \omega^2(1 + A x^2)x = 0 \tag{B.59a}$$

subject to the initial conditions

$$x(0) = 0$$
$$\dot{x}(0) = B \tag{B.59b}$$

The predicted response for the displacement x versus time t for the case $\omega^2=100$, $A=10$, and $B=60$ is shown in Figs. B.8a and B.8b, for the Houbolt and modified Houbolt operators, respectively. Time step sizes of $\Delta t=0.001$, 0.01, and 0.02 are used in each case. For comparison, the time step bound of the central-difference operator for the linear equivalent of the above system is $\Delta t^{CD}_{max} = 2/\omega_{max} = 0.2$.

The following observations are made on the basis of comparison of results presented in Figs. B.8a and B.8b. For the case $\Delta t=0.02$, the results

by the Houbolt operator begin to diverge, whereas the results by the modified Houbolt operator become unstable very early in the timewise response. For the case $\Delta t=0.001$, the results by the modified Houbolt operator show more artificial damping and phase shift than the results by the Houbolt operator. For the case $\Delta t=0.01$, the results obtained by the Houbolt operator are in essential agreement with the results for $\Delta t=0.001$. The results obtained by the modified Houbolt operator illustrate substantial artificial damping and phase shift compared with the results for $\Delta t=0.001$.

The result of primary interest is stability. It appears that the <u>modified</u> Houbolt operator may require a somewhat smaller time step size than the Houbolt operator to ensure stability. Moreover, the question of stability is complicated by the fact that the velocity $\dot{\underset{\sim}{q}}{}^*_m$ in Eq. B.54 represents the post-impact velocity; thus, when impacts occur each cycle (as is characteristic of the time range near fragment rebound), a velocity <u>increment</u> is imposed at each time cycle. Such imposed velocity increments if sufficiently large compared with the actual (pre-impact) nodal velocities, could cause an instability. Such an instability has been observed in a test case of the CIVM-PLATE program; unrealistic and severely oscillatory strain values in the vicinity of the impact location near the time of fragment rebound have been observed. By reducing the time step size, this behavior was eliminated. In general, the guidelines suggested in Subsection 2.5 should be followed; if the above-mentioned instability is found, the time step size used should be reduced.

## B.4 Collision Inspection and Solution Procedure

The collision inspection and solution procedure used in the CIVM-PLATE program will now be discussed. The discussion pertains to all impact cases including the special symmetry cases. If special operations are required for symmetry cases, they are noted in each step. Therefore, unless otherwise noted, the step is assumed to apply to all cases. For the sake of convenience, the following notation (consistent with the variables input in CIVM-PLATE) is used to denote the various symmetry cases:

(1) NSYM=0 refers to general impact cases.

(2) NSYM=1 and IPSS=1 refers to the single symmetry case where the x axis is the line of symmetry.

(3) NSYM=1 and IPSS=4 refers to the single symmetry case where the y axis is the line of symmetry.

(4) NSYM=2 refers to the double-symmetry case.

The following procedure indicated in the flow diagram of Fig. 7 may be employed to predict the motions of the plate and the rigid spherical fragment, their possible collision, the resulting collision-imparted velocities experienced by each, and the subsequent motion of each body:

Step 1: Let it be assumed at time $t_m$ that the displacements $\underset{\sim}{q}^*_m$, and the displacement increments $\Delta \underset{\sim}{q}^*_m$ are known. One can then calculate the strain increments $\Delta \underset{\sim}{\varepsilon}_m$ at all Gaussian stations over the span and through the thickness of the plate.

Step 2: Using a suitable constitutive relation for the plate material, the stress increments $\Delta \underset{\sim}{\sigma}_m$ and the plastic strain increments $\Delta \underset{\sim}{\varepsilon}^p_m$ at corresponding Gaussian stations within each finite element can be determined from the known strain increments $\Delta \underset{\sim}{\varepsilon}_m$. This information together with the appropriate linear extrapolation permits determining the equivalent force vector $\underset{\sim}{F}^{NL}_{m+1}$ in Eq. B.52.

Step 3: Solve Eq. B.52 for the trial nodal displacements, $\overset{T}{\underset{\sim}{q}}{}^*_{m+1}$, and calculate the trial fragment positions $(\overset{T}{x}_f, \overset{T}{y}_f, \overset{T}{z}_f)_{m+1}$ from Eq. B.50a. Trial displacement increments $\Delta \overset{T}{\underset{\sim}{q}}{}^*_{m+1}$ $(= \overset{T}{\underset{\sim}{q}}{}^*_{m+1} - \underset{\sim}{q}^*_m)$ are then calculated for the plate nodes, and for the fragment (e.g. $\Delta \overset{T}{x}_{f_{m+1}} = \overset{T}{x}_{f_{m+1}} - x_{f_m}$). Since impact may have occurred between $t_m$ and $t_{m+1}$, the following sequence of steps may be employed to determine whether or not a collision occurred and, if so, to calculate the corrected post-impact plate-node and fragment <u>velocities</u> at time $t_m$.

Step 4: The current position of the plate nodes and fragment centroid are calculated based on the positions at $t_m$ and the trial displacement increments. The distance (squared) from the fragment centroid to each plate node is then calculated. The minimum of these distances and the corresponding node number are identified. The four elements which share the identified node will be examined for possible plate-fragment collision. For NSYM=1 only those nodes along the line of symmetry are checked and the two elements sharing the identified node are subject to collision inspection. For NSYM=2, this check is not made; instead, element number 1 is identified as the only element subject to collision inspection.

Step 4a: Following the procedure outlined in Subsection B.2.1, each of the elements subject to collision inspection is subdivided into two triangular subregions. Based on the current trial nodal and fragment positions, the penetration distance $d_o$ (Eq. B.10) is calculated for each triangular region. If $d_o>0$, it is determined whether or not the location of maximum overlap $(x_n,y_n,z_n)$ from Eq. B.12 falls within the triangular inspection region. If not, $d_o$ is assigned a negative value to indicate no plate/fragment overlap within the triangular inspection region.

Step 4b: A search is performed to identify the maximum of all $d_o$ values calculated. If all values of $d_o$ are less than or equal to zero, no collision has occurred and Step 7 is followed. For NSYM = 1 and NSYM = 2, if any of the $d_o$ values are positive, the element number and triangular subregion number corresponding to the maximum value of $d_o$ are identified. However, for NSYM = 0, an average of all detected points of overlap is used as the location of effective impact. The following sequence of steps is then followed to define the impact interaction parameters.

Step 5: The plate/fragment impact is assumed to occur at the global coordinates $(x_n,y_n,z_n)$ where the maximum $d_o$ value for NSYM = 1 or NSYM = 2 is calculated, or at the effective point of impact for NSYM = 0. However, if symmetry options are in effect, these values are modified according to the discussion in Subsection B.2.3, as follows.

(1) If NSYM=1 and IPSS=1, set $y_n=0$.

(2) If NSYM=1 and IPSS=4, set $x_n=0$.

(3) If NSYM=2, set $x_n=0$ and $y_n=0$.

Step 5a: Calculate the distance from $(x_n, y_n, z_n)$ to each plate node. If this distance is less than or equal to $L_{eff}$ (Eq. B.15 or otherwise specified), the node is identified as falling within the impact-affected zone on the plate. The proportionality factor $\alpha_i'$ (Eq. B.16) for each impact-affected node is calculated by

$$\alpha_i' = 1 - \frac{|s_i|}{L_{eff}} \tag{B.60}$$

where $|s_i|$ is the distance from the ith impact-affected node to $(x_n, y_n, z_n)$. If a symmetry option is in effect; some of the $\alpha_i'$ must be multiplied by a factor f (see Subsection B.2.3). Thus, each of the impact-affected nodes is inspected and the following values of f apply:

<u>For cases where NSYM=0</u>, no scaling is necessary (f=1).

<u>For cases where NSYM=1</u>: f=2 for all impact-affected nodes in the interior of the plate, and f=1 (no scaling) for impact-affected nodes on the symmetry boundary.

<u>For cases where NSYM=2</u>: f=1 (no scaling) for node number 1 (where lines of symmetry intersect), f=2 for all impact-affected nodes (except node 1) falling on a line of symmetry, and f=4 for all impact-affected nodes in the interior of the plate.

After all $\alpha_i'$ have been calculated (and scaled if necessary), the normalization factor, C, is calculated from Eq. B.18, and the impulse impulse proportionality factor $\alpha_i$ (see Eqs. B.19) for each impact-affected node is calculated from

$$\alpha_i = C\,\alpha_i' \tag{B.61}$$

Step 5b: For NSYM=1 and NSYM=2, the coordinate transformation matrix $\underset{\sim}{T}$ (Eq. B.7) is then defined corresponding to the triangular inspection region in which the maximum plate/fragment overlap was detected. For the cases NSYM=1, $\underset{\sim}{T}$ is shown in Fig. B.5b (for IPSS=1) or Fig. B.6b (for IPSS=4). For the case NSYM=2, no transformation is required (or, for consistency one may define $\underset{\sim}{T}$ to be the identity matrix, $\underset{\sim}{I}$). For the case NSYM=0, $\underset{\sim}{T}$ is defined by Eq. B.15j if more than one impact point is detected; however, if only one location of impact occurs, $\underset{\sim}{T}$ is evaluated from Eq. B.8.

Step 5c: The translational velocity components $(v_{x_i}, v_{y_i}, v_{z_i})$ at each impact-affected node are extracted from the nodal velocity vector $\dot{\underset{\sim}{q}}{}^*_m$ (at time $t_m$) and the components $(\bar{v}_{x_i}, \bar{v}_{y_i}, \bar{v}_{z_i})$ of velocity in the transformed coordinate system (normal and tangent to the plane of impact) at each impact-affected node are calculated from Eq. B.7a, using $\underset{\sim}{T}$ established in step 5b. The fragment translational velocity components $(v_{fx}, v_{fy}, v_{fz})_m$ and rotational velocity components $(\omega_{fx}, \omega_{fy}, \omega_{fz})_m$ at time $t_m$ are transformed using Eq. B.7a yielding components $(\bar{v}_{fx}, \bar{v}_{fy}, \bar{v}_{fz})$ and $(\bar{\omega}_{fx}, \bar{\omega}_{fy}, \bar{\omega}_{fz})$.

Step 5d: The impact interaction constants $S_1$, $S_2$, $A_o$, $B_1$, and $B_3$ are now calculated. For the cases NSYM=0 and NSYM=1, Eqs. B.22 are used to calculate these constants. However, when NSYM=1 and IPSS=1, $S_2$ must be set to zero; when NSYM=1 and IPSS=4, $S_1$ must be set to zero. For the case of NSYM=2, only $A_o$ and $B_3$ are nonzero and are calculated from Eqs. B.42.

Step 5e: The value of $A_o$ is checked. If $A_o \leq 0$, the plate and fragment are moving apart and no collision occurs; in this situation, Step 7 is followed next. If $A_o > 0$, the next steps are followed to calculate the post-impact nodal and fragment velocities.

Step 6: The total impulse imparted to the impact-affected nodes and to the fragment are calculated, and post-impact velocities are calculated according to the following steps.

Step 6a: The components $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ of the total imparted impulse are calculated corresponding to Case I, II, or III as presented in Subsection B.2.2. Note that if NSYM=0, the only nonzero impulse component is $\tilde{p}_N = \tilde{p}_z$ as given by Eq. B.43.

Step 6b: The collision-induced velocity differences (i.e. $v'_{f\bar{x}} - v_{f\bar{x}}$, $v'_{1\bar{x}} - v_{1\bar{x}}$, etc.) of the impact-affected plate nodes and the fragment are then calculated from Eqs. B.19 and transformed back to the global coordinate system by using Eq. B.7b. This yields velocity increments $(\Delta v_{xi}, \Delta v_{yi}, \Delta v_{zi})$ at each impact-affected plate node, and $(\Delta v_{fx}, \Delta v_{fy}, \Delta v_{fz})$ and $(\Delta\omega_{fx}, \Delta\omega_{fy}, \Delta\omega_{fz})$ for the fragment.

Step 6c: The velocity components of the impact-affected plate nodes and fragment are then updated to their post-impact values by

$$\begin{Bmatrix} v_{x_i} \\ v_{y_i} \\ v_{z_i} \end{Bmatrix}_m = \begin{Bmatrix} v_{x_i} \\ v_{y_i} \\ v_{z_i} \end{Bmatrix}_m + \begin{Bmatrix} \Delta v_{x_i} \\ \Delta v_{y_i} \\ \Delta v_{z_i} \end{Bmatrix} \tag{B.62a}$$

$$\begin{Bmatrix} v_{f_x} \\ v_{f_y} \\ v_{f_z} \end{Bmatrix}_m = \begin{Bmatrix} v_{f_x} \\ v_{f_y} \\ v_{f_z} \end{Bmatrix}_m + \begin{Bmatrix} \Delta v_{f_x} \\ \Delta v_{f_y} \\ \Delta v_{f_z} \end{Bmatrix} \tag{B.62b}$$

$$\begin{Bmatrix} \omega_{fx} \\ \omega_{fy} \\ \omega_{fz} \end{Bmatrix}_m = \begin{Bmatrix} \omega_{fx} \\ \omega_{fy} \\ \omega_{fz} \end{Bmatrix}_m + \begin{Bmatrix} \Delta \omega_{fx} \\ \Delta \omega_{fy} \\ \Delta \omega_{fz} \end{Bmatrix} \tag{B.62c}$$

The nodal velocity vector, $\dot{\tilde{q}}^*_m$, is then updated; the components of translational velocity in $\dot{q}^*_m$ corresponding to impact-affected nodes are <u>replaced</u> by the post-impact translational velocity components at those impact-affected nodes as calculated in Eq. B.62a. It

should be emphasized that the nodal velocities, $\dot{\underset{\sim}{q}}^*_m$, and fragment velocities (given by Eqs. B.62b and B.62c) at time $\underline{t_m}$ are now the updated post-impact values. It should also be noted that the effects of impact occurring in the range $t_m \le t \le t_{m+1}$ have been accounted for as updated (modified) nodal and fragment velocities at time $\underline{t_m}$ (i.e. at the beginning of the time step).

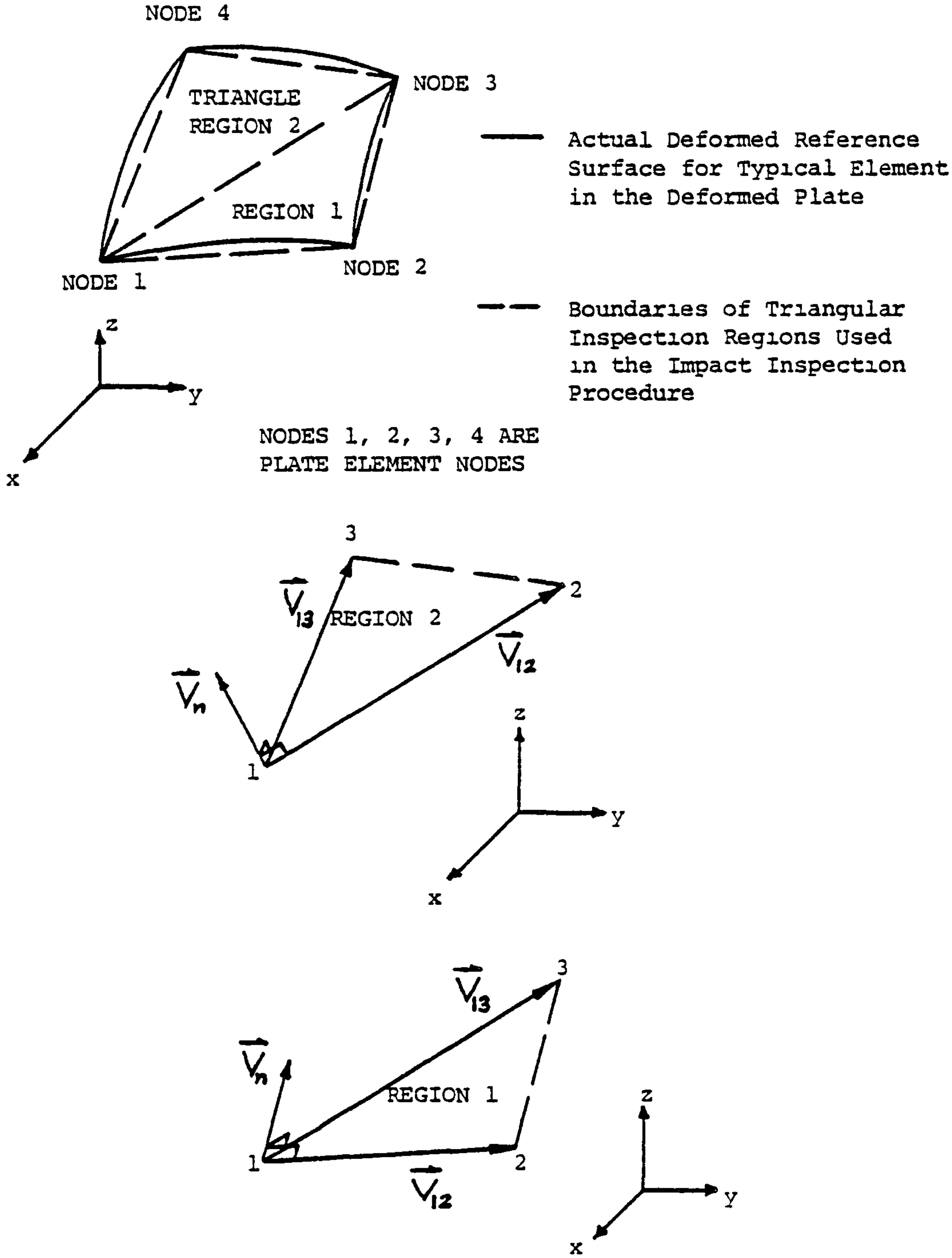
Step 7: The plate node and fragment positions are reduced back to their values at time $t_m$ using $\Delta\overset{T}{\underset{\sim}{q}}{}^*_{m+1}$ for the plate nodes and $(\Delta x^T_f, \Delta y^T_f, \Delta z^T_f)_{m+1}$ for the fragment.

Step 8: If no collision has been detected, Step 9 is followed next. If a plate/fragment collision has occurred, new trial nodal and fragment displacements must be calculated (now including the post-impact nodal and fragment velocities) at time $t_{m+1}$. Thus, steps 3 through 7 are again followed. This "iteration" process will continue until no plate/fragment overlap is detected or until a value $A_o \le 0$ is found.

Step 9: This step is reached when no (further) plate/fragment collisions are detected for the current time cycle. The trial plate node and fragment displacements become the actual (corrected-for-impact) displacements and the fragment and plate node positions at time $t_{m+1}$ are updated based on the actual displacement increments (e.g. $\Delta\underset{\sim}{q}^*_{m+1} = \underset{\sim}{q}^*_{m+1} - \underset{\sim}{q}^*_m$).

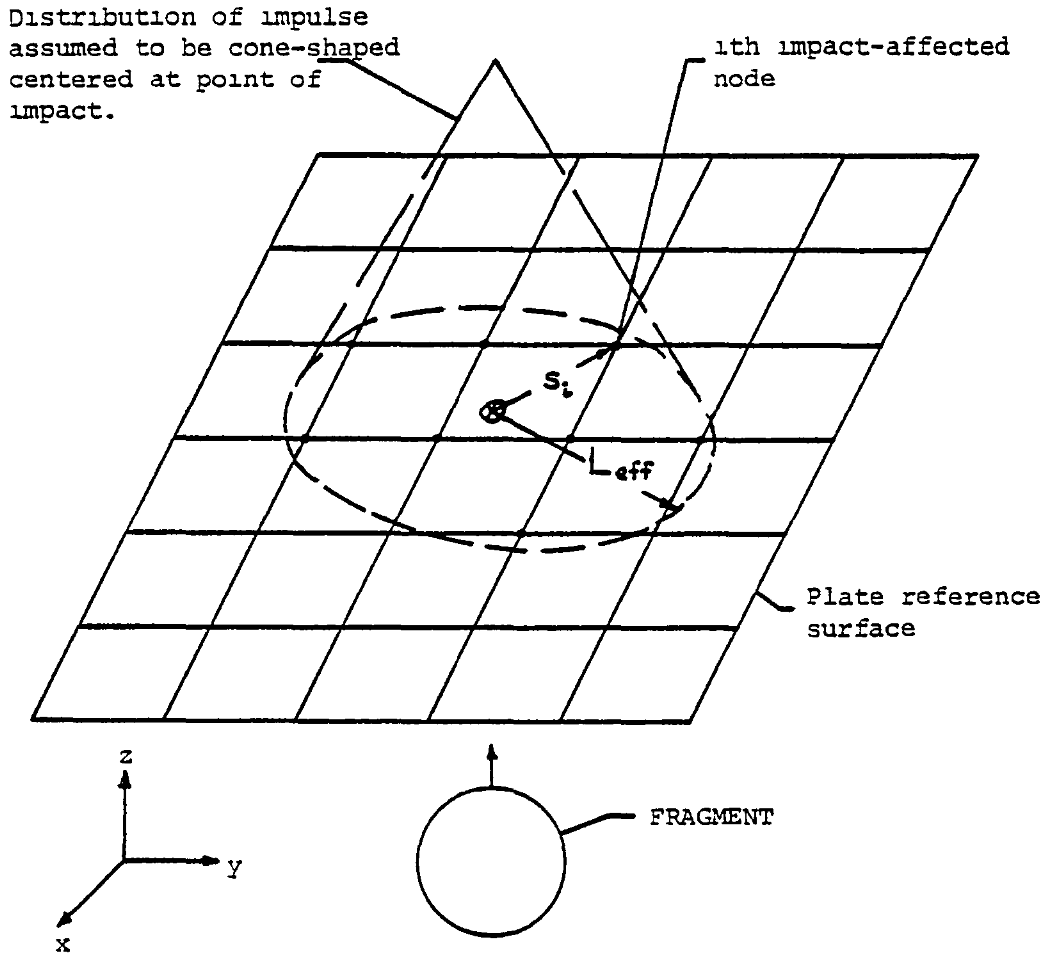
Step 10: The nodal velocity vector $\dot{\underset{\sim}{q}}^*_{m+1}$ (required for the displacement solution for the next time step) is then calculated. If no impacts have occurred during the time step, Eq. B.53 is used. If one (or more) impact has occurred during the current time step, Eq. B.54 is used. The solution for the current time step is now complete; the solution for the next time step (for $t_{m+2}$) is obtained by proceeding to step 1 (with the indices appropriately incremented).

Steps 1 through 10 may be carried out for as many time steps as desired or may be terminated by invoking (and implementing in CIVM-PLATE) the use of a termination criterion such as, for example, the reaching of a critical value of strain at some location on the plate.




NODES 1, 2, 3 ARE TRIANGULAR INSPECTION REGION NODES

FIG. B.1 CONVENTION ADOPTED FOR TRIANGULAR COLLISION INSPECTION REGIONS

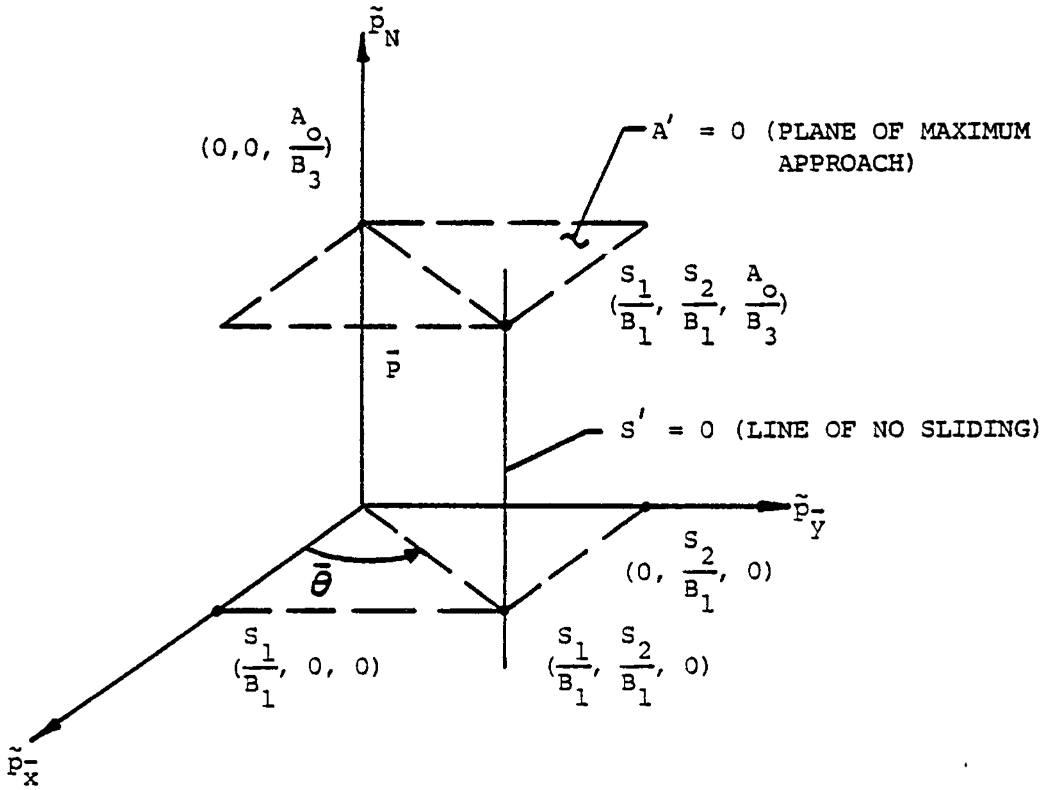



- Impact-affected nodes falling within circle of radius $L_{eff}$ centered at point of impact

⊗ Point of impact (maximum plate/fragment overlap).

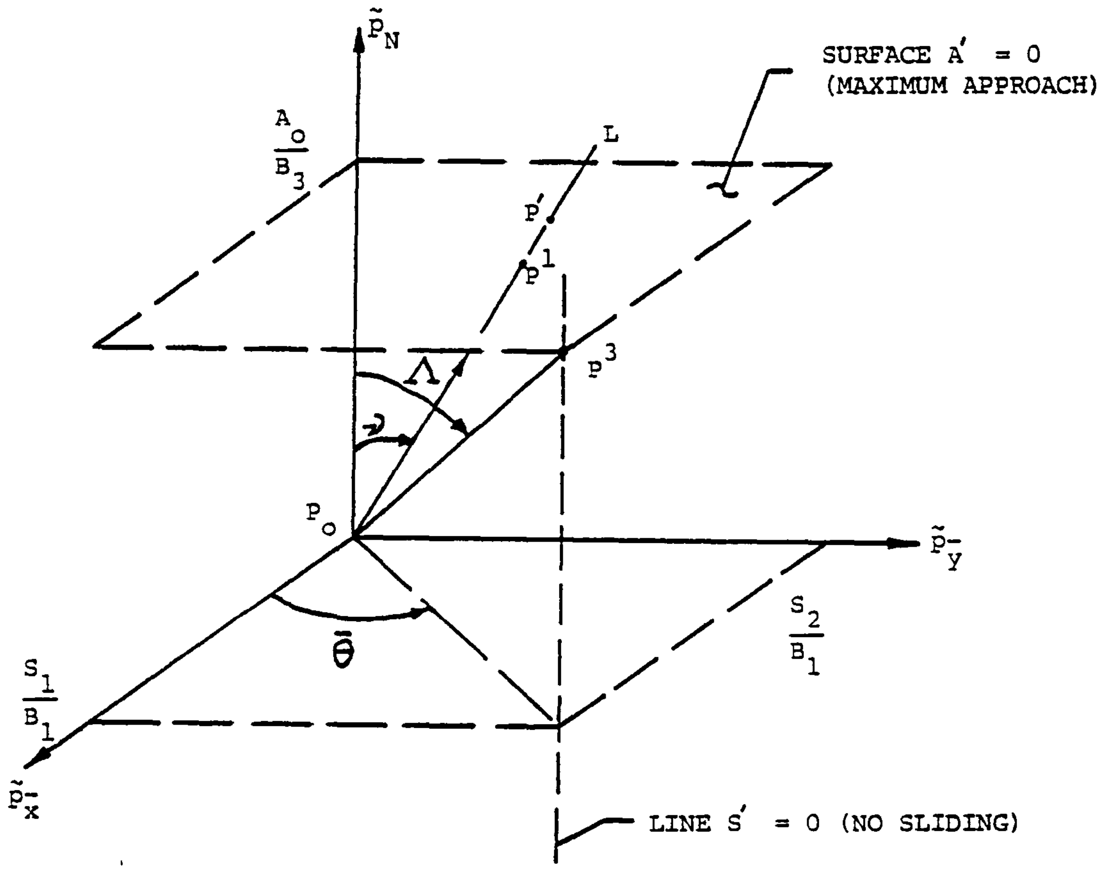
$s_i$ Distance from point of impact to ith impact-affected node.

FIG. B.2 IDENTIFICATION OF IMPACT-AFFECTED NODES AND ASSUMED IMPULSE DISTRIBUTION



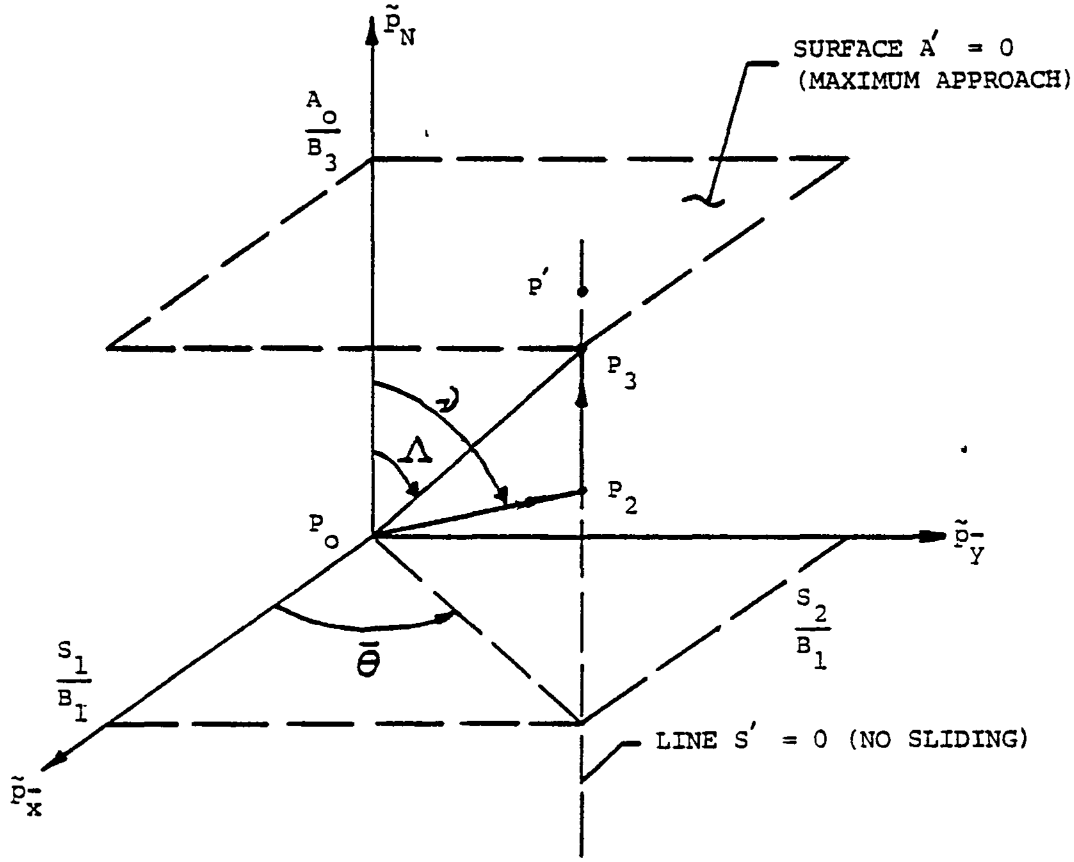

$\bar{P}$ = IMAGE POINT IN $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ COORDINATES

FIG. B.3 DESCRIPTION OF IMPULSE COORDINATE SPACE FOR IMPACT INTERACTION ANALYSIS



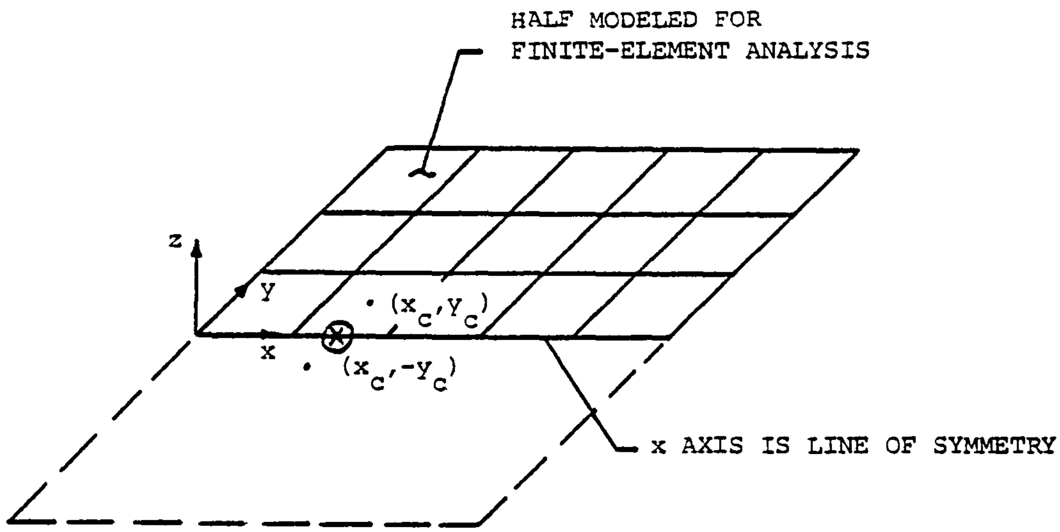

(a) For $\nu < \Lambda$

FIG. B.4 THE TRAJECTORY OF THE IMPAGE POINT $\bar{P}$ IN THE $\tilde{p}_{\bar{x}}$, $\tilde{p}_{\bar{y}}$, $\tilde{p}_N$ PLANE TO DESCRIBE THE IMPULSE STATE AT EACH CONTACT INSTANT




(b) For $\nu \geq \Lambda$

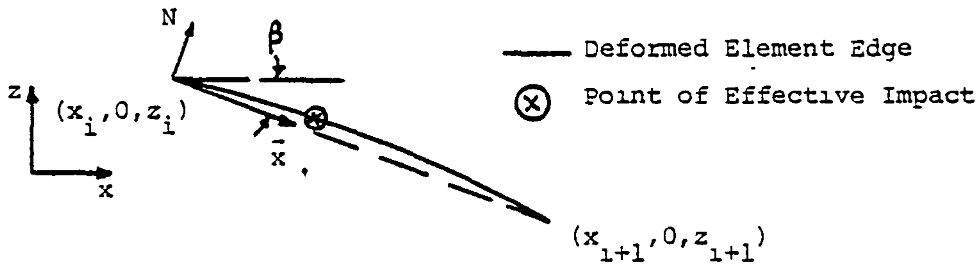
FIG. B.4 CONCLUDED




• POINTS OF SIMULTANEOUS PLATE/FRAGMENT IMPACT

⊗ POINT OF EFFECTIVE IMPACT ON LINE OF SYMMETRY

(a) Finite-Element Model and Location of Effective Point of Impact.




$$\begin{Bmatrix} \bar{x} \\ y \\ N \end{Bmatrix} = [T] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$[T] = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

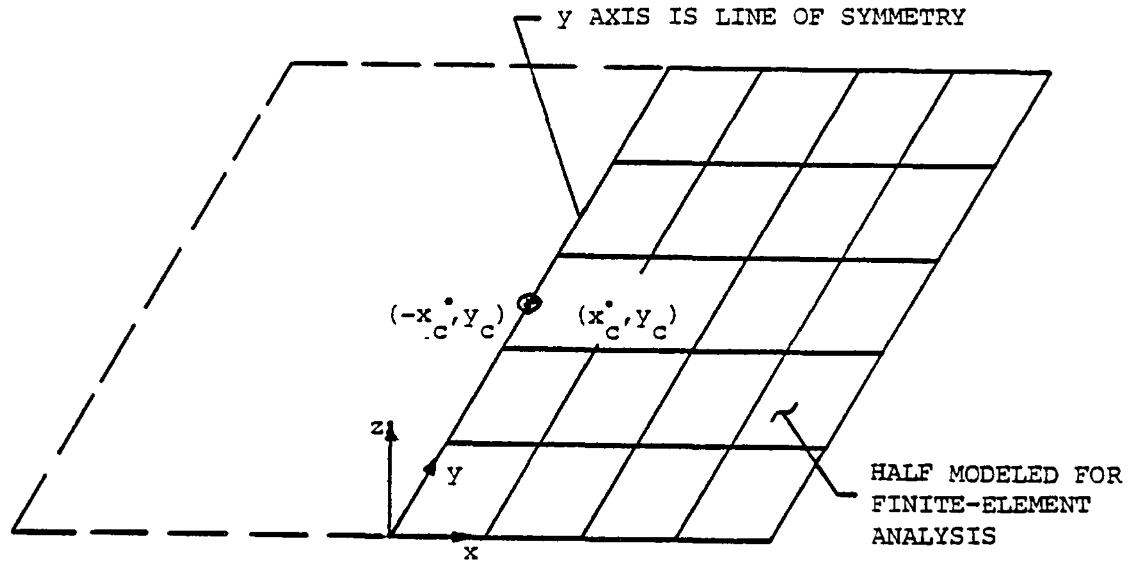
$$\sin\beta = \frac{z_{i+1} - z_i}{\ell}$$

$$\cos\beta = \frac{x_{i+1} - x_i}{\ell}$$

$$\ell = \left[ (x_{i+1} - x_i)^2 + (z_{i+1} - z_i)^2 \right]^{\frac{1}{2}}$$

(b) Coordinate Transformation.

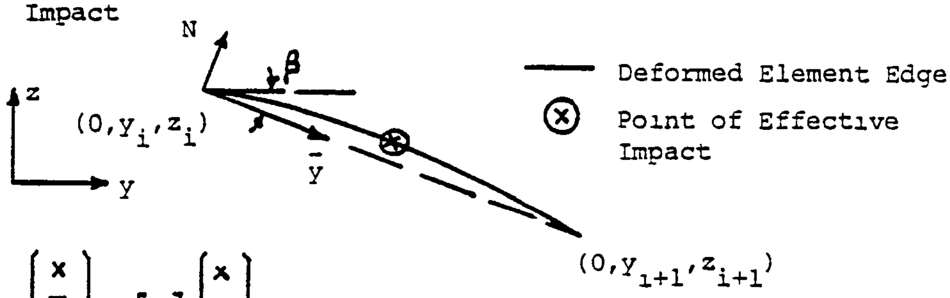
FIG. B.5 CONVENTIONS AND COORDINATE TRANSFORMATION WHEN THE x AXIS IS A LINE OF SYMMETRY




• POINTS OF SIMULTANEOUS PLATE/FRAGMENT IMPACT

⊗ POINT OF EFFECTIVE IMPACT ON LINE OF SYMMETRY

(a) Finite-Element Model and Location of Effective Point of Impact




$$\begin{Bmatrix} x \\ \bar{y} \\ N \end{Bmatrix} = [T] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix}$$

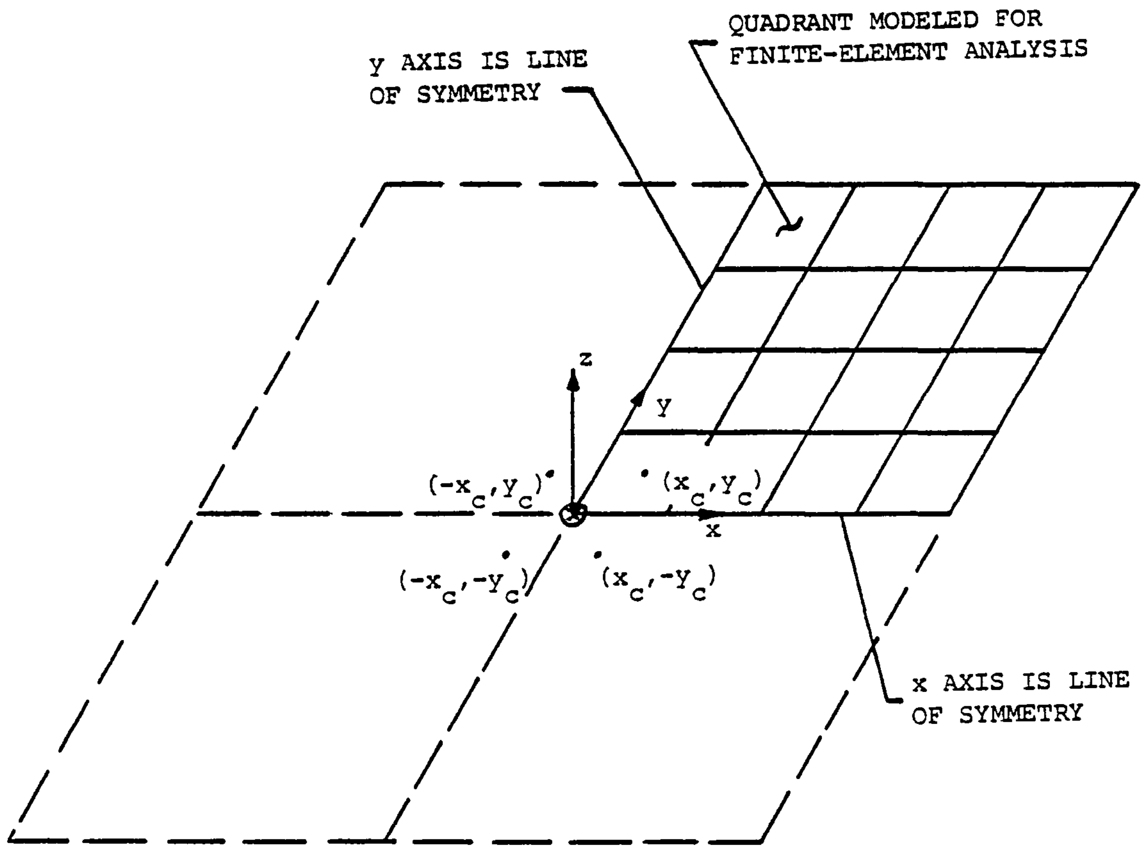
$$\sin\beta = \frac{z_{i+1} - z_i}{\ell}$$

$$\cos\beta = \frac{y_{i+1} - y_i}{\ell}$$

$$\ell = \left[(y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2\right]^{\frac{1}{2}}$$

(b) Coordinate Transformation

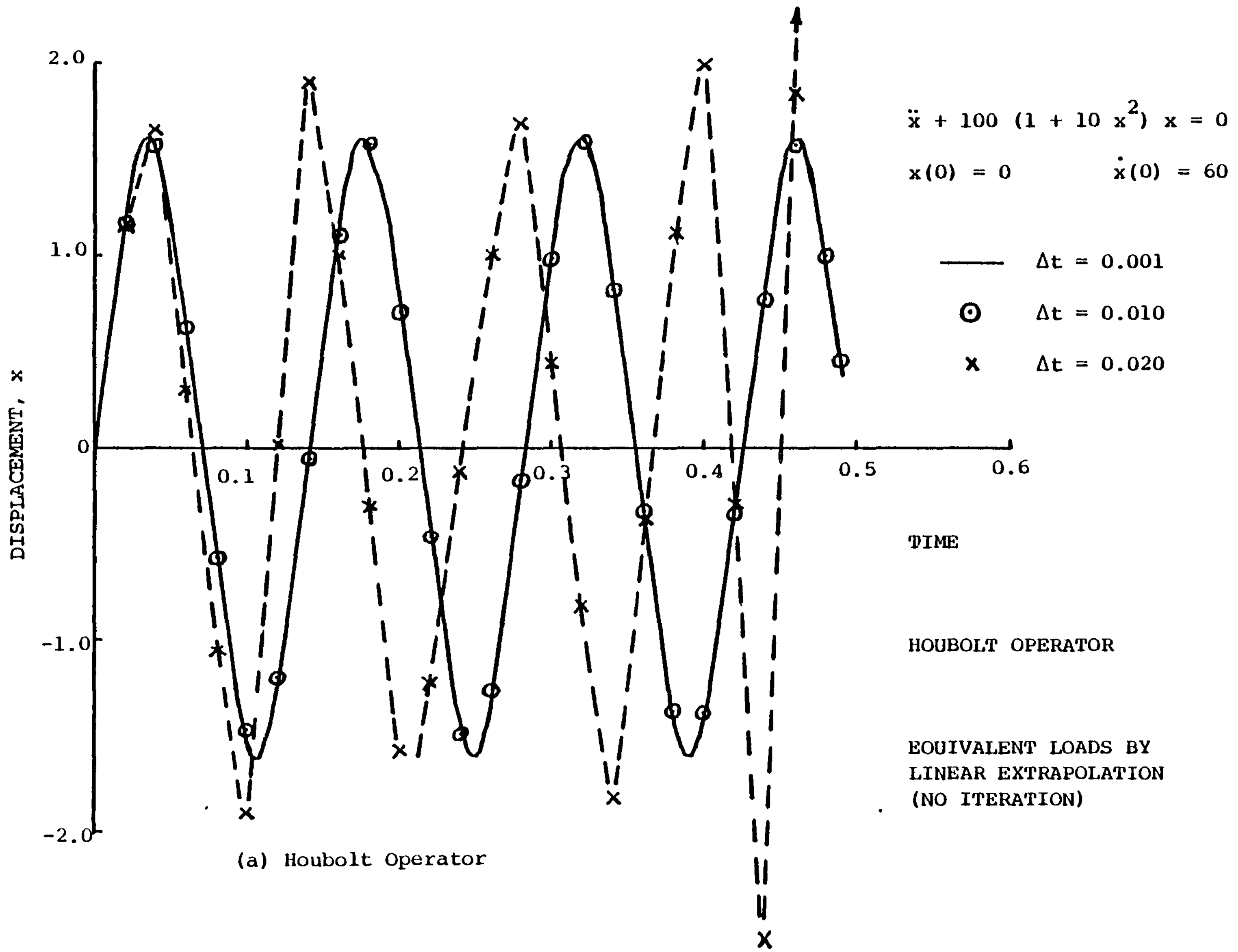
FIG. B.6 CONVENTIONS AND COORDINATE TRANSFORMATION WHEN THE y AXIS IS A LINE OF SYMMETRY




• POINTS OF SIMULTANEOUS PLATE/FRAGMENT IMPACT

⊗ POINT OF EFFECTIVE IMPACT

FIG. B.7 CONVENTIONS AND MODELING FOR DOUBLE SYMMETRY IMPACT CASES




(a) Houbolt Operator

FIG. B.8 PREDICTED RESPONSE OF A ONE DEGREE-OF-FREEDOM NONLINEAR SPRING-MASS SYSTEM SUBJECTED TO AN INITIAL VELOCITY

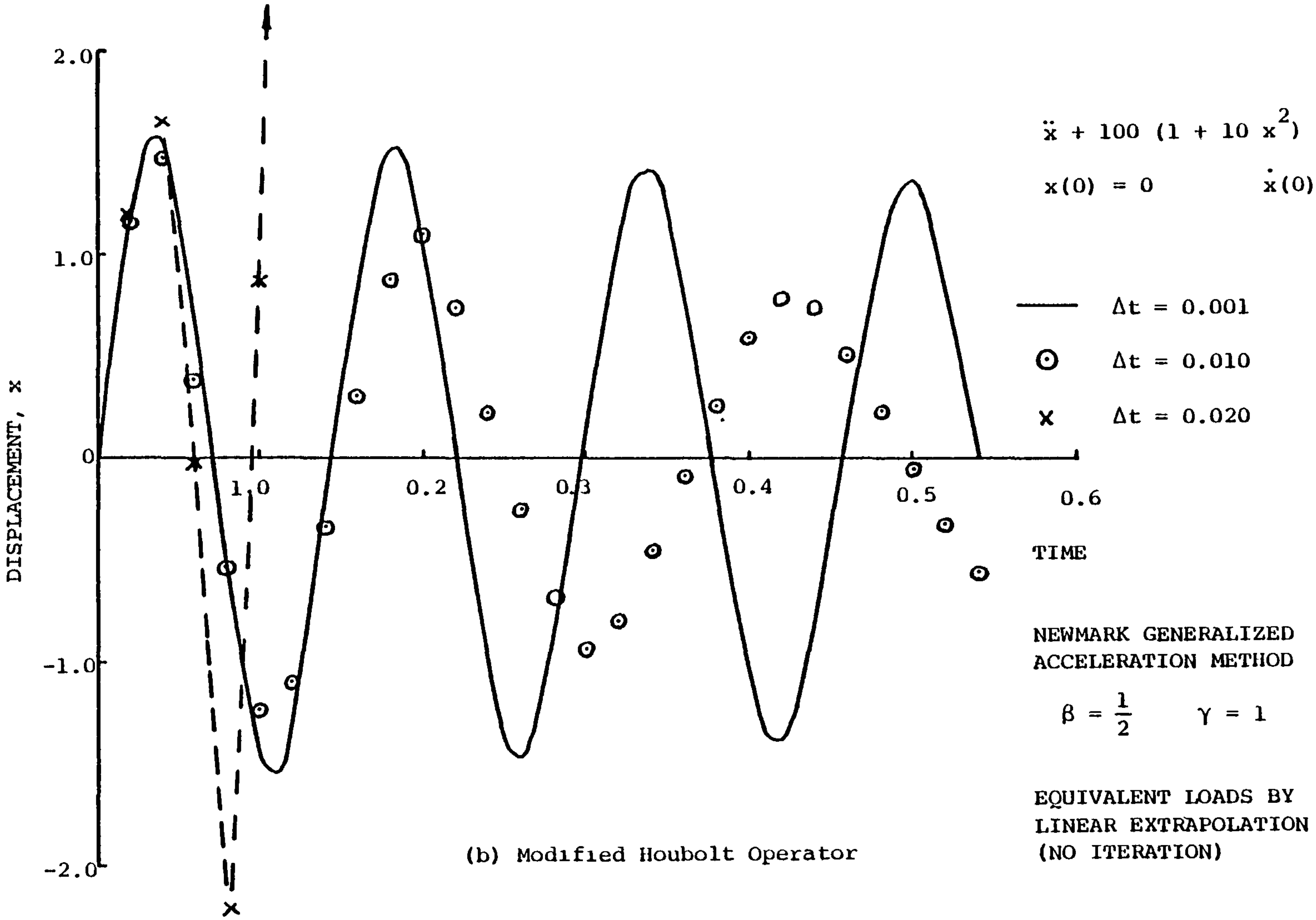



FIG. B.8 CONCLUDED

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